An Approach to Neuro-Fuzzy Feedback control in Statistical Process Control

Thesis Submitted to the University of Wales for the Degree of

Doctor of Philosophy

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February 2001

To my father Wang Mingting, my mother Li yuzhu
and all of the members of my family whose love and support kept me going.

Declaration

Signed .... (candidate)

Date ......1/2/2001......

STATEMENT 1

This thesis is the result of my own investigations except where otherwise stated.

Other sources are acknowledged by footnotes giving explicit references. A bibliography is appended.

Signed .... (candidate)

Date ......1/2/2001......

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Acknowledgements

I would like to express gratitude to my supervisor, Dr. Hefin Rowlands for his patient supervision, invaluable help, friendly advice and encouragement throughout the research. I am also grateful to Prof. Geoff Roberts who has helped me in many ways, especially to sincerely encourage my research work, sufficiently support my study and to improve my presentation skills.

I would also like to thank my colleagues in the Mechatronics Research Centre for their help. Thanks to the University of Wales College, Newport for providing me with a bursary to support this research.

Summary

It is a difficult challenge to develop a feedback control system for Statistical Process Control (SPC) because there is no effective method that can be used to calculate the accurate magnitude of feedback control actions in traditional SPC. Suitable feedback adjustments are generated from the experiences of process engineers. This drawback means that the SPC technique can not be directly applied in an automatic system.

This thesis is concerned with Fuzzy Sets and Fuzzy Logic applied to the uncertainty of relationships between the SPC (early stage) alarms and SPC implementation. Based on a number of experiments of the frequency distribution for shifts of abnormal process averages and human subjective decision, a Fuzzy-SPC control system is developed to generate the magnitude of feedback control actions using fuzzy inference. A simulation study which is written in C++ is designed to implement a Fuzzy-SPC controller with satisfactory results.

To further reduce the control errors, a NeuroFuzzy network is employed to build NN-Fuzzy-SPC system in MATLAB. The advantage of the leaning capability of Neural Networks is used to optimise the parameters of the Fuzzy-$\overline{X}$ and Fuzzy-R controllers in order to obtain the ideal consequent membership functions to adapt to the randomness of various processes. Simulation results show that the NN-Fuzzy-SPC control system has high control accuracy and stable repeatability.

To further improve the practicability of a NN-Fuzzy-SPC system, a combined forecaster with EWMA chart and digital filter is designed to reduce the NN-Fuzzy-SPC control delay. For the EWMA chart, the smoothing constant θ is investigated by a number of experiments and optimised in the forecast process. The Finite Impulse Response (FIR) lowpass filter is designed to smooth the input data (signal) fluctuations in order to reduce the forecast errors. An improved NN-Fuzzy-SPC control system which shows high control accuracy and short control delay can be applied in both automatic control and on-line quality control.

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Chapter 1 Introduction and overview of thesis

Statistical Process Control (SPC) is a useful technique for detecting assignable causes, which lead to “out-of-control” state from a production process. It is widely applied in the industrial and business fields to increase products or services quality and reduce costs (Caulcutt, 1996) (Zorriassatine and Tannock, 1998). However it can not provide the magnitude of adjustments or control action for feedback control. This thesis employs fuzzy logic, neural networks, filter and Exponentially Weighted Moving Average (EWMA) control chart forecasting techniques to develop an NN-Fuzzy-SPC system, which generates the magnitude of control action for automatic feedback control. The simulation results show that under the different process mean shift and variability spread levels, the control system gives high accurate and short delayed control action.

1.1 Introduction

Any company wishes to maintain their customers by their quality products and quality services. It is a key aim of management in every company. A method that uses statistical techniques to monitor and control the production processes for delivering quality products is called SPC. SPC can bring three main benefits: 1. It can reduce incorrect interpretation of management information. 2. It can help engineers to hold the mean on target or change the mean to a more profitable level. 3. By finding and eliminating the assignable causes, engineer can reduce the variability in the process; this will make more acceptable products or services to the customer. SPC can be viewed as an important technique or route to promote the national economy's GNP (Caulcutt, 1996).

SPC is a useful technique to find assignable causes in processes. Nevertheless, the assignable causes are eliminated manually by the engineer's experiences. That is, traditional SPC can not provide the magnitude of adjustments or control action. This weakness or problem means that traditional SPC can not be applied to automatic feedback control.

This thesis focuses on SPC feedback control. Based on sufficient experiments and analysis for SPC zone rules, this thesis employs fuzzy logic technique to build a Fuzzy-SPC system to generate the magnitudes of control actions for adjustment of abnormal pattern or processes. That is, when the assignable causes are detected by SPC zone rules, the feedback control actions, which indicate uncertain processes mean shifts and/or spreads levels, are generated from fuzzy inferences. Furthermore, for different shift and/or spread levels, a neural network is applied to build a NN-Fuzzy-SPC system to optimise the parameters of the fuzzy control system, in order to increase the control accuracy and control capability. A combined forecaster with EWMA chart and finite impulse response (FIR) lowpass filter is designed for the reduction of control delay. For every step of development, sufficient simulation experiments are carried out which show satisfactory results.

This chapter describes related research background and the main themes to the thesis. It involves control charts, pattern recognition (zone rules), feedback control and EWMA forecast. The aim of the research work and an overview of the thesis are also explained.

1.2 Related research background

Statistical process control has taken its place as a central activity in the industrial system, since it was first born as a special discipline in the 1920s (Grant and Leavenworth, 1988). Shewhart publishes his *Economic Control of Quality of Manufactured Product* – outlining statistical methods for use in production and control chart methods in 1931 (Shewhart, 1931). The American Society for Quality Control was formed in 1946. This ASQC organisation promotes the use of quality improvement techniques for all types of products and services, and offers a number of conferences and technical publications (Montgomery, 1997). In the 1950s, the Cumulative Sum (CUSUM) control chart and the Exponentially Weighted Moving Average (EWMA) control chart were introduced by E. S. Page (Page, 1954) and S. Roberts (Roberts, 1959) respectively. In 1956, the Western Electric Handbook (Western Electric, 1956) provided a set of different classes of abnormal patterns as additional action rules or zone rules for Shewart chart that can represent the occurrence of those special or assignable causes.

Since the 1980s, there has been a profound growth in the use of statistical methods for quality control and improvement in America, Europe and Japan. SPC is widely utilised both by the manufacturing and the service sectors. Numerous books and papers are published in the SPC field such as control charts, pattern recognition, and SPC feedback control.

The idea of fuzzy sets was born in 1960s. Professor L.A. Zadeh published the first seminal paper on fuzzy set in 1965 (Zadeh, 1965). Fuzzy sets can be used to describe vague concepts or linguistic variables. Based on fuzzy sets, fuzzy logic theory provides a formal framework to abstract the approximate reasoning characteristics of human decision-marking, and offers an applicable mode of knowledge representation. The source of neural networks can be traced back to the 1940s. Mclulloch and Pitts presented the first neural model (McCulloch and Pitts, 1943). Hebb proposed the first learning rule for adjusting the connecting weights (Hebb, 1949). Rosenblatt developed the first perceptron which is used in model classification (Rosenblatt, 1958). The Back-Propagation algorithm (BP) was proposed by Rumelhart and Mcclelland in 1986. This algorithm can be applied to mapping the non-linear relationships and indicates NN's further research and application future (Rumelhart and Mcclelland, 1986) (Sun, 1997). Neural networks can simulate biological learning capabilities to map the relationship between input and output for complex non-linear systems. Recently, combining the explicit knowledge representation of fuzzy logic with the learning power of neural networks, the NeuroFuzzy model, which is a more useful network or model, is engendered and applied in related application areas (Tanaka, 1995) (Juang and Lin, 1998).

In this research work, fuzzy logic is used to build a Fuzzy-SPC controller to generate the control action based on SPC zone rules. A neural network is used to optimise the parameters of this controller, in order to increase the control accuracy.

1.2.1 Control charts

Some reviews of various aspects of SPC, which focuses on control chart design and application are provided by Montgomery (1980), Vance (1983), Ho and Case (1994) and Woodall (1997). As fundamental tools, Shewhart control charts are used with variable data and attribute data to identify "out-of-control" conditions, which indicate the existence of non-random or special causes if a sample data goes beyond the control limits on control charts. Several typical recent approaches to control charts can be given below.

Delcastillo develops analytic relationships between the design variables of an $\overline{X}$ control chart and the production variables for a single-item stochastic demand production system. Practical applications include finding an $\overline{X}$ chart design to achieve certain inventory service level for a given production policy, or finding a production policy that maximises the service level given an $\overline{X}$ chart design (Delcastillo, 1995).

Mittag and Steimann examine the effect of stochastic measurement error (gauge imprecision) on the performance of Shewhart-type X-S control charts. It is shown that gauge imprecision may seriously affect the ability of the chart to detect process disturbances quickly or, depending on the point in time when the error occurs, the probability of erroneously signalling an out-of-control process state (Mittage and Steimann, 1998).

Chang and Gan proposes a Shewhart $\overline{X}$ chart with modified limits to adapt the integrated circuit (IC) process. It is used to signal the need for adjustment of controllable process variables for improving the process capability (Chang and Gan, 1999).

Costa presents a modification of the $\overline{X}$ chart that allows one to take supplementary samples. The supplementary sample is taken when the current value of $\overline{X}$ falls outside the control limits in order to detect the assignable causes faster (Costa, 2000). Similarly, some adaptive methods through changing sample interval and sample size are proposed by (Runger and Montgomery 1993), (Prabhu et al 1993) and (Zimmer et al 2000).

An approach to p-charts using intelligent methods is proposed by Wang and Raz (Wang and Raz, 1998). The control charts are constructed using linguistic data suitable for situations where quality characteristics can not be measured numerically. The results show that control chart based on linguistic data are significantly more sensitive to process shifts than are conventional p charts. Kim and Byun proposed a fuzzy $\overline{X}$ control chart for the research and development (R&D) productivity evaluation. It enables the manager to efficiently conduct a practical and rational comparison of the R&D personnel's irregular and dissimilar outputs, and a researcher can accept this logical and detailed analysis of his performance with faith (Kim and Byun, 1995).

For the general research of feedback control in this thesis, $\overline{X}$ and R charts are the detectors applied, because they are effective tools, frequently and simply used in wide areas. The zone rules, which are the research focus in this work, are approached based on the traditional $\overline{X}$ and R charts. However, the design ideas which are used in the fuzzy system (chapter 3 and 4) are similar (using fuzzy sets) but different (using fuzzy inference to generate the magnitude of feedback control action) to the related approaches of Wang and Raz (1998) and Kim and Byun (1995).

1.2.2 Pattern recognition

When certain non-random patterns (such as a gradual trend) exist in a process, one or more special causes may be present with no breach of control charts limits. The Western Electric Handbook (Western Electric, 1956) provides a set of different classes of abnormal patterns that can represent the occurrence of these special causes. These classes are also called "Additional Action Rules" (Box and Luceno, 1997), "Zone Rules" (Devor et al, 1992) or "Pattern Recognition" (Montgomery, 1997). In the past few years, many papers have been published on SPC pattern recognition using expert system and neural networks (NN). These efforts are viewed as useful steps towards automatic on-line SPC for continuous improvement of quality and for real-time manufacturing process control (Zorriassatine and Tannock, 1998).

Cheng and Hubele provided useful insight into applications of expert systems to SPC charts and included a brief review and listing of relevant papers (Cheng and Hubele, 1992). Kuo and Mital provide a review of the existing quality control expert systems and recommend a set of quality engineering techniques for developing a knowledge base for an SPC expert system (Kuo and Mital, 1993). An application of expert system in out-of-control pattern recognition has been carried out by Swift and Mize. Once the pattern is identified, the expert system supplies the user with possible causes for the out-of-control condition (Swift and Mize, 1995).

Hwarng and Hubele used the Back – Propagation pattern recognisers to identify unnatural patterns such as cycles and trends exhibited on control charts. This valuable information could be provided for real time process control (Hwarng and Hubele, 1993). Hwarng and Chong used zone rules to train and test NNs to identify non-random situations (Hwarng and Chong, 1995). Pham and Oztemel presented a scheme for using neural networks for discriminating between different types of control chart patterns. A procedure to increase the classification accuracy and decrease the learning time for networks is described (Pham and Oztemel, 1994). Smith formulated the X-bar and R control charts for diagnosis and interpretation by neural networks. The neural networks are trained to discriminate between samples from probability distributions considered within control limits (Smith, 1994). Chang and Aw proposed a neural fuzzy control chart for identifying process mean shifts: the neural network's outputs are classified into various decision regions using a fuzzy set scheme. This system has the ability to identify the magnitude of mean shift (Chang and Aw, 1996).

SPC zone rules have statistical characteristics (Rowlands, 1992). SPC zone rules can provide reasonable indications for assignable causes early, but they are incapable of adjustments or feedback control actions, or at most, they can only give some uncertainty implications. In this research work, based on these uncertainty implications and sufficient experiments, fuzzy logic is employed to generate the feedback control action. That is, some basic SPC zone rules are used as the antecedent conditions in a fuzzy inference system. Through some experiments in section 2.7 and theoretical analysis in section 2.5.2 of chapter 2, it is not advisable to use too many zone rules in a feedback control system if a small type I error probability is expected. As for those applications of NN in pattern recognition, they are significant approaches to enable the use of SPC utility in an automatic system. However their weaknesses are the need for plenty of training data and the difficulty of outcome explanation. In a different manner to those researches of classification of pattern using neural networks, this research work uses neural networks to optimise the fuzzy controller parameters in order to increase control accuracy.

1.2.3 SPC feedback control

Traditional SPC methods such as control charts can give an early warning of a shift in a process mean. SPC zone rule identification can be interpreted as an alarm based testing method (Guh et al. 1999a) (Spanos, 1991). However traditional SPC does not directly define what control actions have to be taken. Definitions of the actions to be taken is normally undertaken by the process and technical staff who understand the processes (Coulson and Cousans, 1987). In the last decade, much research has been undertaken on SPC feedback control and the combination of SPC and engineering process control (EPC) (or automatic process control (APC)) fields. In the first approach, the control action can be postponed, but the magnitude of the adjustment is a critical component for success (Ruhhal and Runger, 2000). In the second approach, SPC is used to monitor the process to detect assignable causes and EPC is used to reduce the effect of predictable quality variation (Janakiram, 1998).

Box presented some formulas for feedback control by manual adjustment (Box, 1991-1992). He pointed out that if this adjustment is repeatedly applied at every step, a series of adjustment equations are obtained and it can be summarised as a proportional integral (PI) control (Box and Luceno, 1997). (Montgomery et al, 1994) and (Janakiram, 1998) proposed integrating EPC and SPC. EPC can be used to minimise deviation from target due to disturbances that occur continuously and are part of the process itself, and SPC applied to the output deviation from target can be used to identify and subsequently eliminate assignable causes. The results demonstrate that the addition of an SPC chart to monitor output deviation from target in a system with active control is a highly effective way to integrate theses two strategies. They conclude that proper use of both SPC and EPC can always outperform the use of either alone.

In the first approach, because the adjustments or control actions are given by the differences between every sample and target values, it will cause frequent adjustments. In the second approach, SPC alarm indicates and eliminates the assignable causes. However, what reasonable magnitude of control action should be taken is a challenge for SPC feedback control. In this thesis, the magnitude of control actions are generated from fuzzy inference, which is designed based on SPC zone rules and substantial frequency distribution experiments. It has the advantages of both quick response and robustness.

1.2.4 EWMA forecast

Roberts introduced Exponentially Weighted Moving Averages (EWMA) for constructing control charts for the mean of a process in 1959. The EWMA chart can be viewed as an addition to the Shewhart charts. EWMA chart can be used for the small shifts and the one-step-ahead forecast in the process (Montgomery, 1997) (Box and Luceno, 1997). In the last decade, the EWMA chart is frequently used in process control applications (Luceno, 1995), the EWMA forecasts are also central to many commercial systems (Johnston and Boylan, 1996).

An application of EWMA technique is developed to the principal components analysis (PCA) and projections to latent structures (PLS) for modelling processes with memory and drift in chemical processes by Wold (1994). He discussed the Exponentially Weighted Moving (EWM) -PCA and EWM-PLS models and the predictive control schemes. Box and Luceno pointed out in the engineering process control field, the mean level of the quality characteristic normally can be assumed to drift over time. If the process disturbance is represented by the integrated moving average (IMA) model, the EWMA of the past data has optimal properties as a forecast of the next observation (Box and Luceno, 1997). Unfortunately, it is sometimes difficult to estimate the smoothing constants needed to update the EWMA of past data (Luceno, 1995). (Johnston, 1993) described the relationship to compute the necessary adjustment to the smoothing constant with simulation results. It will enable the correct weight to be applied to data collected for only part of a normal forecast review interval, and thus prevent over-reaction to very short-term events. An approach to EWMA in business areas confirms that the EWMA forecasting does not remove uncertainty in the business system but sets out to measure and minimise it (Johnston and Boylan, 1994).

It is appropriate to apply the EWMA forecast method in this research work as it has a smoothing function and can minimise the uncertainty. The ideal smoothing constant or parameter is obtained from the results of experiments under various conditions and the optimisation processes in this research work.

1.3 Aim and objectives of project

The aim of this research work is to develop a NN-Fuzzy-SPC intelligent control system. Fuzzy logic is investigated to generate control actions based on SPC zone rules and experimental analysis. A neural network is considered to optimise the fuzzy membership functions. The design of a combined forecaster to reduce the control delay will be investigated. The system designed is required to generate feedback control action to adjust the abnormal patterns or processes with quick response, robustness and high control accuracy.

The project aims to make the following contributions:

- To investigate and analyse the behaviour of five basic SPC zone rules to obtain the frequency distributions of process average shift levels (FDPASL).
- To design a Fuzzy-SPC inference system in which five basic SPC zone rules are applied to design the antecedent parts and the related outcomes of experimental analysis of FDPASL and subjective decisions are used to design the consequent parts.
- Design a Visual C++ program to simulate the Fuzzy-SPC system. Analyse the simulation results for different structures of consequent membership functions.
- To employ neural network techniques to build a NN-Fuzzy-SPC system. The consequent membership functions in the NN-Fuzzy-SPC can be automatically optimised by the neural network, in order to obtain high control accuracy.

- Apply EWMA forecast and design a finite impulse response (FIR) lowpass filter to build a combined forecaster. This new forecaster is used in the NN-Fuzzy-SPC system to reduce the control delay.

1.4 Overview of thesis

- Chapter 2 gives an overview of SPC. Further details of the research background on control charts, zone rules and feedback control sections are described. Sufficient experiments and analysis for the frequency distribution of process mean shift levels are explained.

- Chapter 3 introduces some notions and algorithms of fuzzy systems. It involves fuzzy sets, membership function, fuzzy logic operators, fuzzy approximate reasoning and fuzzy logic control. Related research background and the implication of the investigation to the main theme of the research work are described.

- Chapter 4 exhibits the design of a Fuzzy-SPC system. Based on the outcomes of the analysis discussed in chapter 2 and related notions and algorithms described in chapter 3, a specific fuzzy system for SPC is designed by manual calculation and CAD (MATLAB).

- Chapter 5 describes the C++ simulation study, which is used to assess the Fuzzy-SPC system. As the foundation for real system design, the windows operating environment is designed using Microsoft Foundation Classes (MFC) and Application Wizard (AppWizard). As a primary approach, different shapes of membership functions are used to perform the Fuzzy-SPC system. Their related effects are analysed by statistics.

- Chapter 6 gives the introduction to neural networks and neurofuzzy networks, design and performance of the NN-Fuzzy-SPC system. Under different process mean shifts and process variability spread levels, the NN-Fuzzy-SPC system feedback control these abnormal processes with high accuracy.

- Chapter 7 discusses EWMA forecast behaviour based on experiments for different smoothing constants. A FIR lowpass filter is designed to improve EWMA forecast. The simulation results show that this combined forecaster is successfully applied to the NN-Fuzzy-SPC system to reduce control delay.

- Chapter 8 completes the thesis by explaining the research conclusions, providing the contribution details, discussing the implementation and feasible application of the NN-Fuzzy-SPC system and describing the direction of future work.

Chapter 2 Overview and approach to statistical process control (SPC) patterns

Traditional statistical process control (SPC) can not provide the magnitude of adjustment or control action for feedback control. Based on some basic notions of SPC, related research background and sufficient experiments, this section describes significant discussion and analysis for accurate and robust SPC feedback control.

2.1 Introduction

Quality Control plays an important part in most industrial systems. Its role in providing relevant and timely data to management for decision marking purposes is vital. A method that uses statistical techniques to monitor and control product quality is called Statistical Process Control (SPC) where control charts are test tools frequently used for monitoring the manufacturing process. Engineers or managers can evaluate abnormal process by using SPC Zone Rules in control charts.

As mentioned in chapter 1, the focus of this research work is to develop an feedback on-line controller based on SPC basic zone rules or alarms. Related control charts which are frequently used in SPC are discussed. This chapter provides the foundation for this research work. A brief overview of related statistic notions and methods, which are used in the current chapter and subsequent chapters, along with basic SPC concepts are introduced (section 2.2 and 2.3). Several commonly used control charts and the pattern recognition tool (zone rules) are presented and discussed in section 2.4 and 2.5. Section 2.6 discusses SPC feedback control, where typical methods and applications are presented. Based on a large number of experiments, section 2.7 describes an initial approach for zone rule quantitative behaviour, magnitude comparison and their clustering analysis results.

2.2 Basic statistical methods

SPC is a method based on statistics. To manage or control a product or process, sample data must be drawn from the product or process and analysed using statistical methods. Frequently, the data is assumed to be or is represented as random samples, and the objective of statistical inference is to draw conclusions about a population based on a sample selected from the population (Hwarng and Chong, 1995). Some basic functions are defined for computing the characterisation of the data. For example, the sample mean $\bar{x}$ can be calculated as shown in equation 2.1.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{2.1}$$

and the estimate of the population standard deviation is described in equation 2.2.

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \tag{2.2}$$

where x_i is the sample data and n is the sample size.

The $\overline{x}$ and S statistics describe the central tendency and variability of the sample. A random experiment is a process that yields any one of several possible outcomes in a trial (DeVor, et al, 1992). The sampling distribution can thus be used to characterise this process.

Three distributions commonly used in the statistical analysis are the Normal distribution, Binomial distribution and Poison distribution.

2.2.1 Normal distribution

The most frequently encountered distribution is the normal distribution. It is used so often because many physical measurements of continuous variables provide frequency distributions that closely approximate a normal distribution with larger observations (Central Limit Theorem) (Montgomery and Runger, 1994) (Grant and Leavenworth, 1988b). In statistical process control, $\overline{X}$ and R control charts, which are described in section 2.4.1, are established based on the normal distribution. This distribution has been widely applied to industrial data and can be described by a frequency curve and probability density function $f_X(x)$ (equation 2.3).

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \; -\infty < x < \infty \tag{2.3}$$

where: x is the continuous random variable,

μ and σ are the population mean and the population standard deviation.

As an interpretation of σ, the normal curve indicates that, 68.26% of the population values fall between the limits defined by $\mu \pm 1\sigma$, 95.46% of the values fall between the $\mu \pm 2\sigma$ limits and 99.73% of the values fall within the $\mu \pm 3\sigma$. That is, the population standard deviation measures the distance from the mean. These standard deviation measures are applied in SPC to generate the control limits and several zones for testing the data's random behaviour on the control chart (discussed further in sections 2.4.1 and 2.5.1).

2.2.2 Binomial distribution

The binomial distribution has a wide application in engineering and statistics. It is used for discrete random variables and provides a basis for investigating two complementary categories of major interest. If a process consists of a sequence of n independent trials, the outcome of each trial is either a "success" or a "failure". The typical applications in SPC or quality control are the p-chart and np-chart, which are described in section 2.4.4. Because X is a discrete random variable, $f_X(x)$ is called the probability mass function and is defined by equation 2.4.

$$f_X(x) = \binom{n}{x} p^x (1-p)^{n-x} \tag{2.4}$$

where p is the probability of "success", x is the number of "success" and n is number of experiment. The mean and variance of the binomial distribution are:

$$\mu = np \tag{2.5}$$

$$\sigma^2 = np(1-p) \tag{2.6}$$

2.2.3 *Poison distribution*

The Poisson distribution is applied to any random phenomenon that occurs on a per unit (or per unit area, per unit volume, per unit time, etc.) basis (Kanagawa, et al, 1993). The c-chart and u-chart are applications of the Poison model in SPC. The probability mass function is defined as:

$$f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \ x = 0,1,... \tag{2.7}$$

where $\lambda > 0$.

There are also some other distributions that can be used in quality control. For example, the Hypergeometric distribution is used to design acceptance-sampling procedures. The Exponential distribution is used in reliability engineering as a model of the time to failure of a component or system. In the Weibull distribution which is a very flexible model, the different shapes can be made by adjusting its parameters θ and β. When $\beta=1$, the Weibull distribution reduces to the exponential distribution with mean $1/\theta$. Similarly, the Gamma distribution can assume many different shapes, depending on the values chosen for its parameters γ and λ. If $\gamma=1$, the gamma distribution reduces to the exponential distribution with parameter λ (Montgomery, 1997).

As discussed above, the normal distribution is the most frequently found distribution in industrial process. If the component errors are independent, but equally likely to be positive or negative, then the total error can be shown to have a normal distribution (Montgomery and Runger, 1994). For example, temperature and humidity drifts, vibration, cutting tool wear, rotational speed variations, variations in numerous raw material characteristics and variation in level of contamination, etc. Even though the distribution in the universe or population may not be normal, based on the central limit theorem (Daly et al, 1995), the distribution of the $\overline{X}$ values of samples tends to be close to normal when a large number of samples are taken. The larger the sample size and the closer to a normal the population, the closer will the frequency distribution of averages approach the normal curve (Montgomery, 1997). In this study, as a general approach and related to common problems of industrial processes, the research work is focused on the use of zone rules, which are implemented on $\overline{X}$ and R control charts. Their use is based on the normal curve or distribution. Therefore, many experiments are completed and analysed based on the normal distribution.

2.3 Statistical process control

Under normal conditions, a manufacturing process should produce a product, which is stable and repeatable. The process must be capable of operating with little variability around the target of the product's quality characteristics. Statistical process Control (SPC) is a method that uses statistical techniques to measure, interpret and ultimately control product quality. It uses statistical tools to determine whether to change a process or leave it alone. Improving quality not only decreases cost but also produces more consistent products which will in turn lead to greater customer satisfaction. SPC is directed toward the identification and ultimate removal of the underlying causes of the problem. Hence the central focus of the SPC approach is that both quality and productivity will be enhanced (Montgomery, 1997).

To manage any process and reduce variation, the variation must be traced back to its source – common causes or special causes. Common causes refer to the many sources of chance variation that are always present to varying degrees in different processes. The output of a process that contains only common causes of variation form a pattern that is stable over time and predictable, therefore, it provides the basis for subsequent process improvement. Special causes refer to any assignable factors, which are often irregular, unstable and unpredictable. If special causes are present and affect the process, the process mean or variability will be shifted and will need a corrective control or adjustment action. As to how many control actions (or what magnitudes of adjustments) should be taken adjust this abnormal shift is the basis of the simulation approach in this research work.

The major objective of statistical process control is to observe the occurrence of assignable causes of process shifts in order to take corrective action before many non-conforming units are manufactured. SPC can be applied to any process. It has associated with it seven major tools which are called "the magnificent seven" (Hwarng and Chong, 1995): Histogram, Check sheet, Pareto chart, Cause and effect diagram, Defect concentration diagram, Scatter diagram and Control charts. Of these tools, the control chart, which was developed in the 1920s by Dr. Walter A. Shewhart of the Bell Telephone Laboratories, is the most technically sophisticated and more frequently used. The other charts play an important role in supporting the data collecting and analysis process.

The Shewhart control chart was developed as an on-line problem identification and problem-solving tool and can be summarised by the model of a classical feedback control process (Devor, et al, 1992). This classical feedback control loop consists of observation, evaluation, diagnosis, decision and implementation (Fig. 2.1).

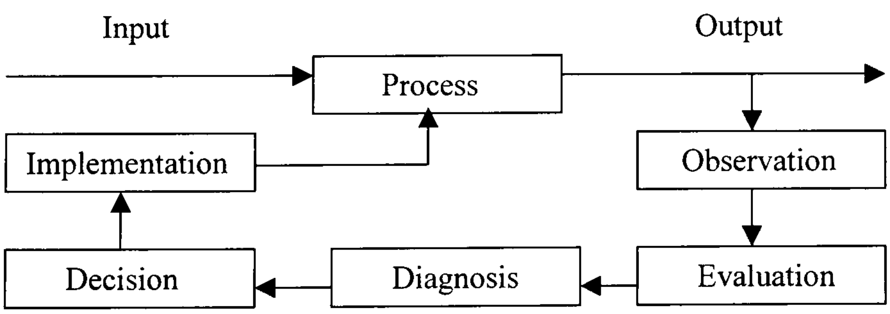

Figure 2.1 Classical Control system of SPC

In the observation phase, the physical system is observed by statistical sampling. In the evaluation phase, the control chart acts as a statistical model that describes how the data should behave if the process is stable and in - control. An additional tool, which is called zone rules, is used for identifying the data behaviour or SPC patterns based on the control chart data. During the diagnosis phase, the cause of the problem based on an analysis of the control chart is determined. The decision phase is used to formulate the appropriate action to eliminate the cause of the disturbance and finally the implementation phase is used to define the specific means to make the correction.

In traditional SPC, many phases are carried out manually. For example, the establishment of control charts and the selection and use of diagnosis tools. Even though some SPC software have been developed recently (Automation Products Ltd., 1995) (Digital Computations Inc., 1997), they can only generate control charts automatically from databases or data files. However, in the decision making phase (Fig. 2.1), which can be viewed as a key phase to achieve automation in SPC, the appropriate control actions are still derived from the process engineer's experience in existing SPC systems. It causes the manual achievement of implementation.

This research work aims to develop an intelligent automatic SPC feedback control method. All manual operations from the observation phase to the implementation phase in SPC can be improved by automating the operations through the use of a computer system. Control charts, which are basic SPC tools, are introduced in the next section. Zone rules are discussed in section 2.5.

2.4 Control charts

A control chart is a production process monitoring technique. It is widely used in many areas for detecting the occurrence of assignable causes or process shifts, and helps managers or engineers to identify the process variability. Normally the control chart contains a centre line (CL) which describes the average value of the product quality characteristics, upper control limit (UCL) and lower control limit (LCL) which indicates the range of the quality variability. These three parameters are determined in the "in – control" state. When a point is plotted outside the control limits, or even if all the points fall inside the control limits but they behave in a systematic or non-random manner, then this is an indication that the process is "out-of-control".

Control charts can be classified as two general types: variable control charts and attributes control charts. The variable control charts are used if the quality characteristic can be measured as a number on a continuous scale of measurement. In this type, the $\overline{X}$ chart is the most widely used for monitoring process central tendency, and an R chart or an S chart used to measure the process variability. The Exponential Weighted Moving Averages (EWMA) chart and the Cumulative summation (CUSUM) chart, can be applied in testing for abnormal processes with small trends (Montgomery, 1997). Attributes control charts are used for quality characteristics that can not be measured on a continuous scale. There are two types of charts in this case: fraction or number of defective (non-conforming) charts and number of defects (non-conformities) charts (Devor et al, 1992). The *p*-chart and *np*-chart belong to the first type: each unit (or item) of product is judged as either conforming or non-conforming (defective) corresponding to whether or not it possesses certain attributes, which gives rise to the *Binomial* distribution. The second type is used to describe the number of defects, errors or faults in a product. *c*-chart and *u*-chart belong to this type which is described by the *Poisson* distribution (Oakland, 1996).

$\overline{X}$ charts, *R* charts and EWMA charts, which are important tools in this research work, are introduced in sections 2.4.1 and 2.4.3 respectively. $\overline{X}$ and *S* charts, CUSUM charts and attribute control charts are briefly mentioned in sections 2.4.2 and 2.4.4. Finally in section 2.4.5, the related research background and research work are described.

2.4.1 X-bar ($\overline{X}$) chart and R chart

Many quality characteristics can be expressed in term of a numerical measurement for data analysis. If the measured quantity is expressed as a number on some continuous scale, it is called a variable. A control chart is used to represent the sample variable or quality characteristics. In statistical language, the Shewhart control chart is a test of an hypothesis (Devor et al, 1992). The sample data is judged to determine whether or not it indicates the presence of a special cause of disturbance.

When dealing with a quality characteristic that is a variable, it is usually necessary to monitor both the mean value of the quality characteristic and its variability. Control of the process average or mean quality level is usually performed with the $\overline{X}$ chart. Process variability can be monitored with the R chart or S chart. The R chart estimates the process standard deviation indirectly by a simpler calculation and is widely used when the sample group size is constant and less than 10 (Hwarng and Chong, 1995). The S chart estimates the process standard deviation directly and it is used for larger or variable sample sizes (Montgomery, 1997). For this research work, as a basic approach, the sample group size is kept constant and takes the value of 5 which is suggested by Grant and Leavenworth (1988b). Therefore, $\overline{X}$ chart and R chart are applied as the testing tools in this research work.

For the $\overline{X}$ chart, the CL represents the process (population) average $\overline{\overline{X}}$ (i.e. average sample mean), the UCL and LCL indicate the range of sample average variability. For the R chart, the CL represents the average range, the UCL and LCL indicate the range of sample range variability. Suppose that a quality characteristic is distributed normally with the mean μ and standard deviation σ, n is sample subgroup size and m is number of sample subgroups, and the sample value is described by x_{ij}. Related parameters can be calculated as:

1. Average of each subgroup $\overline{X}_i$ given by:

$$\overline{X}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n} \tag{2.8}$$

2. Range of each subgroup R_i is given by:

$$R_i = x_{i\max} - x_{i\min} \tag{2.9}$$

3. Process average $\overline{\overline{X}}$ is given by:

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{m} \overline{X}_i}{m} \tag{2.10}$$

4. Average of range $\overline{R}$ is given by:

$$\overline{R} = \frac{\sum_{i=1}^{m} R_i}{m} \tag{2.11}$$

5. For $\overline{X}$ chart, the parameters UCL,LCL and CL are calculated as follows:

$$UCL = \overline{\overline{X}} + 3\sigma_{\bar{x}} = \overline{\overline{X}} + \frac{3}{d_2\sqrt{n}}\overline{R} = \overline{\overline{X}} + A_2\overline{R} \tag{2.12}$$

$$LCL = \overline{\overline{X}} - 3\sigma_{\bar{x}} = \overline{\overline{X}} - \frac{3}{d_2\sqrt{n}}\overline{R} = \overline{\overline{X}} - A_2\overline{R} \tag{2.13}$$

$$CL = \overline{\overline{X}} \tag{2.14}$$

6. The parameters UCL, LCL and CL for R chart are calculated as follows:

$$UCL = \overline{R} + 3\sigma_R = \overline{R} + 3\frac{d_3}{d_2}\overline{R} = D_4\overline{R} \tag{2.15}$$

$$LCL = \overline{R} - 3\sigma_R = \overline{R} - 3\frac{d_3}{d_2}\overline{R} = D_3\overline{R} \tag{2.16}$$

$$CL = \overline{R} \tag{2.17}$$

where d_2 is the expected value of $\frac{\overline{R}}{\sigma}$ and d_3 is the standard deviation of $\frac{\overline{R}}{\sigma}$.

A_2, D_3 and D_4 are constants and can be found in most SPC textbooks (Devor et al, 1992).

Examples of $\overline{X}$ and R charts are given in Figure 2.2..

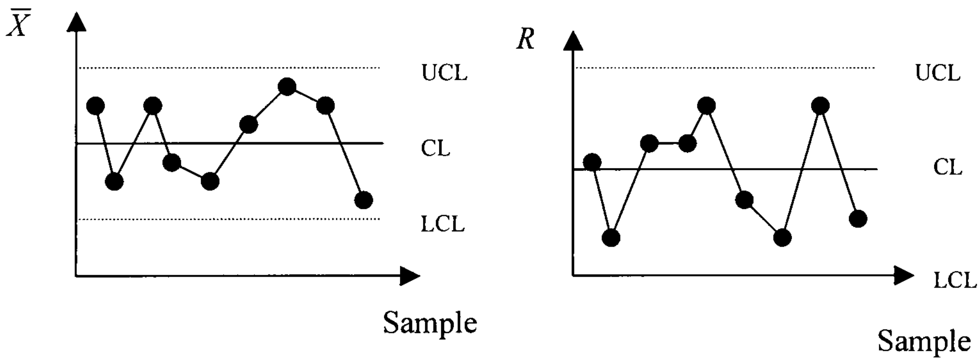

Figure 2.2 $\overline{X}$ and R control charts

2.4.2 X-bar ($\overline{X}$) and S control chart

When the sample size n is larger than 10 or 12, or the sample size n is variable, the $\overline{X}$ and S charts are preferable to replace their counterparts of $\overline{X}$ and R charts. In this case, the range method for estimating variability loses statistical efficiency for moderate to large samples (Montgomery, 1997). S is the sample standard deviation and can be described by equation 2.2 that is mentioned in section 2.2. Related parameters are given by:

$$CL = \overline{S} = \frac{1}{m}\sum_{i=1}^{m} S_i \tag{2.18}$$

$$UCL = \overline{S} + 3\frac{\overline{S}}{c_4}\sqrt{1-c_4^2} \tag{2.19}$$

$$LCL = \overline{S} - 3\frac{\overline{S}}{c_4}\sqrt{1-c_4^2} \tag{2.20}$$

and the parameters for the corresponding $\overline{X}$ chart are given by:

$$CL = \overline{\overline{X}} = \frac{1}{m}\sum_{i=1}^{m} \overline{X}_i \tag{2.21}$$

$$UCL = \overline{\overline{X}} + \frac{3\overline{S}}{c_4\sqrt{n}} \tag{2.22}$$

$$LCL = \overline{\overline{X}} - \frac{3\overline{S}}{c_4\sqrt{n}} \tag{2.23}$$

where

$$c_4 = \sqrt{\frac{2}{n-1}}\frac{[(n-2)/2]!}{[(n-3)/2]!} \tag{2.24}$$

2.4.3 Exponentially weighted moving average (EWMA) chart

The use of Exponentially Weighted Moving Averages (EWMA) for constructing control charts for the mean of a process was introduced by Roberts (Roberts, 1959). If the zone rules (described in section 2.5) are viewed as an addition to Shewhart charts, then the EWMA is another addition to the Shewhart chart. EWMA can be used to detect small shifts in the process (Montgomery, 1997).

EWMA control charts have the following advantages (Mathworks, 1998):

1. They represent smoothed sample means.
2. Can be used for individual measurements when the cost of inspection is high or when expensive destructive testing is involved.
3. Its forecast function can be used for "one-step-ahead" prediction for the process.

The generic formula for the EWMA model is:

$$\tilde{x}_t = \lambda x_t + \theta \tilde{x}_{t-1} \tag{2.25}$$

where $\tilde{x}_t$ and $\tilde{x}_{t-1}$ are EWMA values, and x_t is the current sample value.

λ and θ are smoothing parameters, and $\lambda = 1 - \theta$.

The weight, which indicates the emphasis on the observations, can be calculated by equation 2.26.

$$W_{t-i} = \lambda \theta^i \tag{2.26}$$

where W_{t-i} is the weight associated with the observation x_{t-i}, i is the backwards count of the sequence of number of observations, and $x_{t-i}\big|_{i=0} = x_t$ is the current observation.

Figure 2.3 illustrates that using a larger value of λ (or small θ) results in more emphasis on recent observations. Small λ (or large θ) produces a chart that average the effects of the weights.

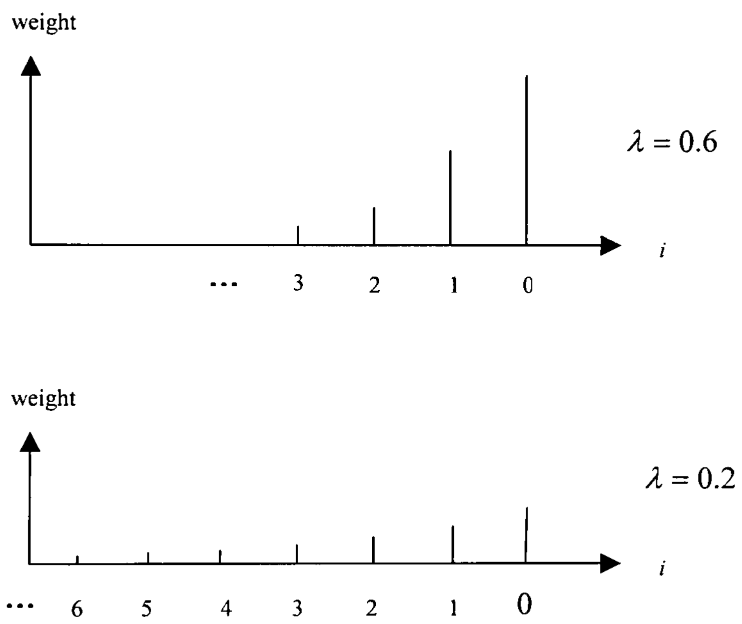

Figure 2.3 Exponential weights for $\lambda = 0.6$ and 0.2

The use of exponential smoothing for forecasting was first arrived at empirically on the grounds that it was a weighted average with the property of giving most weight to the last observation and less to the next-but-last and so on (Montgomery and Runger, 1994). If given recent observation x_t, then an estimate $\hat{x}_{t+1}$ of the next data x_{t+1} can be obtained by:

$$\hat{x}_{t+1} = \lambda x_t + \theta \hat{x}_t \tag{2.27}$$

EWMA statistic can be considered as a "one-step-ahead" prediction for the process (Lucas and Saccucci, 1990), and it allows the system's behaviour to be forecast. Therefore, the EWMA statistic has the property of being useful for feedback control because the prediction can be used to adjust the process (Rius et al, 1998). This advantage of a EWMA chart will be applied in the chapter 7 to forecast the process average shift level at the first observed abnormal point.

2.4.4 Overview of other control charts

1. Cumulative summation (CUSUM) chart

The Cumulative Summation (CUSUM) control chart was first proposed by Page (Page, 1954). Similar to the EWMA chart, the Cusum control chart is a good alternative to detect the small shifts in process mean and variability. The Cusum chart directly combines all the information in the sequence of sample values by calculating the cumulative sums C_i of the deviations of the sample values from a target value.

2. *p*-chart

As was mentioned previously, the *p*-chart belongs to the fraction of defective attribute control charts (Oakland, 1996). The $p-$ chart is used for fraction rejected as non-conforming to specifications where the subgroup size is varying (Grant and Leavenworth, 1988a). The p is estimated by a ratio of the sum of units rejected and the total number of samples inspected.

3. *np*-chart

The np – chart belongs to the number of defective attribute control charts (Oakland, 1996). The np – chart is used for number of non-conforming items where the subgroup size is constant (Grant and Leavenworth, 1988a). The statistics np is the multiplication of sample size n and statistics p, which is mentioned above.

The *np*-chart is a slight modification of the *p*-chart. Some users prefer this chart as the number of defectives can be plotted directly without having to carry out a division to obtain the fraction of defectives (Ledolter and Burrill, 1999).

4. *c*-chart

The *c*-chart is the number of defects (non – conformities) attribute control chart. It is used for number of non-conformities where the subgroup size is constant (Oakland, 1996). This chart usually assumes that the occurrence of non-conformities in samples of constant size is well modelled by the Poisson distribution (Montgomery and Runger, 1994).

Suppose C is the number of defects in a sample of units, where C is a *Poisson* random variable with parameter λ (Montgomery and Runger, 1994). If the parameter λ is unknown, it can be estimated by $\bar{c}$, which is the average of defect numbers for total examined units.

5. u-chart

The u-chart is the number of defects (non–conformities) per unit chart. The u-chart is used for number of non-conformities per unit where the subgroup size is allowed to vary (Oakland, 1996). The statistics u_i is determined as a ratio of the total defects in the ith sample and the ith unit (or subgroup) size. The statistic u is estimated by $\bar{u}$ which is the average of u_i for total samples examined.

2.4.5 Overview of research background in control charting

Many research papers have been published on the construction and application of control charts. Several typical approaches are summarised below. The research background for EWMA chart is discussed with its application in chapter 7.

Krishnan and Gitlow applied $\bar{X}$ and R charts to improve the quality in the treatment of cold gas plasma. By varying the conditions of cold gas plasma treatment, it is possible to obtain a particular effect on the surface of a polymer. They used a cold gas plasma to improve the wettability of the surface of a plastic cuvettes. If the liquid meniscus measurement falls below the LCL on the control chart, the cuvette is not acceptable. If the meniscus data is over the UCL, it is not desirable because it increases the cost. This research work produced benefits to the internal customers of the cold gas plasma treatment process in the form of reduced rework costs from not recycling cuvettes and decreased surface degradation to cuvettes due to fewer cold gas plasma treatments. It also yielded benefits to the external customers of the final product through increased on-time delivery, decreased scrap rates and increased quality (Krishnan and Gitlow, 1997).

Wu proposed a scheme for single $\overline{X}$ control chart. A single $\overline{X}$ chart can obtain the same or even greater detecting effectiveness than the combination of $\overline{X}$ chart and S chart, especially when the magnitude or the frequency of the mean shift is greater than that of the standard deviation shift (or spread). In this scheme, effort spent in designing and maintaining the S chart, which is more difficult to handle and understand than the $\overline{X}$ chart can be spared (Wu, 1994). A computer-aided design is necessary for this single $\overline{X}$ chart.

Some modifications have been proposed to enhance the performance of the conventional control chart. That is, the adaptive control chart has been developed to increase the efficiency of detecting a shift in the process mean. The first approach was based on varying the sample interval. When the process is in control, the samples are taken using a longer sample interval. Conversely, when the process emerges as "out-of-control", a shorter sample interval is selected (Runger & Montgomery, 1993). The second approach focus on adapting the sample subgroup size. The threshold limits are proposed inside the three-sigma control limits that are used to indicate when an increase in the sample size is needed (Prabhu et al, 1993). Zimmer et al presented several adaptive control charts that include changing the sampling interval, the sample size or both according to rules based on the value of the sample statistic. The results show that the adaptive control chart schemes improve the control chart performance and the improvement is relatively modest (Zimmer et al, 2000).

Goh developed an alternative set of decision rules (or patterns) for a low defect process in *c*-chart. These additional rules were based on the Poisson distribution for the number of defects in single samples as well as the Binomial distribution for the occurrence of defects in successive samples. The abnormal patterns or alarms can be obtained through the statistics c_i value and occurrence probability. This approach is useful for the monitoring and control of manufacturing processes that give rise to only a very small number of defects in the products (Goh, 1991).

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In this thesis, the $\overline{X}$ and the R charts are applied to the general research of feedback control. They are effective tools, which are frequently and simply used in a wide range of applications. The use of zone rules, in particular is the research focus of this research work, based on the standard $\overline{X}$ and R charts. The $\overline{X}$ chart is used in chapters 5, 6 and 7 and the R chart is used in chapters 6 and 7 to measure, analyse and control the abnormal behaviour of a simulated process. The *EWMA* chart is also used with an optimal smoothing constant for the forecasting of the process average shift in chapter 7. The applications of other control charts with emphasis on small shifts, adapting the $\overline{X}$ control charts parameters, are left for future work.

2.5 Zone rules

As mentioned in section 2.3, the SPC task in the evaluation phase is the identification of abnormal patterns using control charts and zone rules. When points fall on the chart at random positions between control limits, common causes and no abnormal conditions are indicated and require no control action. When a point falls outside the control limits or a group of points are drawn as some regular patterns, this indicates that some assignable cause or special cause was present and suggests the need for corrective action. Normally this procedure is called control chart interpretation (Zorriassatine and Tannock, 1998) or SPC pattern recognition (Hwarng and Hubele, 1993).

An overview of zone rules is described in section 2.5.1, the number of zone rules used and false alarm probability are discussed in section 2.5.2. In section 2.5.3, the research background in zone rules and the research planning in zone rules are presented.

2.5.1 Overview of zone rules

The zone rule concept is a traditional tool in SPC pattern recognition. Often, the regular or unnatural patterns contain extreme points, too many points near the control limits or points in a run above or below the centreline. They can often be identified by the examination of the charts. Several specific tests called rules or zone rules have been developed for identifying the presence of unnatural patterns in the charts. The tests are performed by dividing the distance between the upper and lower control limits on the $\overline{X}$

charts into C, B and A zones defined by $\pm\sigma_{\bar{X}}, \pm 2\sigma_{\bar{X}}$ and $\pm 3\sigma_{\bar{X}}$ (Devor et al, 1992), (Montgomery, 1997). Several commonly used rules are described in this section.

The Western Electric Handbook (1956) suggests a set of decision rules for detecting unnatural patterns. Specifically, the *Western Electric* rules (WER) conclude that the process is "out of control" if any of the following conditions are met (Montgomery, 1997), (Box and Luceno, 1997):

WER 1: A single point lying beyond the three - sigma limits.

WER 2: Two out of three consecutive points lying beyond the two – sigma limits.

WER 3: Four out of five consecutive points lying beyond the one – sigma limits.

WER 4: Eight consecutive points lying on one side of the target value.

More detailed rules (zone rules) have been proposed by Devor (Devor, et al, 1992):

Zone Rule1: The existence of a single point beyond a control limit signals the process of an out-of-control condition (extreme point *a* in Fig. 2.4). Zone rule1 can be used on both $\bar{X}$ and R chart (WER1).

Zone Rule2: The existence of two of any three successive points in zone A or beyond signals the presence of an out-of-control condition (second abnormal point *b* in Fig. 2.4). Zone rule2 is used on $\bar{X}$ chart only (WER2).

Zone Rule3: The existence of four of any five successive points in zone B or beyond signals the presence of an out-of-control condition (fourth abnormal point *c* in Fig. 2.4). This rule is used on $\overline{X}$ chart only (WER3).

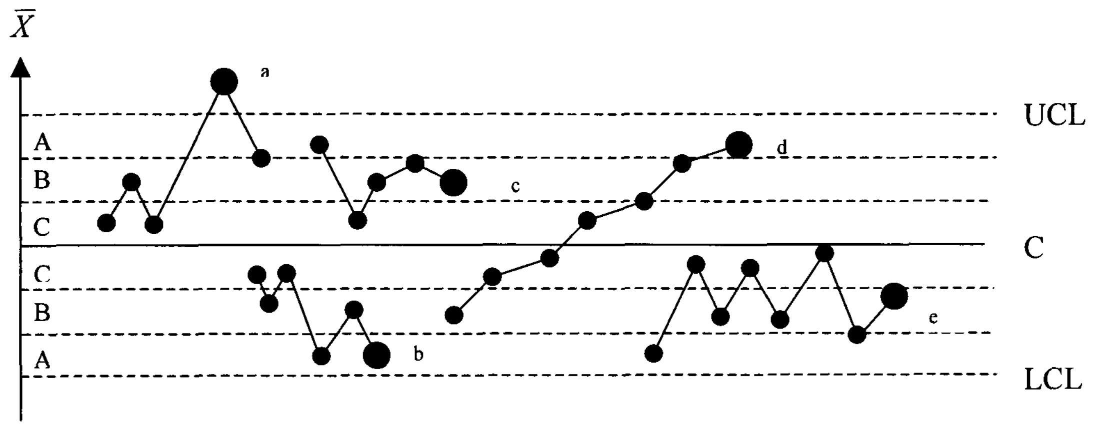


Figure 2.4 The abnormal points

Zone Rule4: When six successive points increase or decrease continuously, a systematic trend in the process is signalled (sixth abnormal point *d* in Fig. 2.4). It can be used on both $\overline{X}$ and R charts.

Zone Rule5: Eight or more successive points either strictly above or strictly below the centreline indicates that the process mean (in $\overline{X}$ chart) or variability (in R chart) has shifted from the centreline (eighth point *e* in Fig. 2.4). This rule can be used on both $\overline{X}$ and R chart (WER4).

Zone Rule6: When 14 successive points oscillate up and down on the chart, a systematic cyclic trend in the process is signalled. It can be used on both $\overline{X}$ and R charts.

Zone Rule7: When eight successive points occurring on either side of the centreline avoid zone C, an out-of-control condition is signalled. It is for $\overline{X}$ chart only.

Zone Rule8: When 15 successive points on the chart fall in zone C only, to either side of the centreline, an out-of-control condition is signalled. It is used on $\overline{X}$ chart only.

Oakland suggests the three *Action*, *Warning* and *Stable* zones which are defined by $\overline{\overline{X}} \pm 3\sigma_{\overline{X}}$, $\overline{\overline{X}} \pm 2\sigma_{\overline{X}}$ and centre line $\overline{\overline{X}}$ for testing the in-control condition (Oakland, 1996):

Rule 1: No mean or range values, which lie outside the Action Limits (same as UCL and LCL shown in Fig. 2.4).

Rule 2: No more than 1 in 40 values between the Warning and Action Limits (shown as zone A in Fig. 2.4).

Rule 3: No incidence of two consecutive mean or range values which lie outside the same Warning Limit on either the mean or the range chart (shown as zone A in Fig.2.4).

Rule 4: No run or trend of five or more which also infringes a warning or action limit (zone A or outside of UCL / LCL in Fig. 2.4).

Rule 5: No runs of more than six samples mean which lie either above or below the Grand Mean (centre line CL showed on Fig. 2.4).

Rule 6: No trends of more than six values of the sample means which are either rising or falling in the stable zone (same to that existing in zone B and C which are shown in Fig. 2.4).

There are also some different types and names of unnatural patterns defined in some published papers. Five abnormal patterns of upward shift pattern, downward shift pattern, upward trend pattern, downward trend pattern and cycle pattern have been recognised by a neural network (Guh and Hsieh, 1999). Six abnormal patterns which includes the five patterns mentioned above and a systematic pattern were identified by Guh et al (1999b). These patterns are contained in zone rule 1 ~ zone rule 8 mentioned above. Some of the major problems associated with the analysis of control chart patterns are summarised by Cheng (Cheng, 1997). For example, a cycle contaminated with noise, a shift being misinterpreted as a trend, detecting a shift in a trend, detecting a shift in a cycle, and detecting a shift in a mixture.

SPC zone rules are useful tool for pattern recognition. Many rules have been developed based on related processes. The question is, is it better to employ more rules in any particular process? Section 2.5.2 addresses this issue.

2.5.2 The problems of zone rules and the number of zone rules employed

Zone rules can be viewed as supplemental sensitising criteria and additional action rules for the Shewhart control chart (Box and Luceno, 1997). Zone rule 1 (section 2.5.1) is called a standard action signal, and the other rules discussed in section 2.5.1 can be called sensitising rules which are used to increase the sensitivity of control charts to a small process shift in order to respond more quickly to the assignable cause.

However it is not good policy to employ more and more sensitising rules for any process. (Champ and Woodall, 1987) found that selecting which rule to use can improve the ability of the control chart to detect smaller shifts, but the "in-control" average run length can be degraded substantially. The average run length (ARL) is defined as the average of the number of samples that are plotted until the plotted process characteristic exceeds the control limits for the first time. However, the more rules used, the higher the probability that the process is stopped unnecessarily where the "in-control" average run length is obviously reduced (Ledolter & Burrill, 1999).

Suppose that k rules have been used and the *ith* criterion has type I error (or false alarm) probability α_i, then the overall type I error or false – alarm probability α for the decision based on all k tests is defined by (Montgomery, 1997):

$$\alpha = 1 - \prod_{i=1}^{k}(1-\alpha_i) \tag{2.28}$$

The error α will increase with an increase in the number of rules k. Therefore, in general, care should be exercised when using several decision rules simultaneously. That is, the sensitising rules need to be used with considerable caution, as an excessive number of false alarms can be harmful to an effective SPC program. If more supplemental rules are applied to the chart, the decision process becomes more complicated, and the inherent simplicity of the Shewhart control chart is lost (Montgomery, 1997).

The first problem of sensitising rules is extremely important for implementation or control process. Based on the discussion above, for general control with a longer "in-control" average run length and small error α, and in order to reduce the process disturbance, this research work employs zone rule 1 to zone rule 5 only, since the majority of these rules are additional action rules which are suitable for feedback control (Box and Luceno, 1997). Furthermore, a previous average position will be calculated in the controller, which is described in chapter 6 in order to compensate and reduce the error α and increase the control robust function.

2.5.3 Research in pattern recognition

In the past few years, many papers have been published on SPC pattern recognition using expert systems and neural networks (NN).

Hwarng and Hubele used the Back – Propagation pattern recognisers to identify unnatural patterns such as cycles and trends exhibited on control charts. This valuable information could be provided for real time process control (Hwarng and Hubele, 1993). Hwarng and Chong used zone rules to train and test NNs to identify non-random situations. They suggested that the identification of non-random patterns, can be achieved using either a special purpose system detecting only one type of pattern but in its variety of forms; or a general purpose system detecting all the possible classes of patterns (Hwarng and Chong, 1995). Pham and Oztemel presented a scheme for using neural networks for discriminating between different type of control chart patterns. A procedure to increase the classification accuracy and decrease the learning time for the Learning Vector Quantization (LVQ) networks is described (Pham and Oztemel, 1994). Smith formulated the X-bar and R control charts for diagnosis and interpretation by neural networks. The neural networks are trained to discriminate between samples from probability distributions considered within control limits and those which have shifted in both location and variance, and to predict future points from processes which exhibit long – term or cyclical drift (Smith, 1994). An alternative approach to SPC using artificial neural network technique and comparing its performance with that of the combined Shewhart – CUSUM schemes has been undertaken by Cheng (Cheng, 1995). This paper is concerned with the detection of gradual trends and sudden shifts in the process mean. The extensive comparison shows that the proposed network has 20-40% faster detection of small process changes than the combined Shewhart-CUSUM control schemes. Guh and Tannock investigated the detection of concurrent patterns where more than one pattern exists simultaneously. The Back – Propagation Network system was used and two evaluation scenarios were evaluated: in the first, unnatural patterns are already present; while in the second, patterns may appear progressively at any time. Numerical results are provided that indicate that the pattern recogniser can perform very well in the first scenario, while it performs effectively but with deficiencies for some specific pattern combinations in the second approach (Guh and Tannock, 1999). Chang and Aw proposed a neural fuzzy control chart for identifying process mean shifts: the neural network's outputs are classified into various decision regions using a fuzzy set scheme. This system has ability to identify the magnitude of mean shifts (Chang and Aw, 1996). An application of an expert system in out-of-control pattern recognition has been carried out by Swift and Mize. The expert system looks for the following patterns of variation: trend, cycle, mixture, shift and stratification. Systematic and statistical significance tests as interpretative rules are used to determine the pattern of variation. Once the pattern is identified, the expert system supplies the user with possible causes for the out-of-control condition (Swift and Mize, 1995).

Some published papers described above, indicate that the neural network technique is a favourite and frequently used technique in pattern recognition. The neural networks can map the non-linear input/output relation through the learning process without any predefined conditions. Pattern recognition is a typical application of NN's, and successful results can be obtained but adequate data are necessary. This is especially true for a general purpose system which is used to detect all the possible classes of patterns or different type of control chart patterns (Hwarng and Chong, 1995) (Pham and Oztemel, 1994). If too large a number of training data are used to build a recognition system or too specific objective applications are developed, their utility and practicability will be debased, even though much significant research has been undertaken such as the normal and shifted pattern (Smith, 1994), gradual trends and sudden shifts (Cheng, 1995) and the detection of concurrent patterns (Guh and Tannock, 1999). Considering the research discussed above and their drawbacks, the use of fuzzy logic to generate magnitude output (not classification number only) may provide a means to overcome these shortfalls. In this case, a more important advantage is that the magnitude outputs generated from fuzzy logic are more accurate than classification numbers which roughly map the relationship between input and output, are determined or obtained from a NN (Chang and Aw, 1996).

Fuzzy logic can map input/output relation in non-linear systems based on human inference logic. The numerical or magnitude outputs can be derived from natural language by fuzzy logic. The fuzzy logic and neural network notions and further applications will be introduced in chapters 3, 4 and 6.

This thesis uses fuzzy subset theory in SPC. Namely, the traditional control chart concepts are kept but fuzzy logic is used to interpret the zone rules. This forms the basis of a new Fuzzy-SPC Evaluation and Control system which results in a numerical control action which can be used as the output instruction in an SPC process or a supervisory control system. The design of fuzzy-SPC controller is described in chapter 4. The simulation of the numeric control action, which is used to adjust the process mean to obtain an improved quality performance of the system, is illustrated in chapter 5.

2.6 SPC feedback control

Statistical process control originated in the discrete parts industry, and is frequently employed for process monitoring. SPC zone rule identification can be interpreted as an alarm based testing method (Guh et al, 1999a) (Spanos, 1991), It can provide the indication when an abnormal point appears. Traditional SPC however does not directly define what control actions have to be taken. Definitions of the actions to be taken is normally undertaken by the process and technical staff who understand the processes (Coulson and Cousans, 1987).

Box presents a formula for feedback control by manual adjustment (Box, 1991):

$$x_t = X_t - X_{t-1} = -\frac{\lambda}{g}(y_t - T) \tag{2.29}$$

where x_t is called adjustment at time t,

X_t is the compensatory variable state at time t,

y_t is measurement value at time t,

T is measurement target and λ and g are coefficients.

He also points out that if this adjustment is repeatedly applied at every step, a series of adjustment equations are obtained which can be summarised as a proportional integral (PI) control (Box and Luceno, 1997). Equation 2.29 indicates that the adjustment x_t can be obtained by computation based on every sample data.

Montgomery et al proposed an integrating Engineering Process Control (EPC) and SPC study. They pointed out that the integration of EPC and SPC has potentially desirable results. EPC can be used to minimise deviation from the target due to disturbances that occur continuously and are part of the process itself, and SPC applied to the output deviation from target which can be used to identify and subsequently eliminate assignable causes. Combined EPC/SPC control results in this study always result in the reduction of overall variability if the system experiences certain external assignable causes. The results demonstrate that the addition of an SPC chart to monitor output deviation from target in a system with active control is a highly effective way to integrate theses two strategies. They conclude that proper use of both SPC and EPC can always outperform the use of either alone (Montgomery et al, 1994).

Box et al compares SPC and engineering process control (EPC). In feedback control, quality practitioners frequently need to adjust processes. Familiar monitoring devices such as Shewhart charts are inappropriate and inefficient for this purpose, but simple ideas from EPC can be readily put to use. They pointed out that it is necessary to clearly distinguish between the detection of signals in process noise by process monitoring and the estimation of the level of the current disturbance needed for compensation by feedback control. Both should be used but not confused (Box et al, 1997).

Janakiram pointed out the main purpose of SPC is to look for assignable causes in the process data. EPC is used to monitor the process output, compare it with the target, and make compensatory adjustments to the process input on a regular basis to keep the output on target. He proposed a combination scheme of SPC and EPC in a powder loading operation for an automobile air-bag initiator. In the blending operation, the flow properties of the powder is a function of density that in turn depends on the humidity (disturbance) and amplitude of vibration of the feeder bowl. Too much powder will lead to delay in firing, and too little powder will lead to insufficient explosion pressure. $\overline{X}$ and R charts are used to monitor the average powder weight at fixed intervals. If the powder weight has drifted to an "out-of control" condition, the amplitude of the vibrator bowl then is adjusted on demand by the EPC of machine. This paper shows effective application of SPC/EPC integration in order to achieve the desired goal (Janakiram, 1998).

As discussed above, it is very significant to combine SPC and EPC technologies to build a hybrid control system in order to improve both the efficiency of SPC adjustment and the control quality in EPC. To achieve this target, this research focuses on the SPC zone rules and control charts to generate the magnitude control action for SPC adjustment. The abnormal phenomenon which is described by sensitising zone rules will be captured, and the specific numerical feedback control action will be generated from the fuzzy inference or fuzzy – SPC controller (chapter 4). After the feedback control, the process behaviour and control errors will be displayed and analysed (chapter 5). If the control result is larger than a predefined limit, a neural network will be used to optimise the parameters of the Fuzzy – SPC controller, until the best control results are obtained (chapters 6 and 7).

Before considering the design of the feedback Fuzzy-SPC controller, the frequency distributions of abnormal process averages which are detected by zone rules should be adopted for building the membership functions.

2.7 The frequency distribution of process average shift levels

This section describes an approach to determine the frequency distribution of process average shift levels which are detected by zone rules 1 to 5 at the first abnormal point (FDPASL at FAP), or the occurrence frequency of the first abnormal point (FAP) tested by each zone rule. This was undertaken in order to answer two questions: Which shift level occurred, and how often did it occur at the first abnormal point (control point)? The outcomes of this analysis are used to build fuzzy consequent membership functions.

A simulation program was executed in MATLAB. 500 random data were generated in software and the subgroup size chosen was 5 where 100 $\overline{X}$ values are tested by SPC zone rule 1~5. The normal random data subject to standard normal distribution (0,1) was used. To simulate a disturbance, the process mean was shifted from 0.1σ to 1.5σ, where σ is population deviation. In these simulations, σ was replaced by its estimate value, which is the process standard deviation (SD). Shifted processes still follow normal probability distribution but have different means. For each shift level, the program ran 100 times to obtain the 100 first abnormal points behaviour which includes computing the zone rule number, process average, the average of 5 data before FAP and the average of 5 data after FAP. This process was repeated three times and a total of 4500 FAPs were obtained and used to compute the average of occurrence frequencies.

Table 2.1~Table 2.3 summarises the FDPASL at the FAP. The sample details are presented in the Appendix A.1. In these tables, columns indicate the different zone rules

(ZR1~ZR5) which are used to test the FAP and rows indicate the different process average shift levels. Table 2.4 shows averages of frequencies in Table 2.1~2.3.

No.1	ZR1	ZR2	ZR3	ZR5	ZR4
0.1SD	0.18	0.13	0.19	0.39	0.01
0.2SD	0.18	0.2	0.23	0.37	0.02
0.3SD	0.17	0.19	0.27	0.29	0.06
0.4SD	0.2	0.22	0.37	0.2	0.01
0.5SD	0.19	0.14	0.45	0.19	0.03
0.6SD	0.23	0.25	0.44	0.06	0.02
0.7SD	0.25	0.38	0.33	0.04	0
0.8SD	0.26	0.43	0.23	0.07	0.01
0.9SD	0.39	0.38	0.17	0.05	0.01
1.0SD	0.59	0.29	0.11	0.01	0
1.1SD	0.65	0.23	0.1	0	0.02
1.2SD	0.65	0.27	0.07	0	0.01
1.3SD	0.8	0.18	0.01	0	0.01
1.4SD	0.81	0.17	0.02	0	0
1.5SD	0.9	0.09	0.01	0	0

Table 2.1 The FDPASL at the FAP in No.1 group

No.2	ZR1	ZR2	ZR3	ZR5	ZR4
0.1SD	0.19	0.16	0.2	0.37	0.08
0.2SD	0.18	0.19	0.25	0.36	0.02
0.3SD	0.12	0.17	0.37	0.31	0.03
0.4SD	0.16	0.24	0.41	0.18	0.01
0.5SD	0.17	0.22	0.45	0.16	0
0.6SD	0.2	0.28	0.39	0.11	0.02
0.7SD	0.26	0.39	0.26	0.07	0.02
0.8SD	0.29	0.48	0.2	0.02	0.01
0.9SD	0.35	0.39	0.2	0.03	0.03
1.0SD	0.38	0.36	0.19	0.04	0.03
1.1SD	0.58	0.24	0.15	0.02	0.01
1.2SD	0.71	0.22	0.07	0	0
1.3SD	0.83	0.14	0.03	0	0
1.4SD	0.83	0.16	0	0	0.01
1.5SD	0.9	0.1	0	0	0

Table 2.2 The FDPASL at the FAP in No.2 group

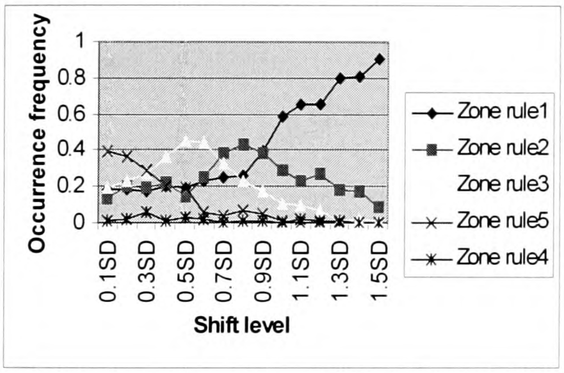
No.3	ZR1	ZR2	ZR3	ZR5	ZR4
0.1SD	0.19	0.17	0.15	0.42	0.07
0.2SD	0.17	0.16	0.25	0.37	0.05
0.3SD	0.18	0.12	0.32	0.31	0.07
0.4SD	0.15	0.21	0.35	0.26	0.03
0.5SD	0.17	0.19	0.43	0.19	0.02
0.6SD	0.16	0.25	0.44	0.12	0.03
0.7SD	0.21	0.36	0.26	0.13	0.04
0.8SD	0.27	0.43	0.22	0.07	0.01
0.9SD	0.3	0.39	0.23	0.06	0.02
1.0SD	0.46	0.29	0.17	0.07	0.01
1.1SD	0.59	0.3	0.1	0	0.01
1.2SD	0.75	0.21	0.02	0.02	0
1.3SD	0.81	0.17	0.02	0	0
1.4SD	0.85	0.14	0.01	0	0
1.5SD	0.92	0.08	0	0	0

Table 2.3 The FDPASL at the FAP in No.3 group

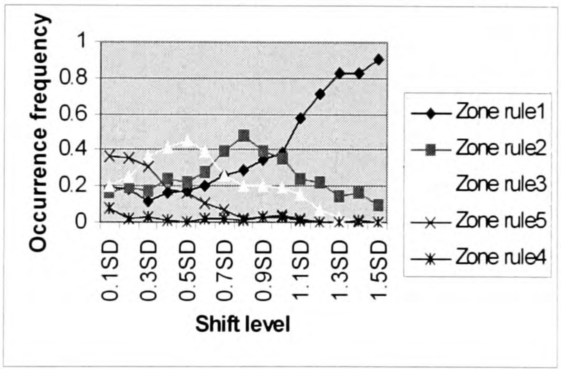
	ZR1	ZR2	ZR3	ZR5	ZR4	Average
0.1SD	0.1867	0.1533	0.1800	0.3933	0.0533	
0.2SD	0.1767	0.1833	0.2433	0.3667	0.0300	
0.3SD	0.1600	0.1767	0.2633	0.3400	0.0533	
0.4SD	0.1700	0.2233	0.3767	0.2133	0.0167	
0.5SD	0.1767	0.1833	0.4433	0.1800	0.0167	
0.6SD	0.1967	0.2600	0.4233	0.0967	0.0233	
0.7SD	0.2400	0.3767	0.2833	0.0800	0.0200	
0.8SD	0.2733	0.4467	0.2167	0.0533	0.0100	
0.9SD	0.3467	0.3867	0.200	0.0467	0.0200	
1.0SD	0.4767	0.3133	0.1567	0.0400	0.0133	
1.1SD	0.6067	0.2567	0.1167	0.0067	0.0133	
1.2SD	0.7033	0.2333	0.0533	0.0067	0.0033	
1.3SD	0.8133	0.1633	0.0200	0	0.0033	
1.4SD	0.8300	0.1567	0.0100	0	0.0033	
1.5SD	0.9067	0.0900	0.0033	0	0	
average	0.4176	0.2402	0.1993	0.1216	0.0187	
SD	0.1465	0.1009	0.1442	0.1423	0.0164	0.1335

Table 2.4 The averages of FDPASL at the FAP

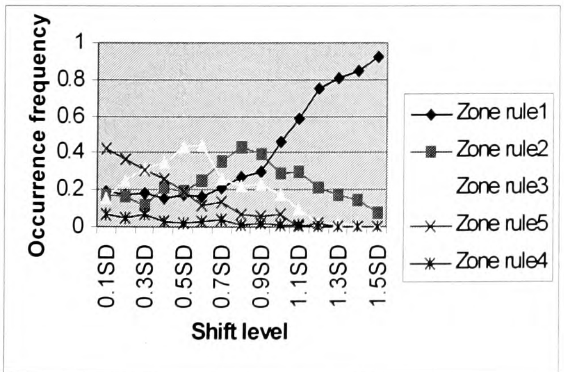
No. (1) ~ No. (3) in Figure 2.5 show the FDPASL at the FAP for Table 2.1~2.3. No. (4) in Figure 2.5 is plotted from Table 2.4. Fig. 2.5 shows that each frequency distribution curve has central tendency. The central values or domains indicate related zone rules, which are used to test the FDPASL at the FAP. The peak values of distribution curves occur at 0.1SD, 0.5SD, 0.8SD and 1.5SD which can be viewed as the central values of four possible shift ranges corresponding to zone rules 5, 3, 2 and 1 respectively. This value is used as the adjustment values range or control range in section 4.2.1 of chapter 4. Since the occurrence frequency of zone rule 4 is too small in this work, there is no particular control range for it. Its control actions depend on other rules (Appendix B.5).



(1)



(2)



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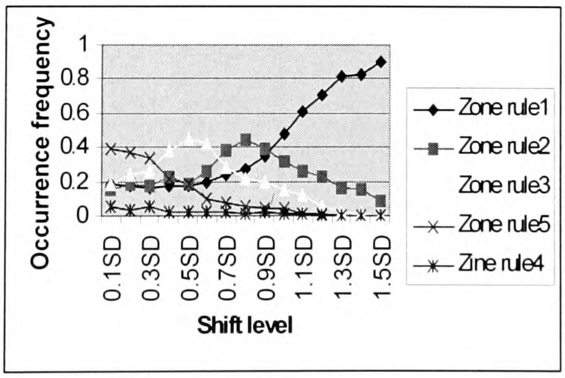


(4)

Figure 2.5 Frequency distributions

Figure 2.5 shows that there are some overlaps below the 0.2 frequency line. These overlaps can cause control errors if only the SPC zone rules are used to generate the control actions. Since the FAP occurs with some randomness, even though it has a central tendency there are some overlaps. Some additional modifications are required to reduce this control error.

Tables 2.5~2.7 describes the improved FDPASL at the FAP based on Tables 2.1~2.3. That is, the average of 5 data after FAP is used to modify the SPC zone rules. The details of the modified frequency data of tables are presented in Appendix A.2. Table 2.8 describes the averages of frequencies in Tables 2.5~2.7.

No.1	ZR1	ZR2	ZR3	ZR5	ZR4
0.1SD	0	0	0.15	0.74	0.11
0.2SD	0	0.03	0.23	0.72	0.02
0.3SD	0	0.06	0.35	0.53	0.06
0.4SD	0	0.11	0.62	0.26	0.01
0.5SD	0.03	0.09	0.71	0.17	0
0.6SD	0.03	0.36	0.53	0.06	0.02
0.7SD	0.1	0.57	0.3	0.03	0
0.8SD	0.18	0.71	0.11	0	0
0.9SD	0.42	0.49	0.07	0.01	0.01
1.0SD	0.64	0.35	0.01	0	0
1.1SD	0.88	0.12	0	0	0
1.2SD	0.91	0.08	0	0	0.01
1.3SD	0.96	0.03	0	0	0.01
1.4SD	0.97	0.03	0	0	0
1.5SD	1	0	0	0	0

Table 2.5 Modified FDPASL at the FAP in No.1 group

No.2	ZR1	ZR2	ZR3	ZR5	ZR4
0.1SD	0	0	0.15	0.77	0.08
0.2SD	0	0.01	0.25	0.72	0.02
0.3SD	0	0.05	0.47	0.45	0.03
0.4SD	0	0.11	0.61	0.27	0.01
0.5SD	0.04	0.16	0.73	0.07	0
0.6SD	0.07	0.32	0.53	0.06	0.02
0.7SD	0.15	0.53	0.26	0.04	0.02
0.8SD	0.17	0.71	0.11	0.01	0
0.9SD	0.4	0.53	0.04	0	0.03
1.0SD	0.62	0.33	0.02	0	0.03
1.1SD	0.78	0.2	0.01	0	0.01
1.2SD	0.9	0.1	0	0	0
1.3SD	0.96	0.04	0	0	0
1.4SD	0.99	0	0	0	0.01
1.5SD	1	0	0	0	0

Table 2.6 Modified FDPASL at the FAP in No.2 group

	ZR1	ZR2	ZR3	ZR5	ZR4
0.1SD	0	0	0.14	0.79	0.07
0.2SD	0	0.01	0.3	0.64	0.05
0.3SD	0	0.02	0.48	0.43	0.07
0.4SD	0.01	0.08	0.61	0.27	0.03
0.5SD	0.06	0.17	0.73	0.03	0.01
0.6SD	0.03	0.4	0.5	0.05	0.02
0.7SD	0.11	0.54	0.3	0.04	0.01
0.8SD	0.13	0.7	0.13	0.03	0.01
0.9SD	0.38	0.52	0.08	0	0.02
1.0SD	0.59	0.39	0.01	0	0.01
1.1SD	0.77	0.21	0.01	0	0.01
1.2SD	0.94	0.04	0.02	0	0
1.3SD	0.98	0.02	0	0	0
1.4SD	1	0	0	0	0
1.5SD	1	0	0	0	0

Table 2.7 Modified FDPASL at the FAP in No.3 group

	ZR1	ZR2	ZR3	ZR5	ZR4
0.1SD	0	0	0.1467	0.7667	0.0867
0.2SD	0	0.0167	0.2600	0.6933	0.0300
0.3SD	0	0.0433	0.4333	0.4700	0.0533
0.4SD	0.0033	0.1000	0.6133	0.2667	0.0167
0.5SD	0.0433	0.1400	0.7233	0.0900	0.0033
0.6SD	0.0433	0.3600	0.5200	0.0567	0.0200
0.7SD	0.1200	0.5467	0.2867	0.0367	0.0100
0.8SD	0.1600	0.7067	0.1167	0.0133	0.0033
0.9SD	0.4000	0.5133	0.0633	0.0033	0.0200
1.0SD	0.6167	0.3567	0.0133	0	0.0133
1.1SD	0.8100	0.1767	0.0067	0	0.0067
1.2SD	0.9167	0.0733	0.0067	0	0.0033
1.3SD	0.9667	0.0300	0	0	0.0033
1.4SD	0.9867	0.0100	0	0	0.0033
1.5SD	1	0	0	0	0
average	0.4044	0.2049	0.2127	0.1598	0.0182
SD	0.4255	0.2327	0.2490	0.2660	0.0235

Table 2.8 The averages of modified FDPASL at the FAP

No.(1)~No.(3) in Figure 2.6 show the modified FDPASL at the FAP summarised in Table 2.5~2.7. No.(4) in Figure 2.6 is the average (Table 2.8). Figure 2.6 shows that the overlaps are much reduced. The shapes of the distribution curves are close to the triangle form. No.(4) in Fig. 2.6 is used as a design basis to build the consequent membership functions in fuzzy inference system in chapter 4.

No.(4) in Figure 2.6 also can be expressed by approximately a normal probability distribution. It is well known that the value of μ determines the centre of the normal probability density function (Montgomery and Runger, 1994), the random data occurring around values of μ have higher probability than at other points.

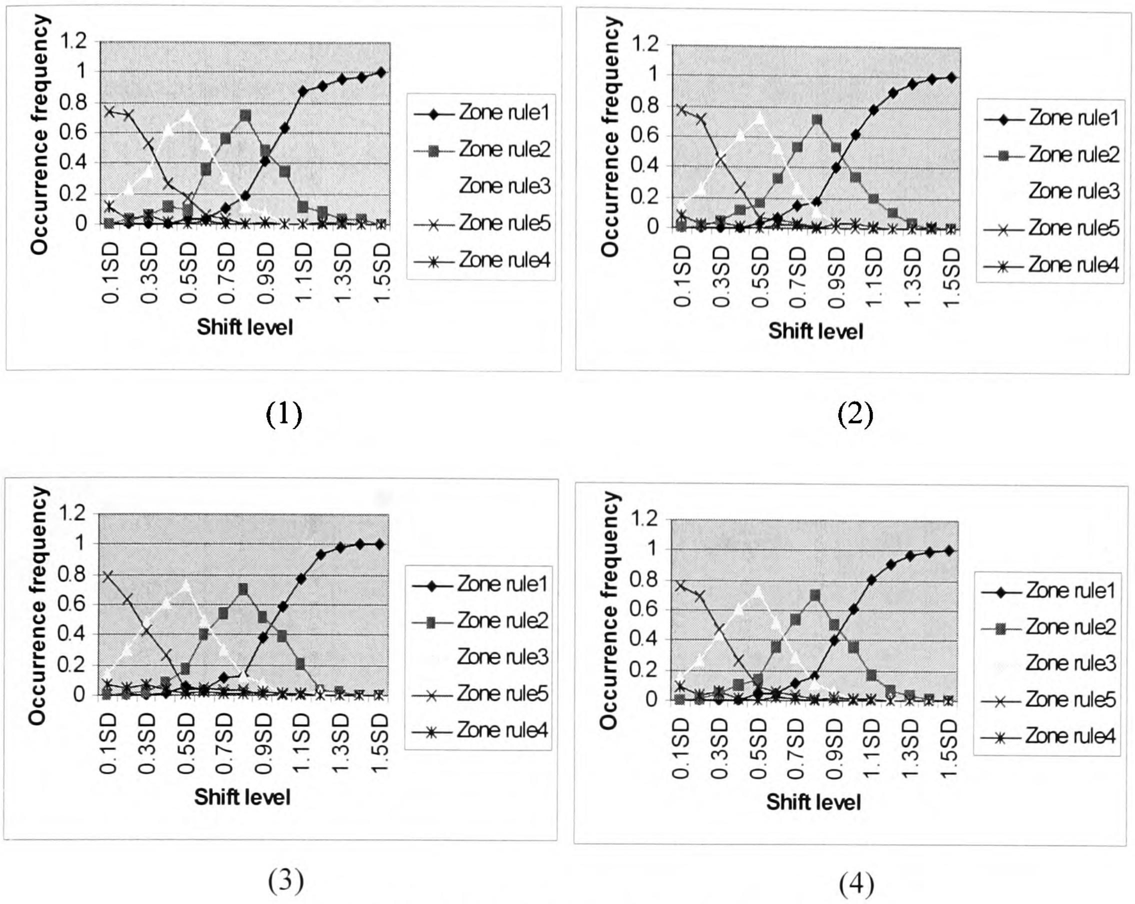


Figure 2.6 Modified frequency distributions

As discussed at the beginning of this section, the random data, which are generated by the simulation program, are subject to normal distribution. The FAP occurrence probability still follows a normal distribution. This is because in a shifted process, FAP occurs around the shifted new centre, which indicates the related testing zone rule has a higher probability.

The normal probability density function is given by:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty \tag{2.30}$$

If the average (0.1335) of the standard deviation, which is shown in Table 2.4 is chosen for process variation σ, and x takes value from 0.2SD to 1.1SD, the central values 0.2SD, 0.5SD, 0.8SD and 1.1SD which are shown in table 2.4 and No.(4) in Fig. 2.5 that indicate related testing zone rules (ZR 5, 3, 2 and 1) are chosen for process mean μ, then the outcomes of $f(x)$ are summarised in the Table 2.9 and plotted in Fig. 2.7.

	ZR1	ZR2	ZR3	ZR5
0.1SD	1.9560E-12	3.2007E-06	0.0336	2.2573
0.2SD	4.0397E-10	0.0001	0.2393	2.9883
0.3SD	4.7603E-08	0.0027	0.9729	2.2573
0.4SD	3.2007E-06	0.0336	2.2573	0.9729
0.5SD	0.0001	0.2393	2.9883	0.2393
0.6SD	0.0027	0.9729	2.2573	0.0336
0.7SD	0.0336	2.2573	0.9729	0.0027
0.8SD	0.2393	2.9883	0.2393	0.0001
0.9SD	0.9729	2.2573	0.0336	3.2E-06
1.0SD	2.2573	0.9729	0.0027	4.76E-08
1.1SD	2.9883	0.2393	0.0001	4.04E-10

Table 2.9 Outcomes of density function

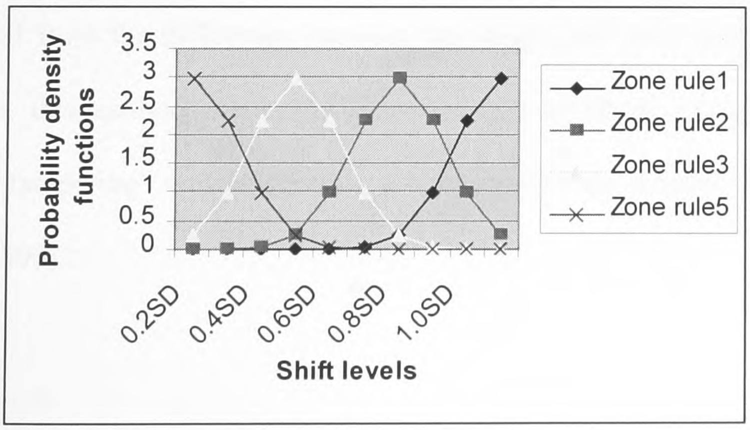


Figure 2.7 Probability density function for ZR 1, 2, 3 and 5

For a comparison between No.(4) in Fig. 2.6 and Fig. 2.7, the distribution curves have similar form.

2.8 Conclusion

Statistical process control is an effective technique in management and process control fields. It can be found in many (perhaps all) industries to maintain and improve product qualities (Caulcutt, 1996). The control charts are prominent tools in SPC. $\overline{X}$ and R control charts are simple and effective tools and are frequently used in many applications to identify "out-of-control" conditions. As additional action rules, zone rules are developed to increase the detection sensitivity of control charts. From this, SPC pattern recognition became a very active research section. The neural network-SPC pattern recognition, which aims to provide on-line measurement and automation are very significant approaches. Their weaknesses are that plenty of training data are required and the results are relatively rough. In the SPC feedback control research section, what magnitude of adjustment or control action should be taken is a big challenge. However, it can be generated from the difference between the target and every sample data. For a random process, this method can easily cause control vibration and incorrect control action such as "tampering" which indicates the reaction to the common causes (Latzko and Saunders, 1995).

The aim of this research work is to give more accurate and robust control action. The control actions are generated from fuzzy inference (chapter 4) which is based on basic SPC zone rules. Before the design of this fuzzy system, how many number of zone rules should be selected and which shift level and how often it occurs (frequency distribution of process average shift levels (FDPASL)) are approached. For the number of zone rules, too many rules will cause an increase in type I error. For FDPASL, particular control action ranges, which are indicated by the FDPASL at the first abnormal point (FAP), are obtained for zone rules 1, 2, 3 and 5. As for zone rule 4, the control action to be taken depends on other zone rules.

2.9 Summary

This chapter introduced some basic knowledge of statistical methods and of statistical process control. As important elements or tools in SPC, control charts are described. Several frequently used types of control chats are summarised and analysed with their advantages and application areas. SPC zone rules are the foundations for SPC pattern identification. It is also the focus in this research work. A discussion proposes that, too many zone rules selected will cause an increase of the type I error. An overview of research background in control chart, pattern recognition and SPC feedback control are explained. Specific behaviour of "out-of-control" conditions in magnitude at different zone rules are important foundations for building fuzzy membership functions and if-then rules. This is approached by FDPASL. Through the simulations of abnormal processes, which are detected by five zone rules, the frequency distributions of process average shift levels are analysed.

Before the design of Fuzzy-SPC controller is described, some related notions of fuzzy logic and the design methods of fuzzy inference system are introduced in chapter 3. These notions and methods will be applied to build the Fuzzy-SPC controller in chapter 4, in order to achieve the SPC feedback control or adjustment.

Chapter 3 Approach to fuzzy logic

Fuzzy logic is based on natural language. Fuzzy logic refers to a logical system for reasoning under uncertainty. This chapter provides on overview to the basis and notions of fuzzy set and fuzzy logic. The design methods for fuzzy control system are also discussed.

3.1 Introduction

Fuzzy logic is an important subject. It is largely due to a wide array of successful applications ranging from consumer products, to industrial process control, pattern recognition, model identification and information system. In a narrow sense, fuzzy logic refers to a logical system that generalises classical two-valued logic for reasoning under uncertainty. In a broad sense, fuzzy logic refers to all of the theories and technologies that employ fuzzy sets, which are classes with un-sharp boundaries (Yen and Langari, 1999). In the basic topic, fuzzy logic involves fuzzy sets and membership function notions, fuzzy relation, composition and related operators. These notions and tools are used to build a fuzzy system frame and to achieve basic operation respectively. The fuzzy inference is a central part, which completes the approximate reasoning in fuzzy system.

As discussed in chapter 2, this thesis focuses on the generating of adjustment or control action based on the interpreting of SPC zone rules. The challenges are that in traditional SPC, there are no ready-made formulas to calculate the on-line adjustments accurately. The control actions are taken by engineers or operators who understand the process. Fuzzy logic can be viewed as a convenient way to map an input space to an output space under the experiences of experts and natural language. It can be used to create a fuzzy system to match imprecise data (Gulley and Jang 1995). These advantages lead to the application of fuzzy logic in this thesis.

This chapter provides an outline of fuzzy sets, fuzzy logic, fuzzy control systems and related research backgrounds. Section 3.2 introduces the concepts of fuzzy sets, membership functions, related fuzzy operators and fuzzy calculations. Section 3.3 describes the fuzzy logic and fuzzy inference, approximate reasoning and its algorithms. Section 3.4 explains the fuzzy control theory and the construct of a fuzzy control system. Finally, the research background in fuzzy systems and the integration of the research work are summarised and discussed in section 3.5.

3.2 Fuzzy set and its operations

3.2.1 Fuzzy set

Classical set theory was developed as the foundation of mathematics by George Contor (1845~1918). It is a collection of objects in a given domain. An object either belongs to the set or does not belong to the set. Therefore, there is a sharp boundary between members of the set and those not in the set (Yen and Langari, 1999).

Professor L.A. Zadeh published the first seminal paper on fuzzy sets in 1965 (Zadeh, 1965). A fuzzy set is a set without a crisp or clearly defined boundary. Fuzzy sets can be used to describe vague concepts or linguistic variables (e.g. fast runner, hot weather. See figure 3.1). It generalises the notion of membership from a black and white binary categorisation in classical set theory into one that allows partial membership. The membership function can be viewed as a possibility distribution of the interested projects (Yen and Langari, 1999). In fuzzy sets, membership becomes a matter of degree that is a number between 0 and 1 (Figure 3.1). A degree of 0 represents complete non-membership and 1 represents a complete membership. In mathematical language, a fuzzy set is characterised by a mapping from its universe of discourse into to the interval [0,1]. This mapping is called membership function $\mu(x)$ of the set while represents the membership of an element x which belongs to a set. The x is also called a linguistic variable (Wang, 1994).

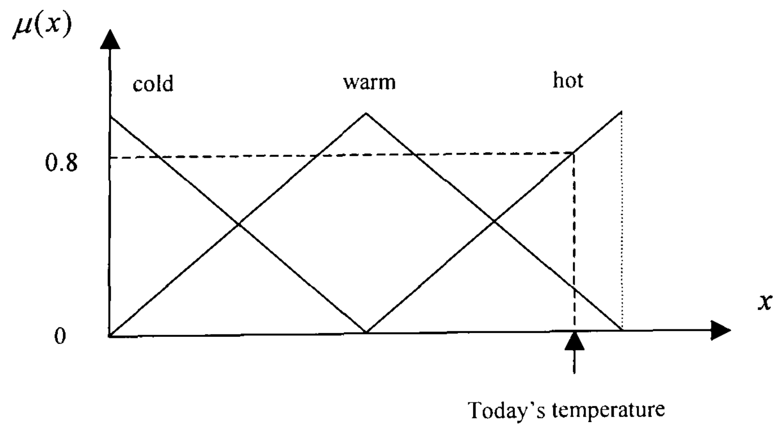


Figure 3.1 Membership function

For a finite universes of discourse while, a fuzzy set can be defined:

$$A = \sum_i \mu_A(x_i)/x_i \tag{3.1}$$

or

$$A = \sum_{x \in X} \mu_A(x)/x \tag{3.2}$$

For infinite universes X,

$$A = \int_{x \in X} \mu_A(x)/x \tag{3.3}$$

In equations 3.1-3.3, the sign "/" is used to distinguish $\mu_A(x)$ and μ in set A, and does not mean a division operator. Also the signs "Σ" and "$\int$" are neither denoting a true sum nor a true integral but have only symbolic meaning for a set (Bandemer and Gottwald, 1995).

3.2.2 Membership function

The membership function is the curve that defines how each point in the input / output space is mapped to the membership value (Figure 3.1). Membership function characterises the fuzziness in a fuzzy set. A fuzzy set can be described by a membership function whose membership values are strictly monotonically increasing, monotonically decreasing or monotonically increasing then monotonically decreasing for elements in the universe of discourse. The membership function embodies the mathematical representation of membership in a set, and the notation used for a fuzzy set is a set symbol, for example the symbol **A,** therefore:

$$\mu_A(x) \in [0,1] \tag{3.4}$$

Several types of basic functions can be used for membership functions: singleton, triangular, trapezoidal, Gaussian curve, generalised bell function, sigmoidal function and polynomial curves.

A singleton membership function is defined by one parameter only (equation 3.5). This model can be used to translate the precise crisp input for fuzzification or to represent the inference solution (Hines, 1997). When variable x is a, its membership $\mu(x)$ takes value 1 (Fig.3.2).

$$\mu(x) = \begin{cases} 1, x = a \\ 0, x \neq a \end{cases} \tag{3.5}$$

Figure 3.2 Singleton function

A triangle membership function is specified by three parameters {a, b, c} (Fig. 3.3) defined by:

$$\mu(x)=\begin{cases}0, & x<a\\ (x-a)/(b-a), & a<=x<=b\\ (c-x)/(c-b), & b<=x<=c\\ 0, & x>c\end{cases} \tag{3.6}$$

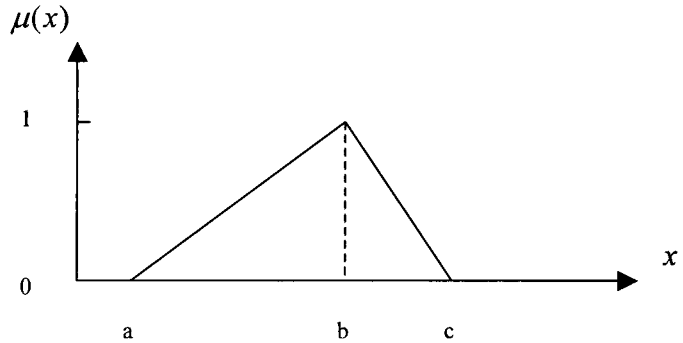


Figure 3.3 Triangle membership function

The triangular model is a simple membership function, where the precise appearance of the function is determined by the choice of parameters a, b, and c. Triangular membership functions have been frequently used in many applications of fuzzy sets including fuzzy controller, fuzzy models and classification schemes (Pedrycz, 1994). The triangular forms also can be used in the business area, if the business models are simpler to specify and easier to visualise.

The trapezoidal function is used in many business and public policy applications (Cox, 1999). The trapezoidal curve depends on four parameters a, b, c and d (Fig. 3.4) defined by:

$$\mu(x)=\begin{cases} 0, & x \le a \\ (x-a)/(b-a), & a \le x \le b \\ 1, & b \le x \le c \\ (d-x)/(d-c), & c \le x \le d \\ 0, & d \le x \end{cases} \tag{3.7}$$

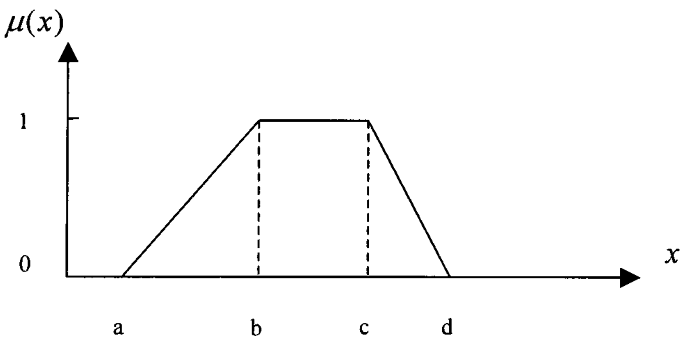


Figure 3.4 Trapezoidal membership function

Gaussian curves and generalised bell membership function have their characteristics of smoothness and concise notation, they are popular methods for specifying fuzzy sets in financial, marketing, transportation and other business models (Cox, 1999). Example curves are shown in figure 3.5 and figure 3.6. The Gaussian curve or exponential curve depends on two parameters σ and c that indicate the width and centre of the curve (Fig.3.5). The equation representing the Gaussian curve membership function is given by:

$$\mu(x) = e^{\frac{-(x-c)^2}{2\sigma^2}} \qquad (3.8)$$

The generalised bell function depends on three parameters a, b and c. Parameter a represents the half width of the curve at the inflection point, value b describes the slope of the curve and value c locates the centre of the curve. The equation representing the Generalised bell curve membership function is given by:

$$\mu(x) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}} \qquad (3.9)$$

Figure 3.5 Gaussian curve membership function

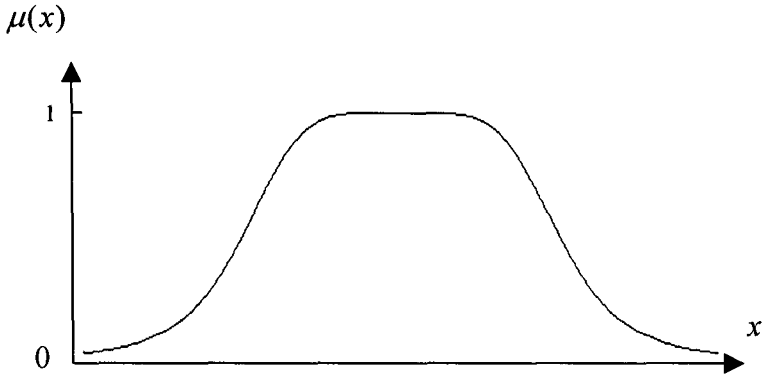


Figure 3.6 Generalised bell curve membership function

Similar to the Gaussian curve and generalised bell curve, the sigmoid curves can be used to indicate the increasing and decreasing of nonlinear surfaces. Sigmoid curves are able to specify asymmetric and closed (i.e. not open to the left or right) membership functions or to combine two sigmoidal functions to form a special model (figure 3.7). The sigmoid function depends on parameters a and c (or a_1 , a_2 and c_1, c_2 for the combined special model) (equation 3.10). Parameter a represents the slope of the curve and parameter c describes the half width of the curve at the inflection point.

$$\mu(x) = \frac{1}{1+e^{-a(x-c)}} \tag{3.10}$$

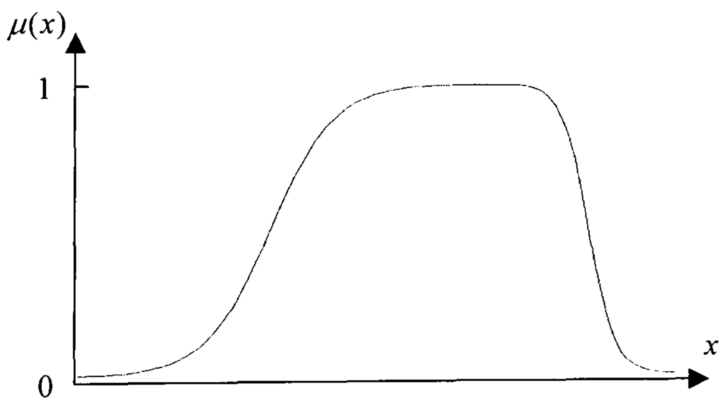


Figure 3.7 Combined two sigmoid curves membership function

3.2.3 *Fuzzy set operations*

In more general terms, the most commonly used fuzzy set operations are defined as Intersection (AND), Union (OR) and Complement (NOT). Their calculations are performed based on the membership functions. For example, consider two fuzzy sets A and B on the universe X, for a given element x of the universe, the basic operations are defined as:

Intersection:

$$\mu_{A\cap B}(x) = \mu_A(x)\ \hat{t}\ \mu_B(x) \tag{3.11}$$

where $\hat{t}$ is the notation of the triangular norm or t-norm, which is a fuzzy conjunction operator (Yen and Langari, 1999). The Minimum and Algebraic Product are most commonly used to calculate the t-norm for a fuzzy intersection (Reznik, 1997, Jager, 1995):

$$\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x) = \min[\mu_A(x), \mu_B(x)] \tag{3.12}$$

or

$$\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x) = \mu_A(x)\mu_B(x) \tag{3.13}$$

Union:

$$\mu_{A\cup B}(x) = \mu_A(x)\,\hat{s}\,\mu_B(x) \tag{3.14}$$

where $\hat{s}$ is the notation of the triangular conorm or s-norm, which is a fuzzy disjunction operator (Yen and Langari, 1999). The Maximum is suggested and most commonly used to calculate the s-norm for a fuzzy union (Zadeh, 1965):

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)) \qquad (3.15)$$

Complement:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \qquad (3.16)$$

3.2.4 *Fuzzy relations*

A further important notion for application purposes is the specifying of fuzzy relations. A fuzzy relation represents the approximate degree of membership between two (binary relation) or more objects. A fuzzy relation can be viewed as a fuzzy set. Formally, the fuzzy relation R between variables x *and* y, whose domains are X and Y, respectively, is defined by a function that maps ordered pairs in $X\times Y$ to their degree in the relation, which is a number between 0 and 1, i.e., R: $X\times Y\rightarrow$ [0,1]. The strength of the mapping is expressed by the membership function of the relation $\mu_R(x,y)$ for ordered pairs from the two universes. More generally, an n-dimension relation R in $x_1, x_{2,} ..., x_n$, whose domains are $X_1, X_{2,} ..., X_n$, respectively, is defined by a function that maps an n-tuple

$< x_1, x_2, ..., x_n >$ in $X_1, X_2, ..., X_n$ to a number in the interval, i.e., *R:* $X_1 \times X_2 \times ... \times X_n \rightarrow$ [0,1] (Yen and Langari, 1999).

The 2-dimensional relation *R* which is used in fuzzy implication is represented in equation 3.17.

$$R = A \times B = \begin{bmatrix} \mu_R(x_1, y_1) & \mu_R(x_1, y_2) & \dots & \mu_R(x_1, y_m) \\ \mu_R(x_2, y_1) & \mu_R(x_2, y_2) & \dots & \mu_R(x_2, y_m) \\ \dots & \dots & \dots & \dots \\ \mu_R(x_n, y_1) & \mu_R(x_n, y_2) & \dots & \mu_R(x_n, y_m) \end{bmatrix} \tag{3.17}$$

where *A* and *B* are fuzzy sets that are defined respectively on the finite universes *X* and *Y* of *n* discrete elements, $X = \{x_1, x_2, ..., x_n\}$ and $Y = \{y_1, y_2, ..., y_n\}$. In other words, relation *R* is defined on the Cartesian space $X \times Y$.

3.2.5 Operations and composition of fuzzy relations

As discussed in section 3.2.4, the fuzzy relation is a fuzzy set that is defined on the Cartesian space $X \times Y$, and the general fuzzy set calculation can be used for the relation. The composition is an important operator based on fuzzy relation calculations.

Suppose R is a fuzzy relation on the Cartesian space $X \times Y$, S is a fuzzy relation on $Y \times Z$ and T is a fuzzy relation on $X \times Z$, then T can be noted by composition (equation 3.18) which is marked by the symbol $\circ$.

$$T = R \circ S \tag{3.18}$$

and its membership can be calculated by:

$$\mu_T(x,z) = \underset{y \in Y}{\vee} (\mu_R(x,y) * \mu_S(y,z)) \tag{3.19}$$

where $\vee$ is the union symbol, and $*$ is a calculation symbol. Commonly, *max* is used for union (section 3.2.3), and "$*$" is frequently defined as intersection or product calculations. Therefore, the max-min composition and max-product composition are formed and described by equation 3.20 and equation 3.21.

1. Max-min composition

$$\mu_T(x,z) = \underset{y \in Y}{\vee} (\mu_R(x,y) \wedge \mu_S(y,z)) = \max_y(\min \mu_R(x,y), \mu_S(y,z)) \tag{3.20}$$

The max-min composition is the most frequently used composition method (Sun, 1997).

2. Max-product composition

$$\mu_T(x,z) = \underset{y \in Y}{\vee} (\mu_R(x,y) \bullet \mu_S(y,z)) = \max_y(\mu_R(x,y)\mu_S(y,z)) \tag{3.21}$$

3.3 Fuzzy logic and fuzzy inference

3.3.1 Fuzzy logic

Fuzzy Logic is a method of common sense or inference based on natural language (Gulley and Jang, 1995). It generalises the notion of truth values from classical logic (i.e., true or false) to a matter of degree. A true value is a number between 0 (false) and 1 (true) that represents a partially true statement. The ultimate goal of fuzzy logic is to form reasonable inferences even though the condition of an implication rule is partially satisfied. This capability is referred to as approximate reasoning. It is analogous to predicate logic for reasoning with precise propositions, and hence is an extension of classical propositional calculus that deals with partial truths (Ross, 1995).

3.3.2 Approximate reasoning

The process of approximate reasoning can be formulated as a compositional rule of inference which is included in the Generalised Modus Ponens (GMP) as a special case. This process is referred to as the fuzzy modus ponens to reflect the characteristic features of vagueness and non-uniqueness of fuzzy premises (Zadeh, 1975).

GMP is generalisation of the method of modus ponens which is frequently used in control engineering (Sun, 1997): when state a is known to be true and a rule which is "if a then b" exists, it is valid to deduce that b is true:

Rule: If x is A Then y is B

Known: x is A'

Reasoning conclusion: y is B'

where x and y are linguistic variables and A, B, A' and B' are fuzzy sets. The fuzzy sets A, B and A' are known but B' can be deduced by compositional rule of fuzzy inference (Chang et al. 1991). There are two types of models commonly used in the approximate reasoning, Mamdani (Mamdani, 1974) and Takagi-Sugeno (Sanchez and Gupta, 1983). In the Mamdani model, the reasoning conclusion is a fuzzy set, and in the Takagi-Sugeno model, the reasoning conclusion is a linear function of the input. The Takagi-Sugeno model is described in section 3.4.1.

Approximate reasoning is achieved in fuzzy logic by two related techniques. One is the meaning of a fuzzy implication rule using a fuzzy relation, and the other is that of obtaining an inferred conclusion by applying the compositional rule of inference to the fuzzy implication relation.

3.3.2.1 Implication

The implication rule is an important concept. It describes a generalised logic implication relationship between two logic formulas involving linguistic variables and imprecise linguistic terms (Yen and Langari, 1999). An implication rule has two parts: The if-part of the rule " x is A" is called the premise or antecedent and the then-part of the rule "y is B" is called the conclusion or consequent. The rule of "If x is A then y is B" represents the fuzzy implication relation between subsets A and B (on the universes of discourse X and Y respectively), and $A \rightarrow B$ as an expression of that value of A implies the value B in a rule base.

There are many methods for realisation of the fuzzy implication. For example, Zadeh's arithmetic fuzzy implication, Zadeh's maximum fuzzy implication, Standard sequence fuzzy implication, Godelian sequence fuzzy implication, Goguen's fuzzy implication, Mamdani fuzzy implication and Larsen fuzzy implication. For applications in industrial processes, the Mamdani fuzzy implication and the Larsen fuzzy implication are frequently used for fuzzy controller applications (Sun, 1997).

1. Mamdani implication

$$\begin{cases} R_C = A \rightarrow B = A \times B = \int_{X \times Y} \mu_A(x) \wedge \mu_B(y)/(x,y) \\ \mu_{R_C}(x,y) = \mu_A(x) \wedge \mu_B(y) \end{cases} \tag{3.22}$$

or

$$t(x \, is \, A \rightarrow y \, is \, B) = \mu_A(x) \wedge \mu_B(y) \tag{3.23}$$

2. Larsen implication

$$\begin{cases} R_P = A \to B = A \times B = \int\limits_{X \times B} \mu_A(x)\mu_B(y)/(x,y) \\ \mu_{R_P}(x,y) = \mu_A(x)\mu_B(y) \end{cases} \tag{3.24}$$

or

$$t(x \, is \, A \to y \, is \, B) = \mu_A(x)\mu_B(y) \tag{3.25}$$

3.3.2.2 The calculation of the conclusion

The fuzzy conclusion is calculated from inputs and the implication or rule base. For example, the conclusion or output B' can be calculated from the premise or input A' and a condition of the fuzzy implication relation or rule base $A \to B$:

$$B' = A' \circ (A \to B) = A' \circ R \tag{3.26}$$

In equation 3.26, if the implication $R = A \to B$ is calculated by R_C, and the *max-min* method is applied for composition represented by "$\circ$", the conclusion $\mu_{B'}(y)$ is given by equation 3.27.

$$\begin{aligned}\mu_{B'}(y) &= \mu_{A'}(x) \circ [\mu_A(x) \to \mu_B(y)] \\ &= \max_x[\min(\mu_{A'}(x), \mu_R(x,y))]\end{aligned} \tag{3.27}$$

3.3.3 *Other related topics*

3.3.3.1 Connective "and"

In the fuzzy implication or inference, the connective *"and"* is frequently used to connect more than one fuzzy proposition. For example, for two inputs and one output, "if x is A and y is B, then z is C". The antecedents "if x is A and y is B" can be viewed a Cartesian product of fuzzy subset $A \times B$ defined on Cartesian space $X \times Y$. If the *min* operator is used for the intersection, their membership function is given by equation 3.28.

$$\mu_{A\times B}(x,y) = \min[\mu_A(x), \mu_B(y)] \tag{3.28}$$

where the fuzzy implication relation can be described by $R = A \times B \to C$, and the conclusion or output C' can be calculated from inference:

$$C' = (A' \times B') \circ R = (A' \times B') \circ [(A \times B) \to C] \tag{3.29}$$

where premises or inputs are noted by A' and B'.

The membership function of output C' is represented by equation 3.30.

$$\mu_{C'}(z) = [\min(\mu_{A'}(x), \mu_{B'}(y)] \circ [\min(\mu_A(x), \mu_B(y)) \rightarrow \mu_C\ (z)] \tag{3.30}$$

3.3.3.2 *Several rules*

In fuzzy logic control, a fuzzy rule base usually contains several rules. If every rule has the same structure: "R_j: if x is A_j and y is B_j, then z is C_j" $(j = 1,2,...,m)$, where m is the number of rules. The total fuzzy implication relation can be determined from equation 3.31 (Sun, 1997):

$$R = \bigcup_{j=1}^{m} R_j \tag{3.31}$$

Therefore, the inference results or conclusion and its membership function for the *j*th rule is given by equation 3.32 and equation 3.33 (see also Appendix B.2).

$$\begin{aligned} C_j' &= (A' \times B') \circ R_j \\ &= (A' \times B') \circ (A_j \times B_j \rightarrow C_j) \\ &= [A' \circ (A_j \rightarrow C_j)] \bigcap [B' \circ (B_j \rightarrow C_j)] \end{aligned} \tag{3.32}$$

$$\begin{aligned} \mu_{C_j'}(z) &= [\min(\mu_{A'}(x), \mu_{B'}(y)] \circ [\min(\mu_{A_j}(x), \mu_{B_j}(y)) \rightarrow \mu_{C_j}(z)] \\ &= \min\{[\mu_{A'}(x) \circ (\mu_{A_j}(x) \rightarrow \mu_{C_j}(z))], [\mu_{B'}(y) \circ (\mu_{B_j}(y) \rightarrow \mu_{C_j}(z))]\} \end{aligned} \tag{3.33}$$

The total conclusion C' and its membership function are given by equation 3.34 and equation 3.35 (see also Appendix B.1).

$$C' = (A' \times B') \circ R = (A' \times B') \circ \bigcup_{j=1}^{m} R_j = \bigcup_{j=1}^{m} (A' \times B') \circ R_j = \bigcup_{j=1}^{m} C'_j \tag{3.34}$$

$$\mu_{C'}(z) = \max\{[(\mu_{A'}(x) and \mu_{B'}(y)) \circ \mu_{R_1}(x,y,z)], \ldots [(\mu_{A'}(x) and \mu_{B'}(y)) \circ \mu_{R_m}(x,y,z)]\} \tag{3.35}$$

These equations are used for the design of the Fuzzy-SPC control system described in chapter 4.

3.3.3.3 *Matching degree*

The inference results for equation 3.33 also can be described by equation 3.36 or 3.37 (Sun, 1997):

If R_C is used for the fuzzy implication,

$$\mu_{C'_j}(z) = \alpha_j \wedge \mu_{C_j}(z) \tag{3.36}$$

or, if R_P is used for the fuzzy implication,

$$\mu_{C'_j}(z) = \alpha_j \mu_{C_j}(z) \tag{3.37}$$

where (see also Appendix B.3 and B.4)

$$\alpha_j = [\max_x(\mu_{A'}(x) \wedge \mu_{A_j}(x))] \wedge [\max_y(\mu_{B'}(y) \wedge \mu_{B_j}(y))] \tag{3.38}$$

α_j can be viewed as a weight or can be called a matching degree of the j^{th} rule (Sun, 1997), (Yen and Langary, 1999). The matching degree is an important notion in the NN-Fuzzy model, which is discussed in chapter 6.

If the antecedent (input) fuzzy sets are determined as singletons, equations 3.36~3.38 can be written as:

$$R_C : \mu_{C'_j}(z) = \alpha_j \wedge \mu_{C_j}(z) \tag{3.39}$$

$$R_P : \mu_{C'_j}(z) = \alpha_j \mu_{C_j}(z) \tag{3.40}$$

where

$$\alpha_j = \mu_{A_j}(x) \wedge \mu_{B_j}(y) \tag{3.41}$$

3.4 Basic theory of fuzzy control

The block diagram shown in figure 3.8 depicts a basic fuzzy logic controller.

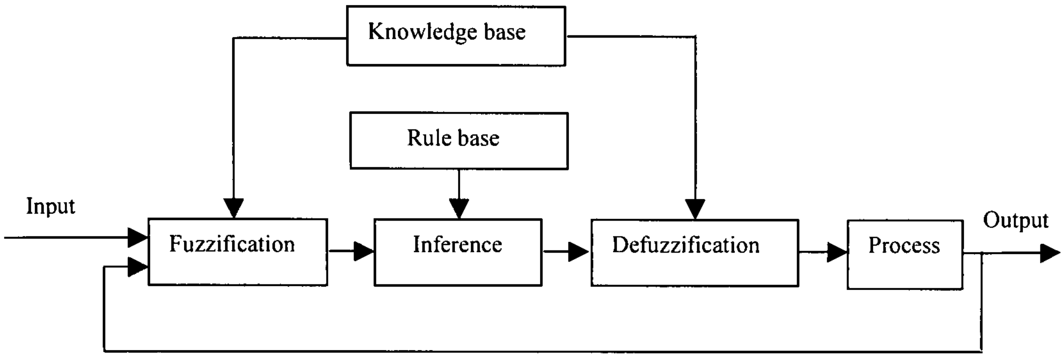


Figure 3.8 A simple fuzzy logic controller

3.4.1 Knowledge base and rule base

A knowledge base contains all the input and output fuzzy partitions that includes the linguistic terms, corresponding input / output variables, and membership functions. In the partition, every input / output linguistic variable (e.g. season) is assigned by variable value or a group of fuzzy linguistic names (i.e. spring, summer, autumn and winter), and every linguistic name corresponds to their fuzzy subset in the same universe of discourse. The number of partition (i.e. the same as the number of fuzzy subsets) is determined by experience (Sun, 1997). If too small a number is chosen, the control accuracy will decrease, if too large a number is chosen, the number of rules will be increased and the performance speed is decreased. In the knowledge base, the membership functions are established by form type (for discrete universe) or function type (for continuous universe).

The rule base contains all the if-then rules based on fuzzy linguistic variables. In this block, how many input (antecedent) and output (consequent) variables are chosen, what and how many fuzzy control rules are used. Normally they are determined by designer's knowledge, experience and specific control objectives. In fuzzy control, the control rules can be divided into two types: Mamdani fuzzy rules and Takagi and Sugeno (T-S) fuzzy rules.

As discussed in section 3.3, Mamdani rule was used in the first reported application by Mamdani in 1974 (Mamdani, 1974). This rule is that in the absence of an explicit plant model and / or clear statement of control design objectives, informal knowledge of the operation of the given plant can be codified in terms of "if-then", or condition – action, rules and form the basis of a linguistic control strategy (Yen and Langari, 1999). In the Mamdani rule, as mentioned previously in section 3.3.2, the antecedent is a fuzzy subset for the input, and the consequent is a fuzzy subset for the output.

The T-S rule was introduced by Takagi and Sugeno in 1983 (Takagi and Sugeno, 1983). In the T-S rule, the antecedent is same as in the Mamdani rule but the consequent of the fuzzy rules are linear function of the controller inputs. For example, if x is A and y is B, then

$$z = p_0 + p_1 x + p_2 y \tag{3.42}$$

where p_0, p_1 and p_2 are constants.

In general, for multiple input and single output (MISO) systems, suppose the input vector is $\boldsymbol{x}$ and linguistic variable set is $\boldsymbol{T}$:

Then:

$$\boldsymbol{x} = [x_1 x_2 \cdots x_n] \tag{3.43}$$

$$\boldsymbol{T}\ (x_i) = [A_i^1, A_i^2, \cdots A_i^{m_i}] \quad (i = 1,2,\cdots,n) \tag{3.44}$$

Where A_i^j $(j = 1,2,\cdots m_i)$ is j^{th} linguistic variable value for x_i, m_i is the number of A_i^j for x_i and n is the number of input variables.

T-S rules can be described by:

If x_1 is A_1^j and x_2 is A_2^j ... x_n is A_n^j, then

$$z_j = f_j(x_1, x_2, \ldots x_n) = p_{j0} + p_{j1}x_1 + \cdots + p_{jn}x_n, \quad (j = 1,2,\ldots,m) \tag{3.45}$$

where m is the number of A_i^j for $\boldsymbol{x}$ or the number of total rules.

The Mamdani rule is good for capturing the expertise of a human operator, but it is awkward to use to design a working controller if the plant model is known. The T-S rule is good for embedding linear controller and continuous switching between these output equations (see section 3.4.3). This becomes very effective when the plant model is known. Also an adaptive capability and mathematical tractability make this type of fuzzy controller a primary choice for nonlinear and / or adaptive control design (Reznik, 1997). When constant consequent are chosen in the T-S rule, it becomes similar to the Mamdani rule.

3.4.2 Fuzzification

Fuzzification is the translation from the numerical input to the fuzzy input (Jager, 1995). Before this procedure is carried out, the sample data should be converted to the input data points (or data point) which are distributed in the universe of discourse.

3.4.2.1 Conversion

Normally real data sampled from different processes can have different values and can be distributed in different ways. They have to be converted to the data points distributed in the universe of discourse for the next operation. If the sample data is x' and $x'_{\min} \le x' \le x'_{\max}$, the universe is $[x_{\min}, x_{\max}]$, and data point x_0 ($x_{\min} \le x_0 \le x_{\max}$) is converted from x', where x_0 will be fuzzified (see section 3.4.2.2). The conversion equations are given by equation 3.46 and equation 3.47 (Sun, 1997).

$$x_0 = \frac{x_{\min} + x_{\max}}{2} + k(x' - \frac{x'_{\min} + x'_{\max}}{2}) \tag{3.46}$$

where

$$k = \frac{x_{\max} - x_{\min}}{x'_{\max} - x'_{\min}} \tag{3.47}$$

If the universe is [-1,1] or [0,1] this conversion can be called normalisation (Ross, 1995).

3.4.2.2 *Fuzzification*

Fuzzification is the process of making a crisp quantity fuzzy. In this process, the data points that are converted from the sample data signals or input observations are translated to suitable linguistic terms, which are defined by the fuzzy set (Yan et al, 1994). That is, the data points are mapped to fuzzy sets on the universe of discourse in order to be used for the fuzzy approximate reasoning calculation (section 3.3.2). Normally there are two possible choices for this mapping (Wang, 1994).

1. Singleton fuzzy set

If the data point x_0 is crisp and precise (Ross, 1995), it can be fuzzified to a singleton fuzzy set A', where its membership function is $\mu_{A'}(x)$ that is described by equation 3.5.

2. Nonsingleton fuzzy sets

When the observations are subject to experimental error, the data points become imprecise or inaccurate (Jager, 1995). That is, $\mu_{A'}(x)$ decreases from 1 as x moves away from x_0 (Wang, 1994). In this situation, data points can be fuzzified to a triangular (equation 3.6) or bell shaped (equation 3.8) fuzzy sets (Sun, 1997). Triangular sets have the advantage of simpler and faster calculation.

3.4.3 Defuzzification and crisp output

The defuzzification process is to produce a crisp output for the fuzzy output, and is used in the Mamdani fuzzy controller only. There are several methods for defuzzification: Centre-of-area/gravity defuzzification, centre-of-largest-area defuzzification, mean of maxima, first-of-maxima defuzzification, middle-of-maxima defuzzification and height defuzzification. The specific defuzzification method can be chosen depending on the context or control problem (Ross, 1995). In process control, the centre-of-area/gravity (COA) is the most popular defuzzification technique, since it takes into account the entire possibility distribution in calculating its representative points (Yen and Langari, 1999):

In a continuous universe of discourse,

$$z_0 = COA(C') = \frac{\int \mu_{C'}(z) z dz}{\int \mu_{C'}(z) dz} \tag{3.48}$$

In a discrete universe of discourse,

$$z_0 = COA(C') = \frac{\sum_{j=1}^{m} \mu_{C'}(z_j) \times z_j}{\sum_{j=1}^{m} \mu_{C'}(z_j)} \tag{3.49}$$

where z_0 is the crisp output, z_j is an element of output fuzzy set or (output) linguistic variable, C' is a fuzzy set conclusion and m is the number of z_j or rules.

The T-S model is different from the Mamdani model. For every if-then rule, the output $z_j = f_j$ (equation 3.45) has a crisp value, and the total crisp output of the inference system is a weighted average, which is described by equation 3.50.

$$z_0 = \frac{\sum_{j=1}^{m} \alpha_j f_j(x_1, x_{2,\dots} x_n)}{\sum_{j=1}^{m} \alpha_j} \tag{3.50}$$

Where α_j is the matching degree of the j^{th} rule, the m is the number of total rules, and n is the number of inputs.

3.5 Research background and related to research work

Fuzzy set theory and fuzzy logic have been successfully applied to many fields. Many papers have been published in pattern recognition, image processing, data analysis, and design of fuzzy control system. For example, in pattern recognition, Boutleux and

Dubuisson used the directional membership functions to diagnose the state associated with data evolution between known functional models (Boutleux and Dubuisson, 1999). In image processing, Jing et al proposed an optical system based on polarisation and area-coded scheme for the fuzzy image process. All of the fuzzy logical functions of two images can be implemented in parallel, and the system can exhibit a high operation speed and a large information throughput (Jing et al, 1998). Hofmeister et al proposed dynamic fuzzy data analysis to cluster objects (i.e. scenarios) which are represented by trajectories over time, in order to reduce complexity by extracting a small set of typical scenarios out of a large set of possible scenarios (Hofmeister et al, 2000). Calcev and Chai et al developed fuzzy controllers for non-linear systems, where the absolute stability conditions of the system are obtained (Calcev, 1998) (Chai et al, 1998). Some related papers on the background theories of fuzzy set and fuzzy logic have been introduced in the previous sections. How they are linked to the research objectives are reviewed in the next section.

3.5.1 Fuzzy set and operation

Much research work has been carried out to explore the use of fuzzy set theory to build a measurement or identification system with linguistic data. Chun, M.H. and Ahn, K.I. demonstrate a potential use of fuzzy set theory and provide its formal procedure in the quantification of the uncertainties of accident progression event trees for a nuclear power plant system (Chun and Ahn, 1992). First, the fuzzy set theory is applied to the simple portion of a given accident progression event tree with typical imprecise and uncertain type of input data. Suitable computational algorithms are then identified. Secondly, the merits of the fuzzy set theory model in the real application are described.

Kahraman et al identified that fuzzy logic theory has the capability to represent vague data and allow mathematical operators and programming to be applied to the fuzzy domain. Fuzzy set theory is primarily concerned with quantifying the vagueness in human thoughts and perceptions. They employed fuzzy set theory in the area of discounted cash flow techniques for justifying manufacturing technologies, which are based on the data uncertainty or risk. The vague data such as interest rate and cash flow are used in the discounted cash flow techniques, and the fuzzy benefit-cost (FBC) ratio method is used to justify the manufacturing technologies. The justified results describe that the FBC method is more efficient in the justification of manufacturing technologies than the traditional approach (Kahraman et al, 2000).

A novel method using a fuzzy practicable interval to characterise non-random error in dynamic measurement has been proposed by Xia, and his colleagues (Xia et al, 2000). The method permits an estimate of the error under the conditions that the number of measurements is very small and the probability distribution is unknown. The feasibility of the method is validated by simulation experiments.

An intelligent method was proposed by Wang and Raz (Wang and Raz, 1998) in which control charts are constructed using linguistic data suitable for situations where quality characteristics can not be measured numerically. The centre line and control limits were transferred to the fuzzy subsets associated with the linguistic data. The linguistic approach was applied to p-charts, and was verified using results obtained from simulated data. The results suggested that control chart based on linguistic data are significantly more sensitive to process shifts than are conventional p charts.

Chang and Aw proposed a fuzzy set scheme in a neural fuzzy control chart, which is applied to detecting and classifying process mean shift. In this neural fuzzy control chart, a neural network (more discussions of neural net are presented in chapter 6) is designed to detect the "out-of-control" situation and the fuzzy set is used to identify the process status based on the neural network (NN) outputs. That is, fuzzy sets can identify and decide which target the neural network output is targeting when the NN output falls into the overlapping interval (Chang and Aw, 1996).

Each of papers, which are reviewed above, employs fuzzy set theory to represent uncertain or vague events. Fuzzy set theory is used to provide quantification of the uncertainties or imprecise types of input data, human thoughts and perceptions or quality characteristics. These approaches cause significant raising of measurement or diagnosis accuracy. In this thesis, fuzzy set theory is also employed to represent vagueness but in a different application. This approach provides a mapping method in quantification level for uncertain relation between SPC zone rules and control actions (more details in section 3.6).

3.5.2 The design and tuning of membership functions and if-then rules

An overview of membership function generation techniques for pattern recognition has been discussed by Medasani and his colleagues. They provided seven methods which are heuristics, probability to possibility transformations, histograms, nearest neighbour techniques, neural networks, clustering and mixture decomposition. They considered that there is no single best method, and the choice of the method depends on the particular problem. In many decision-making application, membership functions of fuzzy sets are based on subjective perceptions of vague or imprecise categories rather than on data or other objective entities involved in the given problem. The problem of assigning numbers to subjective perceptions of vague categories is a matter of mathematical psychology and requires the utilisation of various techniques of the theory of measurement and scaling (Medasani et al, 1998).

Lotfi and Tsoi provided an adaptive membership function scheme for general additive fuzzy systems. The proposed scheme can adapt a proper membership function for any non-linear input-output mapping, based upon a minimum number of rules and an initial approximate membership function. This parameter adjustment procedure is performed by computing the error between the actual and the desired decision surface (Lotfi and Tsoi, 1996)

Wong and Chen proposed a fuzzy controller design using if-then rules adjustment. The idea is that an adaptive law is used to indirectly regulate the output fuzzy variables in the fuzzy controller according to the pre-labelled if-then rules. The determination of if-then rule selection is formed by an analytical parameter equation with multiple input variables of the fuzzy controller. In the adjustment procedure, the if-then rules are first labelled, and then an adaptive law is used to tune the injected parameters in the IF-part to appropriately determine each output fuzzy variable under the situation of lack of any expert knowledge of the plant (Wong and Chen, 1999).

Three typical application papers, which are introduced above, involve the generation of membership function, adaptive membership function and tuning of if-then rules. In this research work, the subjective perception and probability-possibility methods are used to build antecedent and consequent membership functions (chapter 4). Both adaptive membership functions and if-then rules methods can increase the fuzzy control accuracy. The adaptive membership function method is chosen as it has the advantage of visibility, which is convenient to analyse.

3.5.3 Fuzzy logic control

Since the invention of the first fuzzy controller which was published in 1974 (Mamdani, 1974), quantities of control applications have been achieved in many areas. In most cases fuzzy control can be viewed as an interpolation of a partially specified control function in a vague environment, which reflects the indistinguishability of measurements or control values. Klawonn shows that equality relations turn out to be the natural way to represent such vague environments and develops suitable interpolation methods to obtain a control function. The Mamdani model and triangular membership functions are applied in this method, which is successfully used in a case study of engine idle speed control for the Volkswagen Golf GTI (Klawonn et al, 1995).

Bioprocesses have been operated according to the judgement of experts who are skilled operators and have long experience. These experiences can be described by the linguistic if-then rules. Fuzzy inference is one of the powerful tools to incorporate linguistic rules into computation algorithms for application to process control. Honda and Kobayashi categorised fuzzy control into two types of bioprocesses. The first one is direct fuzzy control of process variables such as sugar feed rate in fed-batch culture and broth temperature in batch operation. The second one is indirect control: the phase recognition is first done by fuzzy inference using process variables such as sugar concentration, pH and so on and then the control strategies constructed in each phase are used for the process operation (Honda and Kobayashi, 2000).

Bremner described an application of a model-predictive controller, based around a fuzzy relational model to a grain dryer, which has large non-linear disturbances in a whisky distillery. The dryer process data are sampled and identified in order to construct the fuzzy relational model. The results of on-line controller trials show that good control performance is achieved, especially when compared with the previous manually controlled system (Bremner and Postlethwaite, 1997).

Ying analysed a simple fuzzy controller with two inputs (error and rate of change of error), linear and non-linear defuzzification algorithms. This fuzzy controller is precisely equivalent to a conventional linear PI (proportional-integral) controller if a linear defuzzification algorithm is used. The simulation shows that the performance of the fuzzy controller is almost same as the PI controller when the first-order and second-order linear processes are selected. More importantly, the simulation result illustrates that the fuzzy controller is stable when a non-linear process model is controlled (Ying et al, 1990).

The importance of improving product quality at continuous hot-dip galvanising lines steadily grows. Wagner described the revision of a conventional non-adaptive control strategy towards a modern solution using methods of computational intelligence. The already existing feedforward control is complemented by a neural process model and a neural-fuzzy controller replaces the previously used conventional process controller. The neural process model is optional and is used for model-based control so that the process inherent measurement dead time is avoided. The new control arrangement is adaptive, and guarantees a more constant coating (Wagner and Kochs, 1998).

3.6 Conclusion and overview of the research work in fuzzy logic

Fuzzy set theory and fuzzy logic are emerging as very powerful techniques for application where uncertainty is prominent. This is largely due to a wide array of successful applications ranging from consumer products, to industrial process measurement and control, to data analysis, pattern recognition and quality control areas. Compared to classical set theory, fuzzy sets can express the vague concepts by its membership functions. This has the advantage of enabling the many linguistic variables based on natural world and human thought to be operated in a quantified way, using mathematical methods. Fuzzy inference based on fuzzy logic operations is a central element of a fuzzy control system. This basic structure can be designed through various types of input/output variable numbers, membership function shapes and if-then rules for mapping the relationship between inputs and outputs in a complex system.

The fuzzy set theory and fuzzy logic control are applied in this research work as it can be used to achieve the control of processes with uncertain and vague relationships between inputs and outputs, as previously discussed. The control chart patterns, which are discussed in chapter 2, are interpreted in quantified expression by fuzzy sets and the related control actions are generated by fuzzy approximate reasoning in the fuzzy inference system which is designed in chapter 4.

The Mamdani model is used for fuzzy-SPC inference, which is described in chapter 4. The numerical expression can be generated for SPC $\overline{X}$ control chart pattern identification (chapters 4, 5 and 6) and R control chart (chapter 6) by fuzzy set theory, triangular and trapezoidal membership functions, which are frequently applied in process control, classification and random schemes (Pedrycz, 1994) , (Sun, 1997), but in a different way to previous work which concentrated on p-chart interpretation (Raz and Wang, 1990) and fuzzy classifier only (Chang and Aw, 1996). In the approximate reasoning, the antecedent part, the numerical expression (membership function) of control chart pattern or SPC zones (fuzzy set) and Mamdani implication (section 3.3.2) will be used to calculate the consequent part by max-min composition. This is because Mamdani implication and max-min composition satisfy the requirements of GMP and are easy to compute. Connective "and" (section 3.3.3.1) and union calculations (section 3.3.3.2) are also used for multiple input and several if-then rules applied in the Fuzzy-SPC system, in order to obtain the crisp output as the numerical control action.

The Takagi-Sugeno (T-S) model is also applied in this research as it has the adaptive capability and computational tractability (Reznik,1997). The application is explained in chapter 6.

3.7 Summary

This chapter introduced some basic concepts of fuzzy set and fuzzy logic control such as the fuzzy set, membership function and related operators. The membership function characterises the fuzziness of variables in a fuzzy set. Fuzzy relation and composition are also introduced, as they are very important operators in approximate reasoning. Fuzzy approximate reasoning is the central part in fuzzy inference. This reasoning process involves fuzzy implication which uses the fuzzy relation operator to build the general logic implication and inferring conclusion, which is calculated by the composition operator. Finally the features of this research work and the general structure of a fuzzy control system are explained.

In this chapter, fuzzy set theory, fuzzy logic and design methods as the general notions are briefly introduced. In chapter 4, based on knowledge and conclusions of chapter 2 and chapter 3, a specific fuzzy system is designed for SPC zone rules and SPC feedback control.

Chapter 4 Design of Fuzzy-SPC control system

4.1 Introduction

Fuzzy logic is a technique, which represents the uncertainty of events based on certain mathematical computation. In this chapter, aspects of the Fuzzy-SPC system are described such as Fuzzification, membership functions, fuzzy reasoning and defuzzification. This is designed in two ways: manual calculations in mathematics and a computer aided design (CAD) using the Fuzzy Logic Toolbox in MATLAB. The related design results are exactly same.

Since the subject of fuzzy set is viewed as an approach to the mathematical representation of everyday language (Zadeh, 1965), (Dombi, 1990), it is frequently applied in many area such as control or recognition which possess vague or uncertain relationships that can be represented by natural language between inputs and outputs. For an utilisation of this characteristic of fuzzy sets, this chapter applies fuzzy sets methodology to express the mathematical mapping based on the vague and uncertain relationship between the SPC zone rules and a suitable control action.

In this chapter, section 4.2 describes the manual design of a Fuzzy-SPC control system. Through this manual design procedure, it is convenient to disclose the design details for a fuzzy system. The input variable conversion and input/output membership functions are discussed in section 4.2.1. Section 4.2.2 describes the if-then rules and simplification preconditions. Section 4.2.3 refers to the defuzzification approach. The calculation of fuzzy inference and an illustrative example given in section 4.2.4. Section 4.3 describes a further design in the MATLAB fuzzy toolbox. Three editors which are used for the system design, membership functions design and if-then rules design are described in section 4.3.1~4.3.3 respectively. The fuzzy inference solutions or outputs are obtained in the MATLAB Rule Viewer, which is described in section 4.3.4. The inference solutions are used to build a fuzzy control table. Section 4.3.5 shows the I/O surface in three dimensions as a graphical expression of fuzzy inferences (for SPC zone rule 2), which are designed both by manual calculation and by using MATLAB CAD.

4.2 Design of Fuzzy-SPC control in mathematics

The fuzzy logic method is employed in conjunction with SPC to develop a new method to represent the characteristic of a product, to analyse the behaviour and tendency of the process, and to generate the output instructions to the operator, engineer or to an automatic control system where appropriate. This provides an improvement on existing techniques of evaluating abnormal behaviours using zone rules by providing a numeric evaluation of the abnormal behaviour. The Mamdani model and the Multiple Input / Single Output (MISO) model (from section 3.4.1) are used in this chapter. The Mamdani model is good for capturing the expertise of operators or engineers, and can provide a standard structure (Sun, 1997) to build a primary SPC control system. Its fuzzy set output will be used to analyse the behaviour of control actions, which is explained in detail in chapter 5. The MISO (eight inputs / one output) model is chosen to represent the SPC zone rules. Using the MISO model, a process condition (output) can be identified through the testing of several (1~8) samples of data (inputs) (section 2.5.1). The Fuzzy-SPC control model is designed on the discrete universe of discourse.

4.2.1 Fuzzification and I/O fuzzy sets

As discussed in section 3.4.2.1, the data of the input signal may have different ranges as they are sampled from different process, and this has to be converted to the universe of discourse. If the sample data is x' and $x'_{\min} \leq x' \leq x'_{\max}$, the universe of discourse is $[x_{\min}, x_{\max}]$, and x' is converted to data point x_0 ($x_{\min} \leq x_0 \leq x_{\max}$) that will be fuzzified.

Suppose the sample data has a range (0~99) generated by computer software, fuzzy membership can take on a value between 0 and 1 and x_0 takes in the discrete universe of discourse [-14, +14] (this interval is chosen for convenience and ease of calculation; the basic scale for calculating is integer) that is based on $\pm$ A, $\pm$B, $\pm$C and $\pm$ OUT as linguistic terms.

So,

$$k = \frac{x_{\max} - x_{\min}}{x'_{\max} - x'_{\min}} = \frac{14-(-14)}{99-0} = 0.2828 \tag{4.1}$$

and

$$x_0 = \frac{x_{\min} + x_{\max}}{2} + k(x' - \frac{x'_{\min} + x'_{\max}}{2}) = \frac{-14+14}{2} + k(x' - \frac{0+99}{2})$$
$$= 0.2828x' - 13.9986 \qquad (4.2)$$

Because the data point x_0 is converted from precise or accurate sample data x', x_0 can be translated to (or expressed by) a singleton fuzzy set A', with its membership function $\mu_{A'}(x)$ in the fuzzification (Ross, 1995) where x is the element of the singleton fuzzy set or input linguistic variable (section 3.4.2.2).

Triangular membership functions are suitable to represent random variables (Sun, 1997) such as in SPC, which are used to define the fuzzy sets for input data (Sancho-Royo and Verdegay, 1999). Fig.4.1 illustrates eight possible fuzzy subsets associated with the terms " – OUT, -ZA, -ZB, -ZC, ZC, ZB, ZA, OUT " corresponding to zones A, B, and C in SPC Zone Rules described previously in chapter 2. The eight fuzzy subsets cover the horizontal position in the universe of discourse, which ranges from -14 to $+14$.

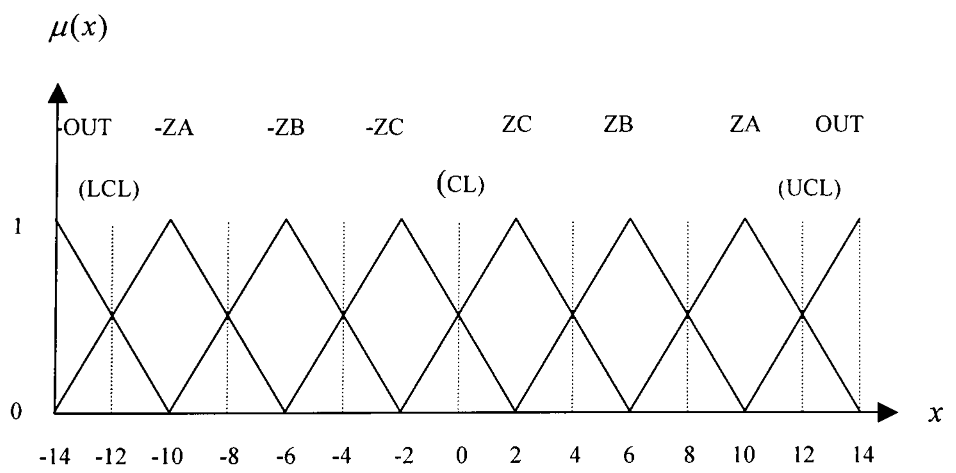


Figure 4.1 Antecedent membership function

The membership functions associated with each linguistic term are defined for the input variables from the process.

In Fig. 4.1, converted data points

$$X \in \{ -14, -13, \ldots -1, 0, 1, \ldots, 13, 14\} \tag{4.3}$$

are represented by the following discrete ranges:

$$X=\{[-14.5, 13.5], [13.5, 12.5], \ldots [-1.5, -0.5], [-0.5, 0.5], [0.5, 1.5], \ldots [12.5, 13.5], [13.5, 14.5]\} \tag{4.4}$$

where linguistic term

$$T(x)= \{ -OUT, -ZA, -ZB, -ZC, ZC, ZB, ZA, OUT \} \tag{4.5}$$

and x is an element or input linguistic variable (input variable in short).

The antecedent fuzzy sets can be expressed as:

$$-OUT=\frac{1}{-14}+\frac{0.75}{-13}+\frac{0.50}{-12}+\frac{0.25}{-11}+\frac{0}{-10}+\cdots\frac{0}{0}+\frac{0}{1}+\cdots+\frac{0}{14} \tag{4.6}$$

$$-ZA=\frac{0}{-14}+\frac{0.25}{-13}+\frac{0.50}{-12}+\frac{0.75}{-11}+\frac{1}{-10}+\frac{0.75}{-9}+\frac{0.50}{-8}+\frac{0.25}{-7}+\frac{0}{-6}+\ldots+\frac{0}{14} \tag{4.7}$$

$$-ZB=\frac{0}{-14}+\cdots+\frac{0}{-10}+\frac{0.25}{-9}+\frac{0.5}{-8}+\frac{0.75}{-7}+\frac{1}{-6}+\frac{0.75}{-5}+\frac{0.5}{-4}+\frac{0.25}{-3}+\frac{0}{-2}+\cdots+\frac{0}{14} \tag{4.8}$$

$$-ZC=\frac{0}{-14}+\cdots+\frac{0}{-6}+\frac{0.25}{-5}+\frac{0.5}{-4}+\frac{0.75}{-3}+\frac{1}{-2}+\frac{0.75}{-1}+\frac{0.5}{0}+\frac{0.25}{1}+\frac{0}{2}\cdots+\frac{0}{14} \tag{4.9}$$

$$ZC = \frac{0}{-14} + \cdots + \frac{0}{-2} + \frac{0.25}{-1} + \frac{0.5}{0} + \frac{0.75}{1} + \frac{1}{2} + \frac{0.75}{3} + \frac{0.5}{4} + \frac{0.25}{5} + \frac{0}{6} + \cdots + \frac{0}{14} \quad (4.10)$$

$$ZB = \frac{0}{-14} + \cdots + \frac{0}{2} + \frac{0.25}{3} + \frac{0.5}{4} + \frac{0.75}{5} + \frac{1}{6} + \frac{0.75}{7} + \frac{0.5}{8} + \frac{0.25}{9} + \frac{0}{10} + \cdots + \frac{0}{14} \quad (4.11)$$

$$ZA = \frac{0}{-14} + \cdots + \frac{0}{6} + \frac{0.25}{7} + \frac{0.5}{8} + \frac{0.75}{9} + \frac{1}{10} + \frac{0.75}{11} + \frac{0.5}{12} + \frac{0.25}{13} + \frac{0}{14} \quad (4.12)$$

$$OUT = \frac{0}{-14} + \cdots + \frac{0}{10} + \frac{0.25}{11} + \frac{0.5}{12} + \frac{0.75}{13} + \frac{1}{14} \quad (4.13)$$

Figure 4.2 shows the membership functions for the output variables, which correspond to the outcomes of the analysis in section 2.7 of chapter 2. The triangular and trapezoid membership functions are chosen for P(N)Z, P(N)S, P(N)M and P(N)B consequent subsets which indicate different shift levels tested by different zone rules.

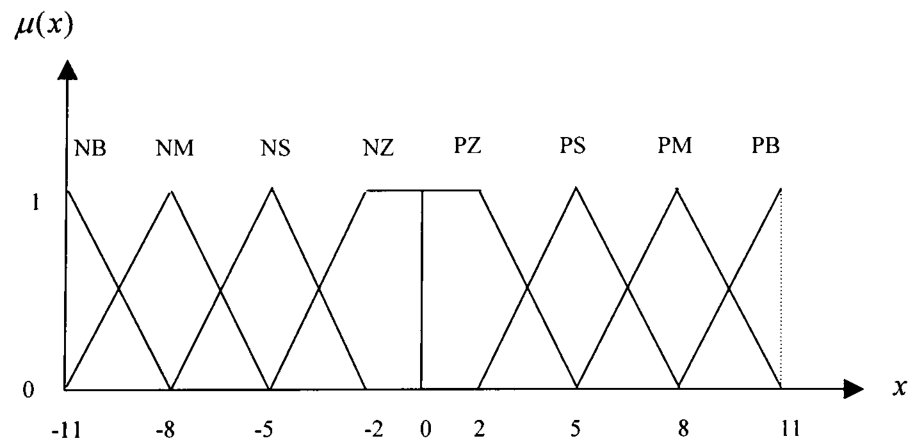


Figure 4.2 Consequent membership function

The universe of discourse for the output is given by equation 4.14.

$$Z \in \{-11,-10,\ldots,-2,-1,0,1,2,\ldots,10,11\} \tag{4.14}$$

where linguistic term T(z) is given by:

$$T(z) = \{ NB, NM, NS, NZ, PZ, PS, PM, PB \} \tag{4.15}$$

where

NB: Negative Big; NM: Negative Medium; NS: Negative Small; NZ: Negative Zero; PZ: Positive Zero; PS: Positive Small; PM: Positive Medium; PB: Positive Big. They are chosen based on the characteristic of the zone rules. z is an element of the output fuzzy set or output linguistic variable (output variable in short).

The linguistic terms T (z) are used to describe different output values from the fuzzy system. They involve the following fuzzy subsets:

$$NB = \frac{1}{-11} + \frac{0.67}{-10} + \frac{0.33}{-9} + \frac{0}{-8} + \cdots \frac{0}{0} + \cdots + \frac{0}{11} \tag{4.16}$$

$$NM = \frac{0}{-11} + \frac{0.33}{-10} + \frac{0.67}{-9} + \frac{1}{-8} + \frac{0.67}{-7} + \frac{0.33}{-6} + \frac{0}{-5} + \ldots + \frac{0}{11} \tag{4.17}$$

$$NS = \frac{0}{-11} + \cdots + \frac{0}{-8} + \frac{0.33}{-7} + \frac{0.66}{-6} + \frac{1}{-5} + \frac{0.66}{-4} + \frac{0.33}{-3} + \frac{0}{-2} + \cdots + \frac{0}{11} \tag{4.18}$$

$$NZ = \frac{0}{-11} + \cdots + \frac{0}{-5} + \frac{0.33}{-4} + \frac{0.67}{-3} + \frac{1}{-2} + \frac{1}{-1} + \frac{1}{0} + \frac{0}{1} + \cdots + \frac{0}{11} \tag{4.19}$$

$$PZ = \frac{0}{-11} + \cdots + \frac{0}{-1} + \frac{1}{0} + \frac{1}{1} + \frac{1}{2} + \frac{0.67}{3} + \frac{0.33}{4} + \frac{0}{5} + \cdots + \frac{0}{11} \tag{4.20}$$

$$PS = \frac{0}{-11} + \cdots + \frac{0}{2} + \frac{0.33}{3} + \frac{0.67}{4} + \frac{1}{5} + \frac{0.67}{6} + \frac{0.33}{7} + \frac{0}{8} + \cdots + \frac{0}{11} \tag{4.21}$$

$$PM = \frac{0}{-11} + \cdots + \frac{0}{5} + \frac{0.33}{6} + \frac{0.67}{7} + \frac{1}{8} + \frac{0.67}{9} + \frac{0.33}{10} + \frac{0}{11} \tag{4.22}$$

$$PB = \frac{0}{-14} + \cdots + \frac{0}{8} + \frac{0.33}{9} + \frac{0.67}{10} + \frac{1}{11} \tag{4.23}$$

4.2.2 If – then rules / fuzzy reasoning

Usually, the if - then rules can be designed by an experienced expert (Huang et al, 1995). In fact, SPC zone rules represent a summary of people's experiences from the manufacturing processes, but also have a statistical basis. As discussed in section 4.2, eight inputs / one output structure is used to represent the SPC zone rules 1 to 5 (the reason of using of 5 zone rules is discussed in section 2.5.2 of chapter 2) in the fuzzy approximate reasoning. Suppose x_i, $(1 \le i \le 8)$ is the input linguistic variable, which is translated as the input singleton fuzzy set from crisp input data point x_0 (equation 4.2), then "if-then" rules can cause a fuzzy set output C' which is inferred by approximate reasoning. This procedure involves a composition operation for input singleton fuzzy set and the antecedent-consequent implication (see section 3.3.2).

For a fuzzy controller, the if-then rules should contain all possible combinations of antecedents. If the fuzzy-SPC system (eight inputs / one output) considers all possible combinations to fully describe the 5 zone rules, it will have a large number of if-then rules. In this system, the whole fuzzy reasoning / if-then rules (ITR) have been simplified to 124 rules using the following preconditions:

1. Because the Minimum operator is used for AND in the antecedent fuzzy sets, there is no relationship between the position or number of "beyond limit points" in an antecedent of the rule and the inference result obtained. Therefore, it is not necessary to contain all possible position of "beyond limit points" and all the number of "beyond limit points" in the if-then rules. For example for zone rule 3, "The existence of four of any five successive points in zone B or *beyond*,..." can be described by "if x_1 is ZA (Zone A) and x_2 is ZB (Zone B) and x_3 is ZB and x_4 is ZB then ..." (ITR_8). In the fuzzy antecedent of the rule, it is not necessary to also contain "if x_1 is ZB and x_2 is ZA and x_3 is ZB and x_4 is ZB then ...", "if x_1 is ZA and x_2 is ZB and x_3 is ZA and x_4 is ZB then ..." and so on.

2. Similarly for zone rule 5, the rule ITR_{109} (See Appendix B.5) can represent all possible combinations of -ZA and –ZC (-Zone C). It is not necessary to also contain "If x_1 is -ZA and x_2 is -ZA and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC and x_7 is -ZC and x_8 is -ZC ..." or "If x_1 is -ZA and x_2 is -ZA and x_3 is -ZC and x_4 is -ZC and x_5 is -ZA and x_6 is -ZA and x_7 is -ZC and x_8 is -ZC ..." and so on.

3. For any variable with an uncertain value, it can be identified by the programme in simulation or for a real system. For example in zone rule 2, "the existence of two out of any *three* successive points in zone A,..." is described by "if x_1 is ZA and x_2 is ZA then ..." which is according to the antecedent of the fuzzy rule. As for the value of third variable x_3, it is identified by the control programme. That is, "x_2" is a nominal variable only in fuzzy if-then rules, it can be assigned by x_2 or x_3, if any of them falls in Zone A.

4. The sequential search is operated from zone rule 1 to zone rule 5 ($ITR_1 \rightarrow ITR_{124}$). Some rules, which have the same situation of antecedent with the rules already operated, can be omitted. For example in zone rule 5, if the inputs fall in the –ZA and –ZB only, for any range of input variable **x**, the abnormal situations can be identified by zone rule 2 or zone rule 3 (ITR_3, ITR_5 or ITR_6).

5. In order to enhance the control accuracy, a modification is carried out (The reason is discussed in section 2.7 of chapter 2) in the program for every zone rule.

The simplified fuzzy reasoning / if-then rules *(ITR)* that is used for zone rule 1 ~ zone rule 3 are defined as below. Other rules for zone rule 4 and 5 are given in Appendix B.5.

For zone rule 1:

ITR_1 : If x_1 is –OUT then z is NB;

ITR_2 : If x_1 is OUT then z is PB;

For zone rule 2:

ITR_3 : If x_1 is -ZA and x_2 is -ZA then z is NM;

ITR_4 : If x_1 is ZA and x_2 is ZA then z is PM;

For zone rule 3:

ITR_5 : If x_1 is –ZB and x_2 is –ZB and x_3 is -ZB and x_4 is –ZB then z is NS;

ITR_6 : If x_1 is –ZA and x_2 is –ZB and x_3 is -ZB and x_4 is –ZB then z is NS;

ITR_7 : If x_1 is ZB and x_2 is ZB and x_3 is ZB and x_4 is ZB then z is PS;

ITR_8 : If x_1 is ZA and x_2 is ZB and x_3 is ZB and x_4 is ZB then z is PS;

4.2.3 Defuzzification

Defuzzification is a calculation method used to convert the value that is described in the fuzzy set to a crisp output value. The popular method of Centre-Of-Area (COA) is used to calculate the crisp variable z_0 in this research work, as it has advantages in continuity and unambiguity. (Drianckov et al, 1993), (Reznik, 1997).

4.2.4 Calculation of fuzzy inference

Suppose the fuzzy set of the output is C' for this MISO system,

$$C' = (x_1 \text{ is } A_1' \text{ and...and } x_n \text{ is } A_n') \circ R = (A_1' \text{ and...and } A_n') \circ R \tag{4.24}$$

$$R = \bigcup_{j=1}^{m} R_j \tag{4.25}$$

$$R_j = (A_1^j \text{ and...and } A_n^j) \rightarrow C_j \tag{4.26}$$

where $\circ$ is the composition operator, $\rightarrow$ is the implication, R is the fuzzy relation, A_i^j $(i = 1,2,...,n, j = 1,2,...m)$ denotes the fuzzy sets in antecedent ($\pm ZA$, $\pm ZB$ and $\pm ZC$), C_j is the conclusion for the j^{th} rule, n is number of input variables and m is the total number of if-then rules.

If the Cartesian product notation "×" is used to express the connective AND (or conjunction) in the antecedent (section 3.3.3.1), and fuzzy intersection is used to calculate the AND operation, the overall conclusion or output C' is given by:

$$C' = (A_1' \times A_2' \times \cdots \times A_n') \circ R = (A_1' \times A_2' \times \cdots \times A_n') \circ \bigcup_{j=1}^{m} R_j \tag{4.27}$$

$$=\bigcup_{j=1}^{m}(A_1' \times A_2' \times \cdots \times A_n') \circ [(A_1^j \times A_2^j \cdots \times A_n^j) \to C_j] \qquad \text{(also see Appendix B.1)}$$

$$=\bigcup_{j=1}^{m}[A_1' \circ (A_1^j \to C_j)] \bigcap \cdots \bigcap [A_n' \circ (A_n^j \to C_j)] \qquad \text{(also see Appendix B.2)}$$

The crisp variable z_0 is given by:

$$z_0 = \frac{\sum_{j=1}^{m} z_j \mu_{C'}(z_j)}{\sum_{j=1}^{m} \mu_{C'}(z_j)} \tag{4.28}$$

Where z_j is an element of output of the fuzzy set C' and m is the number of z_j.

In this Fuzzy-SPC application which is a general approach, the minimum (or min) operator is chosen for the fuzzy intersection in equation (4.27). Because it has simple computing and robustness advantages – when one membership function is smaller (or larger) with respect to the other, it has no influence on the resultant membership function (Reznik,1997). For the same reasons, the max-min method is applied for the composition operation which is noted by "$\circ$". For the implication operation, Mamdani (R_c) and Larsen (R_p) (equation 3.22 and equation 3.24) are appropriate to be used for the General Modus Ponens criterion I, which was discussed as fundamental modus ponens in section 3.3.2 of chapter3. In this approach, the Mamdani implication R_c is chosen for the "$\to$" operator, as it is simpler to compute than R_p (Harris et al, 1993), (Sun, 1997). For the convenience of calculation in the discrete universe of discourse, the fuzzy sets and fuzzy relation are expressed by a fuzzy vector and fuzzy matrix respectively. An example of the calculation for the fuzzy inference is given below.

Suppose $x_1 = x_2 = -9$

In ITR_1 and ITR_2, $C_1^{'} = C_2^{'} = 0$

The ITR_3 *(j=3)* contains two inputs variables x_1 and x_2, their implication R_j^i (a 29×23 matrix) and inference $C_i^{\prime j}$ which contains 23 elements (from -11 to $+11$) are given in equations 4.29 and 4.30.

$$R_1^3 = A_1^3 \to C_3 = A_{-ZA} \to C_{NM} \tag{4.29}$$

$$= \begin{bmatrix} 0 \\ 0.25 \\ 0.5 \\ 0.75 \\ 1 \\ 0.75 \\ 0.5 \\ 0.25 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \wedge \begin{bmatrix} 0 & 0.33 & 0.67 & 1 & 0.67 & 0.33 & 0 & \cdots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0.25 & 0.25 & 0.25 & 0.25 & 0 & \cdots & 0 \\ 0 & 0.33 & 0.5 & 0.5 & 0.5 & 0.33 & 0 & \cdots & 0 \\ 0 & 0.33 & 0.67 & 0.75 & 0.67 & 0.33 & 0 & \cdots & 0 \\ 0 & 0.33 & 0.67 & 1 & 0.67 & 0.33 & 0 & \cdots & 0 \\ 0 & 0.33 & 0.67 & 0.75 & 0.67 & 0.33 & 0 & \cdots & 0 \\ 0 & 0.33 & 0.5 & 0.5 & 0.5 & 0.33 & 0 & \cdots & 0 \\ 0 & 0.25 & 0.25 & 0.25 & 0.25 & 0.25 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_1'^3 = A_1' \circ R_1^3 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad \cdots \quad 0] \circ R_1^3 \tag{4.30}$$

$$= [0 \quad 0.33 \quad 0.67 \quad 0.75 \quad 0.67 \quad 0.33 \quad 0 \quad \cdots \quad 0]$$

Because $x_1 = x_2 = -9$, then

$$C_2'^3 = C_1'^3 \tag{4.31}$$

$$C_3' = C_1'^3 \bigcap C_2'^3 = [0 \quad 0.33 \quad 0.67 \quad 0.75 \quad 0.67 \quad 0.33 \quad 0 \quad \cdots \quad 0] \tag{4.32}$$

In other rules,

$$C_4' = C_5' = \cdots = C_{124}' = 0, \tag{4.33}$$

Therefore,

$$C' = \bigcup_{j=1}^{124} C_j' = C_3' \tag{4.34}$$

$$= [0 \quad 0.33 \quad 0.67 \quad 0.75 \quad 0.67 \quad 0.33 \quad 0 \quad \cdots \quad 0]$$

$$Z_0 = \frac{0.33(-10) + 0.67(-9) + 0.75(-8) + 0.67(-7) + 0.33(-6)}{0.33 + 0.67 + 0.75 + 0.67 + 0.33} \tag{4.35}$$

$$= -8$$

For each zone rule, the crisp output variable z_0 which is calculated for different schemes of input variables x_i ($1 \le i \le 8$), are summarised as a control table or rule base (see chapter 3 section 3.4.1). The control table (Table 4.1) gives the crisp variables, which are the outputs or conclusions from the fuzzy inference system according to SPC zone rule 2. In table 4.1, x_1 and x_2 (which are described in section 4.2.2) are denoted by $|x_1|$ and $|x_2|$ which are assigned to the interval [0, 14]. The calculation results summarised in Table 4.1 are also verified in section 4.3.

$\lvert x_2\rvert$ \ $\lvert x_1\rvert$	$0 \sim 8$	9	10	11	12	13~14
$0 \sim 8$	0	0	0	0	0	0
9	0	8.00	8.00	8.12	8.41	0
10	0	8.00	8.00	8.12	8.41	0
11	0	8.12	8.12	8.12	8.41	0
12	0	8.41	8.41	8.41	8.41	0
$13 \sim 14$	0	0	0	0	0	0

Table 4.1 Control table ($|z_0|$) for zone rule2

4.3 An application of fuzzy logic toolbox in MATLAB

MATLAB is a technical computing environment for high – performance numeric computation and visualisation (Mathworks, 1998). It is an advanced interactive software package specially designed for scientific and engineering numerical computation. It is powerful and comprehensive tool for performing numerical analysis, matrix computation, signal processing and all kinds of computations and scientific data visualisation (Part-Enander et al, 1996).

The Fuzzy Logic Toolbox is a collection of functions built in the MATLAB numeric computing environment. It provides tools for the user to create and edit fuzzy inference systems within the framework of MATLAB. Its various functions can be used to implement several models and to compare and analyse their results (Von Altrock, 1995).

The fuzzy logic toolbox provides five Graphical User Interface (GUI) tools for building, editing and observing fuzzy inference systems, they are the Fuzzy Inference System (FIS) editor, Membership Function Editor, Rule Editor, Rule Viewer and Surface Viewer.

For further calculations and design, the fuzzy – SPC control system can be easily created in this toolbox. The control table is directly obtained from the Rule Viewer tool and the elaborate manual calculation can be omitted. The tools mentioned above are briefly described in sections 4.3.1 to 4.3.5.

4.3.1 The fuzzy inference system (FIS) editor

The FIS Editor displays general information about a fuzzy inference system. For example, the number of input and output variables, types of inference rule or model and decision of the fuzzy operator method.

As in section 4.2, the system has eight inputs variables and one output variable which are designed to represent Zone Rules 1~5. The Mamdani model is used for approximation, *min* and *max* are used for "AND" and "OR" operations separately, *max* is used for aggregation of "several rules" and the Centre-Of-Area (COA) is used for defuzzification as discussed in section 4.2.3.

4.3.2 The membership function (MF) editor

The Membership Function Editor is used to define the shapes of all the input and output membership functions associated with each variable. The types and number of membership functions can be chosen in the MF editor. As in section 4.2, the input/output memberships take on a value between 0 and 1. The input variable x (section 4.2.1) is assigned to the interval [-14, +14] that is based on linguistic terms $\pm$OUT, $\pm$ ZA, $\pm$ZB, $\pm$ZC. The output variable z is assigned to the interval [-11, +11] based on PB/NB, PM/NM, PS/NS, PZ/NZ. The triangular and trapezoidal membership functions are used to represent the SPC inputs and outputs.

4.3.3 The rule editor

The rule editor is for editing the list of rules that defines the behaviour of the system. It can contain a large editable text field for displaying and editing rules. The 124 if-then rules mentioned in section 4.2.2 and Appendix B.5 are contained in this Rule Editor.

4.3.4 The rule viewer

The Rule Viewer is a MATLAB-based display of the fuzzy inference program, and it is used as a diagnostic tool that can show which rules are active or how individual membership function shapes are influencing the results. The crisp results of fuzzy inference z_0 can be obtained as "output1" from this rule viewer by selecting input variable $\boldsymbol{x}$ in Figure 4.3. Figure4.3 shows the calculation result when the input variables x_1 and x_2 are assigned to –9, which correspond to the value in the Table 4.1.

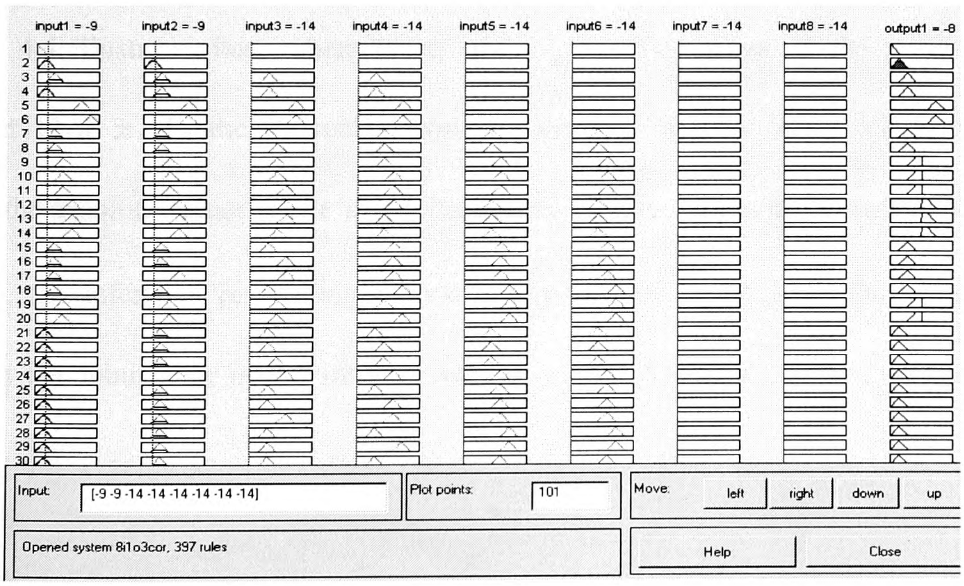


Figure 4.3 Rule viewer for Fuzzy – SPC controller (part 1: ITR_1 ~ ITR_{30})

4.3.5 The surface viewer

The surface viewer shows two or three-dimensional curves that presents the mapping from the input to the output. This figure displays how the output depends on any one or two of the inputs. Figure 4.4 describes the I/O surface for the two inputs and one output model. It is a graphical representation of the fuzzy reasoning for ITR_3 and ITR_4 (or for SPC zone rule 2. See section 4.2.2), which is summarised in the Table 4.1. Figure 4.4 is a diagonally symmetrical graph, and the $Area_1$ and $Area_2$ are ITR_3 and ITR_4 working spaces respectively.

$$Area_1 = input_1 \times input_2 = x_1 \times x_2, \quad +8 < x_{1,2} \leq +12 \tag{4.36}$$

$$Area_2 = input_1 \times input_2 = x_1 \times x_2, \quad -8 > X_{1,2} \geq -12 \tag{4.37}$$

Figure 4.4 illustrates that, when $input_1$ and $input_2$ take values in the lower interval $(\pm 9 \sim \pm 10)$ of $\pm ZA$, the inference *output* obtained is *Medium* value ± 8 in PM/NM. When the absolute values of the $input_1$ and $input_2$ increase from the zone A centre value $|\pm 10|$, the inference result *output* value is generated to be gradually enhanced. It reflects the robustness in the interval, which is close to upper or lower limits in the control chart.

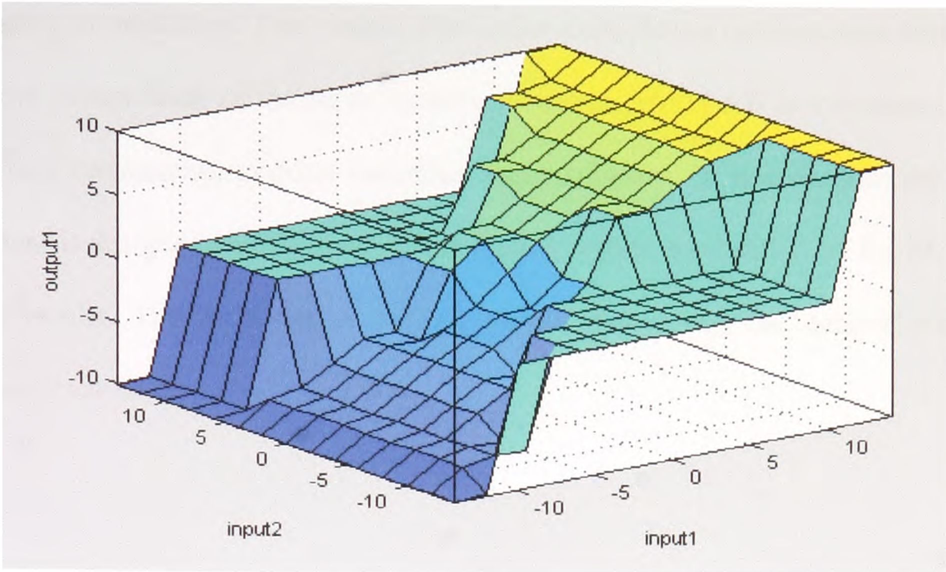


Figure 4.4 Surface viewer for two input and one output

4.4 Conclusion

Fuzzy set theory and fuzzy logic are useable and flexible techniques for expressing vague concepts. For the membership functions, many types of functions can be chosen to build different systems. Based on the analysis results of section 2.7 of chapter 2, the triangular and trapezoidal membership functions are used in the design of a Fuzzy-SPC system. These two types of membership functions also have the advantage of simple calculation. Fuzzy reasoning is a very important element in a fuzzy system. Based on SPC zone rules 1 to 5, the related if-then rules are determined to represent the input/output relationship in this approach. For if-then rules, it is necessary to simplify the rules and reduce the rule number in order to increase the system's working speed. In this simplification procedure, some experiments and analysis are required.

Fuzzy logic is a technique used to describe the uncertainty of events based on certain mathematical computation. This chapter uses some examples of mathematical methods to explain the design ideas of the fuzzy system. However, MATLAB is a powerful tool to design many engineering systems including fuzzy schemes. In this chapter, the Fuzzy-SPC system is designed and the manual calculation results are verified by the MATLAB executed results. The input-output Surface viewer shows that the control scheme is preliminarily satisfied.

4.5 Summary

This chapter represents the design and discussion of a Fuzzy-SPC control system. Triangular and trapezoidal membership functions are used for the antecedent and consequent parts. A series of preconditions for the simplification of if-then rules, which correspond to SPC zone rules was proposed. The COA method of defuzzification was chosen in the fuzzy inference and some reasons for the selection of operators are also explained. An application of the fuzzy toolbox in MATLAB for verifying manual calculations of the fuzzy inference solutions and further designing of the Fuzzy-SPC system is also described. More detailed fuzzy inference solutions are obtained from the Rule Viewer in the fuzzy toolbox of MATLAB and these solutions will be used to build a fuzzy control table or if-then rule base in chapter 5. Finally as an example of a graphical expression for "if-then" rules ITR_3 and ITR_4 (SPC zone rule 2), the input/output surface by three-dimensional curves are shown in the Surface Viewer.

As discussed previously, the Fuzzy-SPC system is designed in this chapter. How well it works should be assessed through an application. In the next chapter, the control behaviour is simulated in a C++ system, and the control results analysed. Based on these primary approaches, the tuning of membership functions for investigation and improvement of the Fuzzy-SPC system is also considered.

Chapter 5 The simulation study written in C++

A Fuzzy-SPC system was implemented in simulation written by C++ language. To achieve a reduction of control errors, the effect of varying membership function shapes are investigated. Based on the results of this approach, a set of membership function schemes are designed and used to control the abnormal processes with high control accuracy.

5.1 Introduction

This chapter describes simulation studies, which were used to assess the Fuzzy-SPC system. Two types of C++ languages were used to design the simulation systems.

Borland C++ was used to design a basic simulation system based on the DOS environment. This basic system was used to implement general operations such as generating abnormal processes and using a Fuzzy-SPC controller with a standard triangular and trapezoidal membership functions to control it. The control errors are calculated in this section.

Visual C++ was applied to build a major simulation system based on the Windows environment to generate process data and control the abnormal data points by the different group of membership functions. Two types of experiments were performed in this study.

One was an investigation into effective ways to change the triangle and trapezoidal shapes of the fuzzy consequent membership functions, which influence the control results for the abnormal processes. The other experiment generated the control results distribution via a set of preferred triangular and trapezoidal membership function schemes, which were determined by the results generated from the first set of experiments. For increased control accuracy, the preferred shape of the membership functions can be selected.

The C++ language and Object – Oriented Programming (OOP) is briefly introduced in section 5.2. The structure of the simulation systems which were written in two types of C++ language are described in section 5.3 and section 5.5, the process charts and analyses of the control results are given in section 5.3, section 5.6 and section 5.7.

5.2 C++ language and OOP functions

The related concepts of the C++ language will be briefly introduced in this section as they are used in the fuzzy-SPC simulation studies. C++ is an expanded version of C. It was initially called "C with Classes" as a high – level and Object – Oriented Program (OOP) language.

Objected-oriented programming is a way of approaching the job of programming. It adopts the best ideas of structured programming. OOP allows the user to easily decompose a problem into subgroups of related parts. These parts can then be translated into self-contained units, which are called objects. An object-oriented programming language has three characteristics: encapsulation, polymorphism and inheritance (Schildt, 1997):

- Encapsulation can be viewed a device that binds together code and data and that keeps them safe for the interference or misuse from outside. An object is a logical entity that encapsulates both data and the code that manipulates that data. In an object, some of the code and /or data may be private to this object and inaccessible to anything outside the object. An object provides the protection against some other, unrelated part of the program accidentally modifying or incorrectly using the private members of the object.

- Inheritance is the process by which one object can acquire the properties of another object, which is in the same father-classification. For example, a red pen is part of the pen class, which in turn is part of the stationery class. The classification can be used to simplify programming via the inheritance, an object need only define those qualities that make it unique within its class.

- Polymorphism can be explained by the phrase "one interface, multiple methods". It can be viewed as an attribute that allows one interface to be used with a general class of action. The specific action selected is determined by the exact nature of the situation. An example of polymorphism is the thermostat. It can work in the same way on different furnaces: gas, oil, or electricity. The polymorphism can reduce the complexity of a program by allowing the same interface to be used to specify a general class of actions.

5.3 The OOP simulation study written in Borland C++ 5

5.3.1 Borland C++ 5 and simulation operators

Most C++ compilers include a full programming environment: editor, compiler, linker and debugger, which are called Integrated Development Environment (IDE). One of the most popular C++ IDE is Borland C++ (Wilks, 1994). The basic Fuzzy – SPC simulation OOP system is written in Borland C++ 5. This system can simulate a random process, generate the random data in any running time with different values, uncontrolled $\overline{X}$ chart and zones can be drawn on the screen and abnormal points marked by sample number. The controlled pattern can be shown as the results after applying the fuzzy inference (Fuzzy – SPC controller).

The main functions of the program are:

- Create data to represent a normal and an abnormal process;
- Calculate the distribution of the data;
- Create related data text files on disk for analysis;
- Calculate the average, standard deviation, CL, UCL, LCL and boundaries between zones A, B and C;

- Inspect and interpret the process data by zone rules 1, 2, 3, 4 and 5;
- Produce the automatic control signal from the control table and transfer it to the SPC control chart;
- Plot $\overline{X}$ charts with and without controlled action for comparison.

5.3.2 OOP simulation program structure

Figure 5.1 illustrates the structure of the OOP simulation program. There are three classes that provide the data and code *encapsulated* in this system.

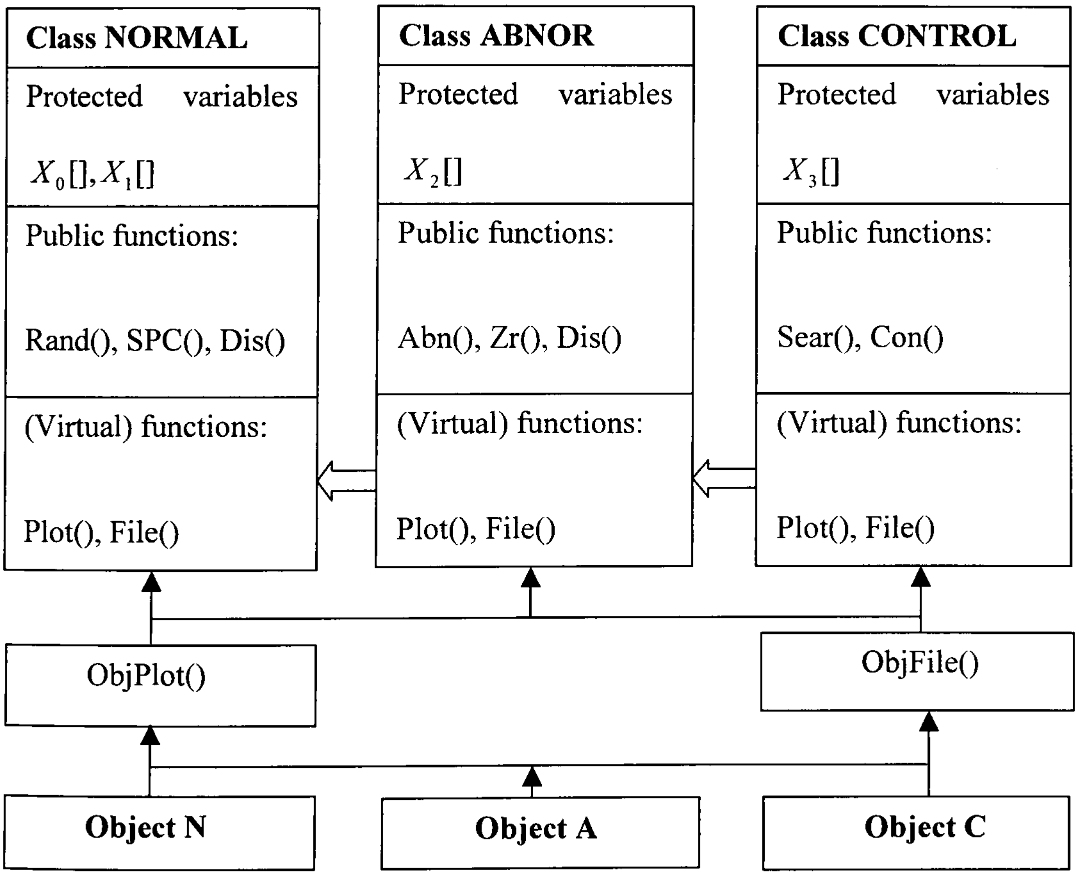


Figure 5.1 The OOP structure of the simulation program

Class NORMAL is used for managing the normal random data and calculating the SPC parameters. Class ABNOR is used for generating and testing the abnormal data. Class CONTROL is used for searching the fuzzy control table, and controlling the abnormal process. In class NORMAL, 500 (the number can be changed through the user interface) random data points with normal distribution are held in an array $X_0[]$, their subgroup average ($\overline{X}$) (the size is chosen as 5) are held in an array $X_1[]$. They are defined as the *protected* member variables, which can be *inherited* (Cai, 1994). The *public* member functions include Rand(), SPC() and Dis(), which can be used to access the member variables from outside this class and to complete their tasks. For example, generating random data, calculating SPC parameters and calculating the data distributions. The *virtual functions* Plot() and File() are used to complete the plotting of charts and saving text files. They can be applied by different object as the *polymorphism* attribute via the *inheritance* process.

The class ABNOR is a *derived class* from class NORMAL by the *Inheritance* method (He et al, 1994). The *protected* member variables $X_0[]$ and $X_1[]$ in the base class NORMAL can be accessed in the *derived* class ABNOR. The member functions Abn(), Zr() and Dis() are used to create the abnormal data, test the abnormal data by zone rules and calculate the distribution of the abnormal data via member variable $X_2[]$. The member functions Plot() and File() are *virtual* functions in the *inheritance* process.

The class CONTROL is a *derived class* from class ABNOR. Similarly, the *protected* member variable $X_2[]$ in class ABNOR can be accessed from the *derived* class CONTROL. The member functions Sear() and Con() are used to search the fuzzy control table and to generate the control action for the abnormal process. Adjusted process data are held in the member variable $X_3[]$. Plot() and File() are virtual functions.

The three objects "N", "A" and "C" have their separate classes - NORMAL, ABNOR and CONTROL to achieve different operations. As the *polymorphism* characteristic, the *virtual* functions can be determined to complete different specific actions via one interface (Schildt, 1997). For example, to plot different charts for object "N", "A" and ""C" via one interface ObjPlot() only, or to save different files via one interface ObjFile() only.

5.3.3 The simulation and analysis

A sample of ten $\overline{X}$ charts in Fig. 5.2 ~ Fig. 5.6 illustrate the executed outputs of the fuzzy system in which an abnormal process is simulated, tested and adjusted by the fuzzy inference system. In the upper charts of Fig. 5.2 ~ Fig. 5.6, abnormal points (i.e., points which contravene the zone rules) are marked by double rings, a vertical line and the number of the sample. The upper charts are uncontrolled and the lower charts are controlled and adjusted by control outputs of the fuzzy inference at the first abnormal points which are marked by the large vertical lines. It can be seen that to the right of the large vertical lines, subsequent abnormal points (marked with double rings) are improved to normal data in the lower charts.

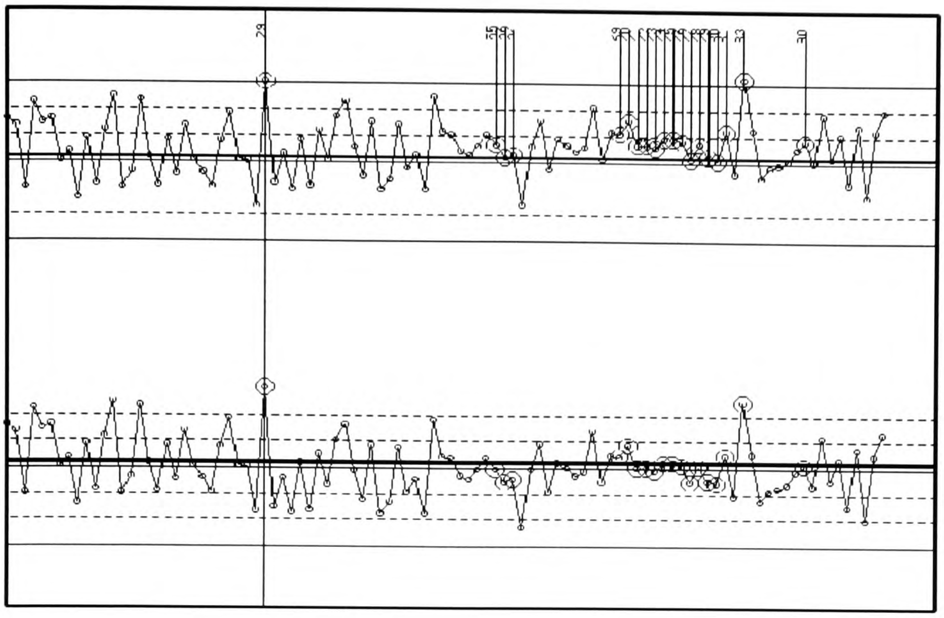


Figure 5.2 Controlled in zone rule 1

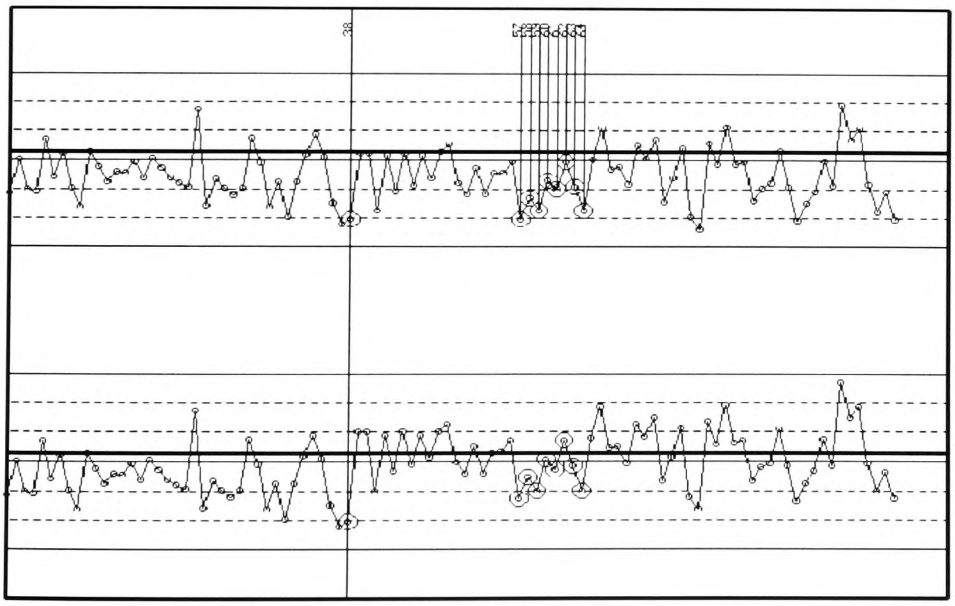


Figure 5.3 Controlled in zone rule 2

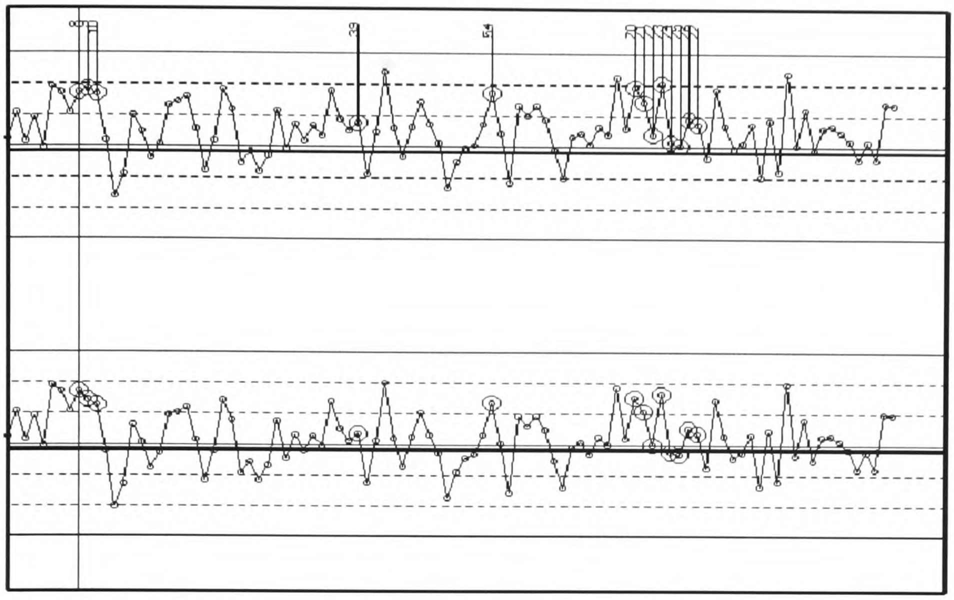


Figure 5.4 Controlled in zone rule 3

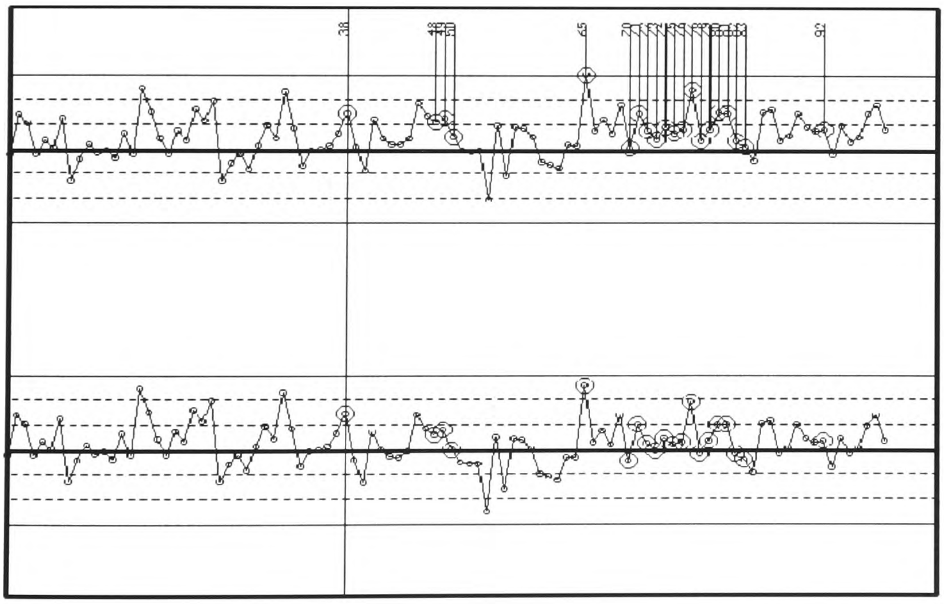


Figure 5.5 Controlled in zone rule 4

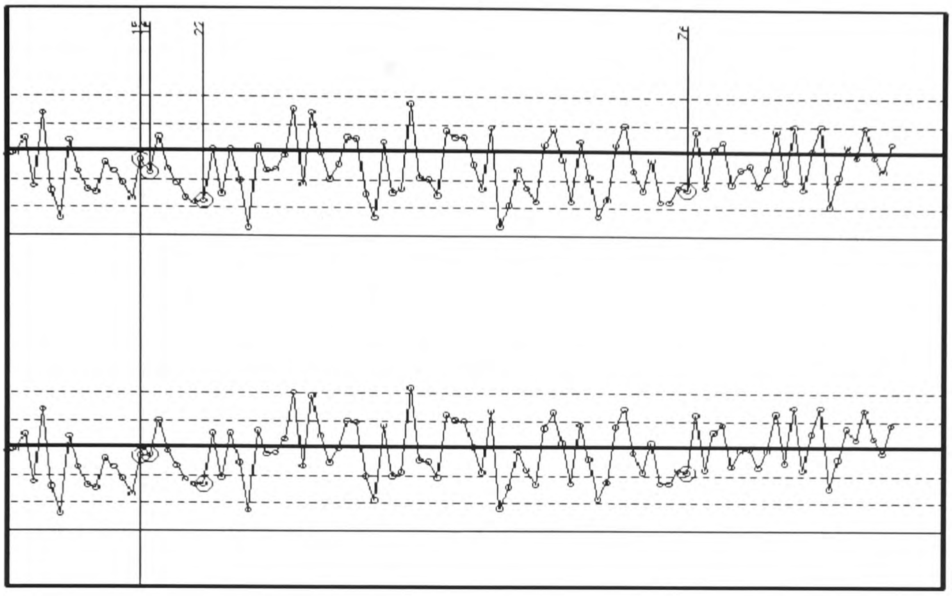


Figure 5.6 Controlled in zone rule 5

In this simulation study, 500 random data points were generated by function RAND () for each run and the subgroup size chosen was 5 with the $\overline{X}$ chart plotted for 100 $\overline{X}$ values. The abnormal process is software generated by shifting 0.1 times the universe of discourse for process average (Box, 1997) and the fuzzy system tests and adjusts the abnormal process automatically. Abnormal data from Fig. 5.2 ~ Fig. 5.6 are summarised in table 5.1. The sample number for the first abnormal point (control point) and successive abnormal points are given in columns 2 and 3 and the zone rule number that is used for the appropriate test is indicated in column 4.

Figure	Number of First abnormal point	Numbers of Successive Abnormal points	Zone rules
5.2	28		1
5.2		83	1
5.2		55, 56, 57, 69~81	4
5.2		90	5
5.3	38		2
5.3		57~65	4
5.4	8		3, 4
5.4		9, 10,	3, 4
5.4		39, 70~77	4
5.4		54	5
5.5	15		4
5.5		16	4
5.5		22	5
5.5		76	3
5.6	38		5
5.6		49	3, 4
5.6		48, 50, 92, 70~83	4

Table 5.1 Summary of simulation details for one run

The control results are expressed as a scalar for Fig. 5.2 ~ Fig. 5.6 and summarised on the second row in Table 5.2, which is marked by No. 1 ~ No. 5 in column "experiments numbers". The simulated control results are normalised to [0,1]. In column 2, "CAPA" describes the controlled abnormal process averages, "NPA" describes the normal process averages and "Error" is calculated from the difference between controlled abnormal process average and normal process average. "ZR1" ~ "ZR5" in columns 3 ~ 7 represent SPC zone rule 1 to 5 which are used to test the abnormal processes. The results show that the maximum absolute value of control error is less than 0.037 when the mean shift level is 0.1. It indicates that the Fuzzy-SPC system successfully adjusts and improves the process for the simulated data.

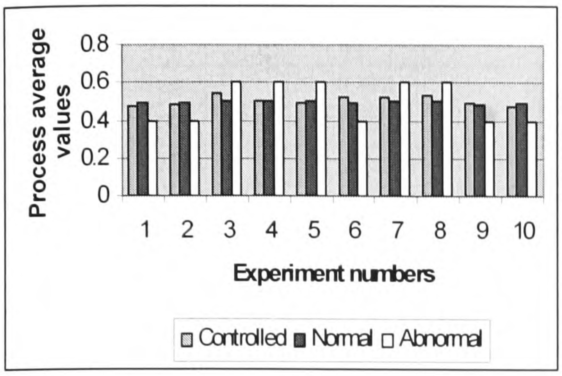
Experiments number	Items	ZR1	ZR2	ZR3	ZR4	ZR5
1~5	CAPA	0.4732	0.4818	0.5409	0.5056	0.4987
	NPA	0.4962	0.4943	0.5057	0.5011	0.5006
	Error	-0.0230	-0.0125	0.0352	0.0045	-0.0019
6~10	CAPA	0.5211	0.5243	0.5291	0.4974	0.4710
	NPA	0.4905	0.5013	0.5066	0.4831	0.4925
	Error	0.0306	0.0230	0.0225	0.0143	-0.0215
11~15	CAPA	0.5037	0.4890	0.4521	0.4857	0.5084
	NPA	0.4919	0.4794	0.4731	0.4887	0.5057
	Error	0.0118	0.0096	-0.0210	-0.0030	0.0027
16~20	CAPA	0.4887	0.5194	0.4590	0.4828	0.5077
	NPA	0.4999	0.5043	0.4941	0.4956	0.5077
	Error	-0.0112	0.0151	-0.0351	-0.0128	0.0000
21~25	CAPA	0.4939	0.5057	0.5477	0.5071	0.4919
	NPA	0.5057	0.5087	0.5212	0.4901	0.4909
	Error	-0.0118	-0.0030	0.0265	0.0170	0.0010
26~30	CAPA	0.4916	0.4892	0.4591	0.5100	0.4835
	NPA	0.5044	0.4851	0.4956	0.4765	0.4936
	Error	-0.0128	0.0041	-0.0365	0.0335	-0.0101
31~35	CAPA	0.5072	0.5223	0.4618	0.5146	0.4902
	NPA	0.4911	0.5023	0.4947	0.5074	0.4956
	Error	0.0161	0.0200	-0.0329	0.0072	-0.0054
36~40	CAPA	0.4926	0.5155	0.4563	0.4829	0.5102
	NPA	0.5040	0.5035	0.4852	0.4934	0.5074
	Error	-0.0114	0.0120	-0.0289	-0.0105	0.0028
41~45	CAPA	0.5195	0.5112	0.4653	0.4711	0.4937
	NPA	0.5043	0.5101	0.4885	0.4945	0.4999
	Error	0.0152	0.0011	-0.0232	-0.0234	-0.0062
46~50	CAPA	0.4899	0.5088	0.4596	0.5132	0.5091
	NPA	0.5055	0.5030	0.4846	0.5018	0.5127
	Error	-0.0156	0.0058	-0.0250	0.0114	-0.0036

Table 5.2 Summary of simulations results for 50 runs

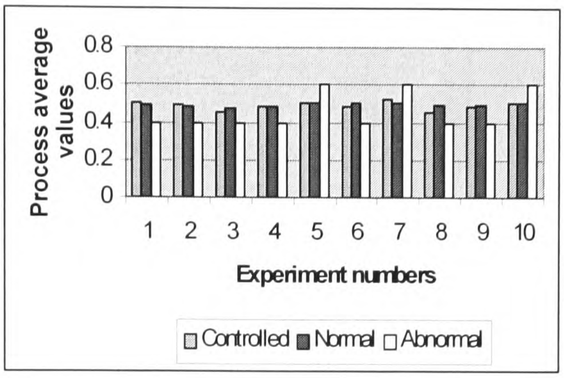
For the primary analysis, table 5.2 summarises the results of 50 simulation performances, which are used to investigate the repeatability and variation of the control effects.

Figure 5.7 illustrates normal process averages, abnormal process averages, controlled abnormal process averages and their control errors for 50 simulation performances which are summarised in Table 5.2. In the Fig. 5.7 (1)~(5), "abnormal" process averages are chosen randomly as 0.4 (*LAbnormal*) or 0.6 (*UAbnormal*), "Normal" process averages have some small random values around 0.5. The abnormal process averages have been shifted approximately $|\pm 0.1|$ which is equal to 10% of the universe of discourse (23% of one side of the chart testing area $\frac{UCL - LCL}{2}$) from the centre line.

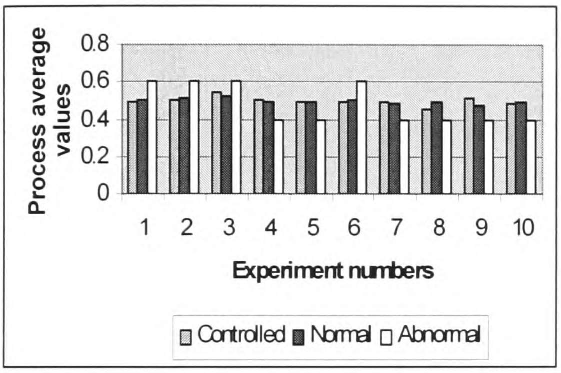
Figure 5.7 describes the "Controlled" abnormal averages, which are much improved and closer to normal data after the control actions. The positions of the controlled abnormal process averages also have a small wave, and it follows the random positions of the normal average points. This random characteristic will be used to investigate the effect of the control action by hypothesis testing in section 5.6.3. The related statistical parameters are calculated and summarised in Table 5.3.



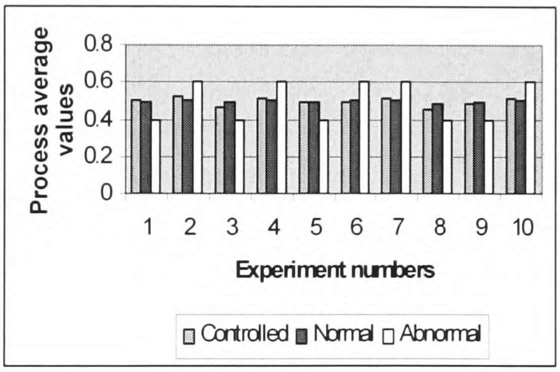
(1)



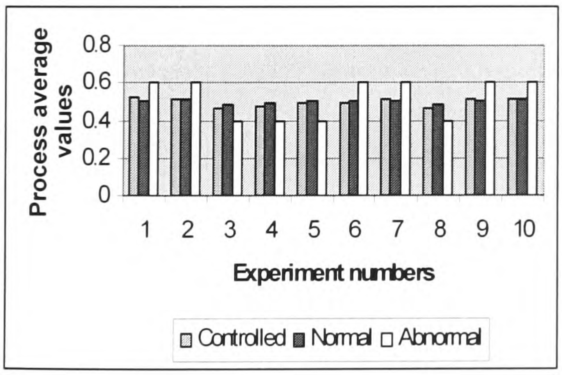
(2)



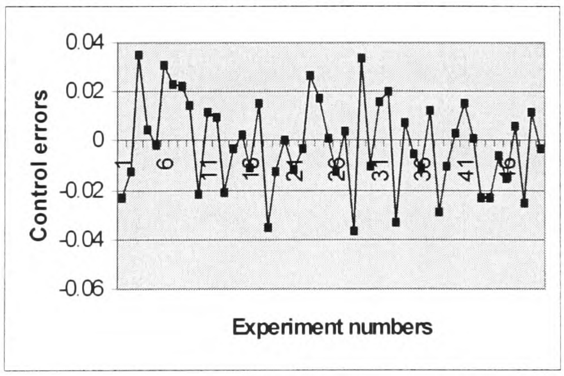
(3)



(4)



(5)



(6)

Figure 5.7 50 control results and related errors

Item	*CAPA*	*NPA*	$Error_1$	$Error_2$
Average	0.4963	0.4975	0.0173	0.0149
S.D.	0.0223	0.0097	0.0205	0.0103
Max	0.5477	0.5212	0.0361	0.0365
Min	0.4521	0.4731	0.0000	0.0000

Table 5.3 The statistical parameters of control results

In table 5.3, the parameter "Average" represents the mean of the controlled abnormal process averages (CAPA). NPA describes the mean of the normal process averages. $Error_1$ and $Error_2$ describe the error average which are calculated by the following equations:

$$Error_1 = \sqrt{\frac{\sum_{i=1}^{50}(NPA_i - CAPA_i)^2}{50}} = 0.0173 \tag{5.1}$$

$$Error_2 = \frac{\sum_{i=1}^{50}|NPA_i - CAPA_i|}{50} = 0.0149 \tag{5.2}$$

For other parameters, the *S.D.*, *Max* and *Min* describe the standard deviation, maximum value and minimum value of the process averages and errors. They describe the range for related random variables CAPA, NPA and Errors.

Finally, if the height of the adjusted space for the abnormal process average that lies between *UAbnormal* and *LAbnormal* which are mentioned previously is viewed as full-scale deflection (f.s.d.), the relative control errors (average) RCE can be calculated as a percentage of f.s.d. (Bentley, 1995):

$$RCE_1 = \frac{Error_1}{|UAbnormal - LAbnormal|} \times 100\% = \frac{0.0173}{|0.6 - 0.4|} \times 100\% = 8.7\% \qquad (5.3)$$

$$RCE_2 = \frac{Error_2}{|UAbnormal - LAbnormal|} \times 100\% = \frac{0.0149}{|0.6 - 0.4|} \times 100\% = 7.5\% \qquad (5.4)$$

5.4 Visual C++6.0 and Microsoft foundation classes (MFC)

Microsoft Visual C++ is a powerful and complex tool for building 32 – bit applications for Window 95 and Window NT (Gregory, 1997). The central part of Visual C++ package is the Developer Studio which is an Integrated Development Environment (IDE). The Developer Studio is used to integrate the development tools and the Visual C++ compiler. Users can create a windows program easily by using the tools which involves the wizards provided as part of the Developer Studio and the Microsoft Foundation Classes (MFC) which is a set of predefined classes or library.

A wizard is a tool that helps guide the user through a series of steps (Williams, 1998). The Developer Studio provides several wizards that are used to simplify developing the windows programs. The most commonly used wizards are AppWizard and ClassWizard.

AppWizard is used to create three type of basic frames of a windows program: Single Document Interface (SDI) application, Multiple Document Interface (MDI) application and dialog based application. A Single Document Interface (SDI) application has only one document open at a time. A Multiple Document Interface (MDI) application such as Excel or Word, can open many documents at once. A dialog – based application does not have any documents or menus (Gregory, 1997).

ClassWizard is used to define the classes in a program generated by AppWizard. By the use of ClassWizard, new classes can be added to the user's project. Functions also can be added to control how messages are received by each class. ClassWizard also helps manage controls that are contained in dialog boxes by enabling the user to associate an MFC object or class member variable with each control (Williams, 1998).

The Microsoft Foundation Classes (MFC) is a set of predefined classes or library that allows Windows programming. This library provides support for all of the frequently used Windows Application Program Interface (API), including windowing functions, messages, controls, menus, dialog boxes, Graphics Device Interface (GDI) objects (fonts, brushes, pens and bitmaps), object linking, Single Document Interface (SDI) and Multiple Document Interface (MDI) (Pappas and Murray, 1994).

AppWizard can generate many programs with very useful characteristics such as Toolbars, Status bars, Windows views, the help functions and completed menus which include open and save file, print and print view. By using the AppWizard and MFC, the user can concentrate on the more important parts and avoid writing code for building the

Windows environment.

5.5 The improved Fuzzy-SPC simulation in Visual C++ 6.0

The simulation system of Statistical Process Control was built in Visual C++ 6 Developer Studio using AppWizard and MFC. The Single Document Interface (SDI) was applied to build this simulation which is called "FuzzySPC".

Figure 5.8 illustrates the outline of the application program. It is consists of four classes: CFuzzySPCApp, CmainFrame, CFuzzySPCView and CFuzzySPCDoc.

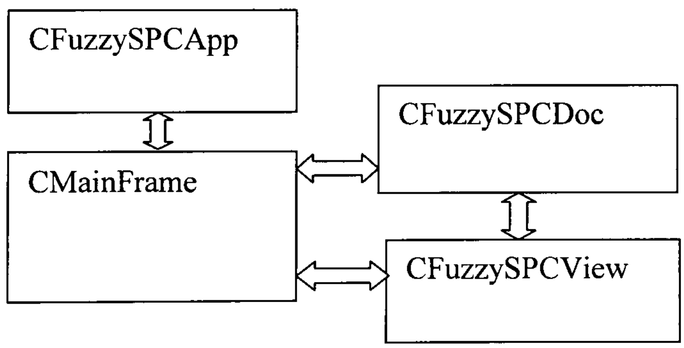


Figure 5.8 Outline of application program

The CFuzzySPCApp is an application class, which is executed firstly in the windows program (Holzner, 1997). It is derived from CwinApp, the base class of MFC. It manages the overall program, for example, to perform initialisation, clearing and displaying the main window.

CmainFrame is a frame class, derived from CframeWnd of base class. It is used to manage the main program window, for example, to display the title, menu bar, toolbars, Status bars and window maximum and minimum keys, etc.

The FuzzySPCDoc is a document class which is used to hold process data and to deal with the file I/O operation. FuzzySPCDoc is derived from MFC Cdocument base class. In the FuzzySPCDoc class, several member functions are designed to manipulate the SPC process data. For example, the member function SpcCal ( ) calculates the process average, upper control limit, lower control limit and the zone boundaries. The member function SpcZR ( ) is used to test the abnormal process data by SPC zone rules 1~5. There are seven and nine fuzzy controllers, which have separate and different fuzzy control bases (or tables) in the class FuzzySPCDoc for different experiments (section 5.6 and section 5.7). The control tables can be obtained from MATLAB rule viewer which corresponds to different fuzzy membership functions (see Fig. 5.9, Fig. 5.16 and Fig. 5.17). The file I/O operation is achieved by member function OnFileOpen ( ) and OnFileSave ( ).

The CFuzzySPCView is a view class, which is used to display program data, graphics on the screen and printer functions. The CFuzzySPCView is derived from MFC CView base class. In the CFuzzySPCView class, member functions ShowPens1(CDC *pDC) and ShowPens2 (CDC *pDC) are designed for drawing the normal process and abnormal process in the $\overline{X}$ charts. Member functions ShowPens01 (CDC *pDC) ~ ShowPens07 (CDC *pDC) and ShowPens01 (CDC *pDC) ~ ShowPens09 (CDC *pDC) are designed for plotting the abnormal process to be controlled by related controllers (section 5.6 and section 5.7) on the $\overline{X}$ charts. The CDC is a draw class, derived from the MFC CObject class. Class CDC can perform many graphics operator (Chen, 1998) and it is referenced via point *pDC in the ShowPens functions.

In this Visual C++ application, the output fuzzy sets of the universe of discourse are changed to [0, 1] for tuning calculation convenience, and some new operators are added to the simulation system (Section 5.3.1). These are:

- Create related data text files I/O on disk for analysis;
- Search several different fuzzy control tables and transfer their control instructions.

5.6 The Visual C++ simulation for varying of membership functions

The aim of this work is to develop an effective way to change the triangle and trapezoid shapes of the membership functions, in order to determine a set of consequent membership functions with suitable form or shape to control the abnormal processes. In this section, the shapes of triangles and trapezoids are changed in three basic ways, the sample and control data are obtained from the performance of the Visual C++ simulation systems. The respective control effects for different shapes of membership functions are identified and compared via the hypothesis *t*-test. After the *t*-test, the most effective way to generate the control result is selected.

Optimal control and optimisation techniques are major contents and tools in random process control (Zheng and Zhu, 1991). The parameters of the fuzzy controller can be adjusted or tuned as an optimisation procedure to achieve good control performance for the complex model of objects which have uncertain conditions (such as a random process) (Reznik, 1997) (Li 1994). It is a frequently used method to refine membership functions in the optimisation of the fuzzy controller. For quick and effective performance in practice, it is necessary to investigate the effects of membership function parameters (section 5.6) before tuning or adjustment (section 5.7). This is in order to evaluate the effect of the membership function and to reduce the total number of membership functions used (Li, 1997).

5.6.1 Basic investigation for varying triangle shape

As discussed in chapter 3, the triangular and trapezoidal functions are determined by the choice of parameters a, b, c and a, b, c, and d (Fig. 3.3 and 3.4). Varying the triangle and trapezoid shapes can be classified as changing the steepness of slope (Boston, 1997), position and width of the function shape. Figure 5.9 illustrates seven different membership functions shapes, which were generated to investigate the effect of the membership function on the simulation results.

(1)
(2)
(3)
(4)
(5)
(6)
(7)

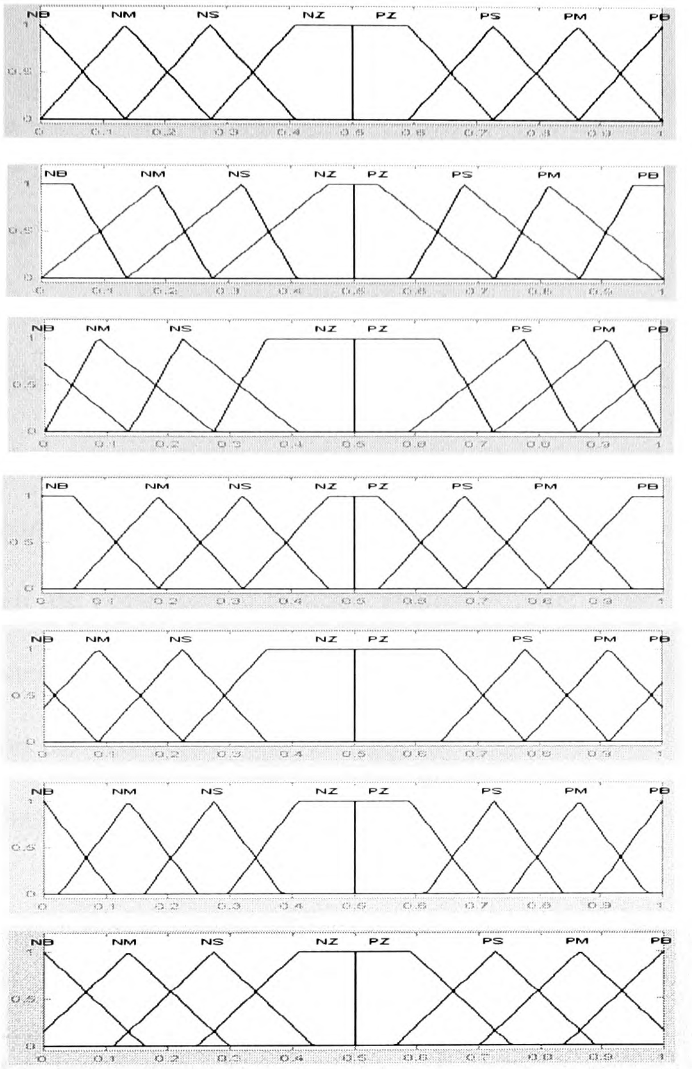


Figure 5.9 Seven types of membership function shapes

No.(1) in Fig.5.9 is a standard triangular membership function, No. (2) and No. (3) illustrate changes to the slope: the peaks of the triangles have been inclined by an average of 0.05 in different directions, No. (4) and (5) represent different positions, No. (6) and No. (7) illustrate different width of the triangular function shapes. The positions and the widths have been changed by 0.05. The value 0.05 is used as a deviation value for the initial approach.

5.6.2 Simulation of seven basic type of membership functions

A sample of nine $\overline{X}$ charts shown in Fig. 5.10 ~ Fig. 5.14 were used as examples to illustrate the executed outputs in which an abnormal process is simulated, tested and controlled by different output membership functions using the fuzzy zone rule 5. Other experiments which uses zone rule 1 ~ zone rule 4 to test and control the abnormal process were summarised in table 5.4. In Fig. 5.10 ~ 5.14, chart (1) represent a normal process and chart (2) is an abnormal process, with abnormal points marked by large circles, vertical lines and number of zone rule used to detect the abnormality. Charts (3) ~ (9) describe the abnormal charts which are controlled at the first abnormal point (control point) of chart (2). The process is tested again after the first control action. It can be seen that to the right of the control point on the charts (3) ~ (9), subsequent abnormal points are reduced and improved to normal data.

(1)

(2)

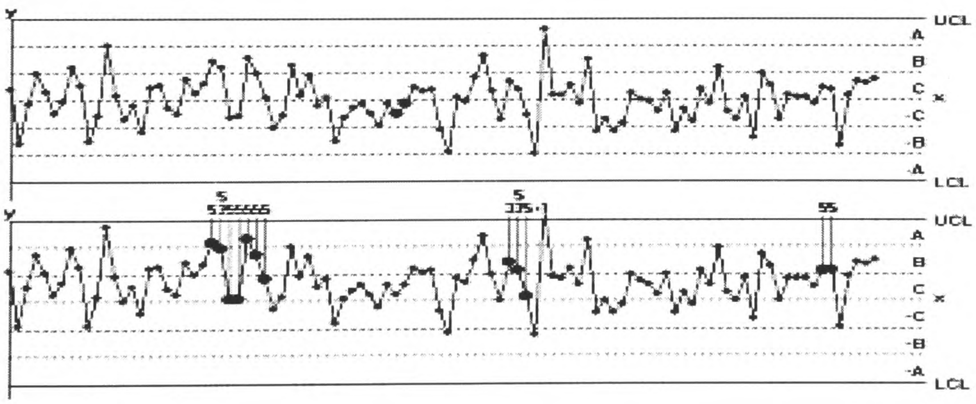


Figure 5.10 Normal and abnormal processes

(3)

(4)

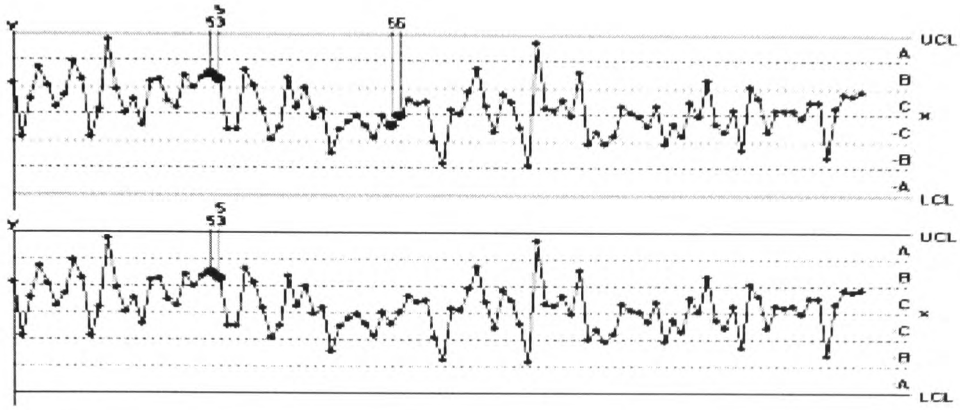


Figure 5.11 Controlled using membership functions No.1 and No.2

(5)

(6)

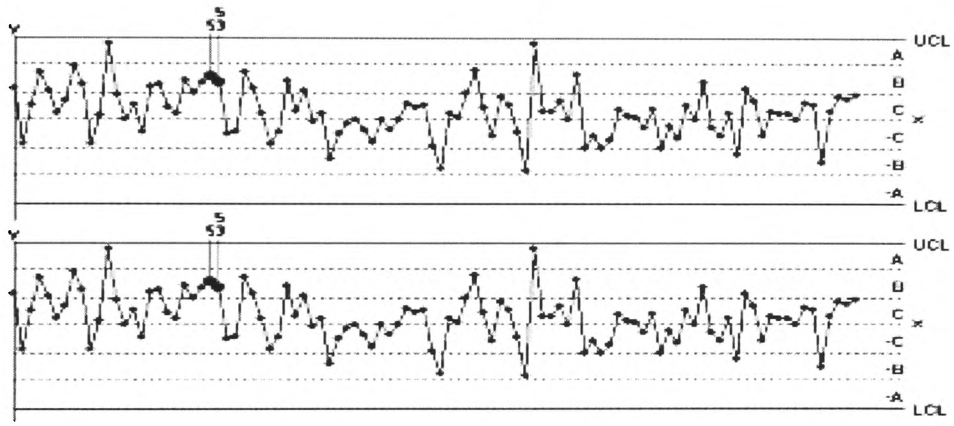


Figure 5.12 Controlled using membership functions No.3 and No.4

(7)

(8)

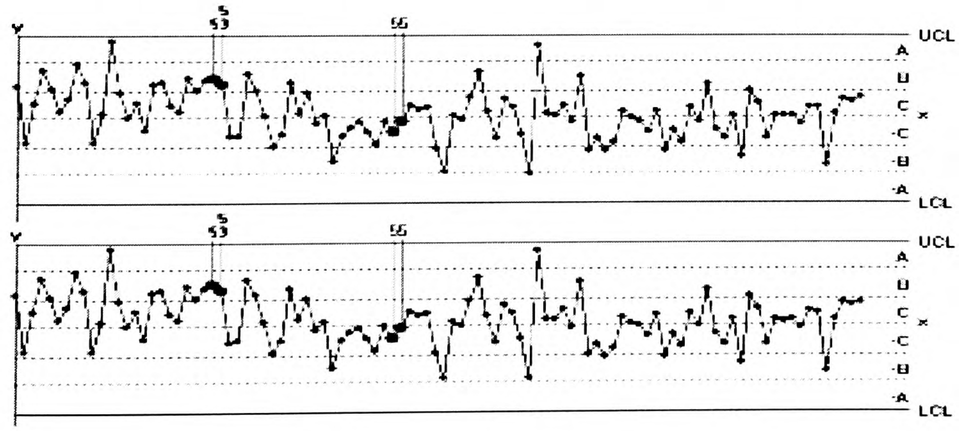


Figure 5.13 Controlled using membership functions No.5 and No.6

(9)

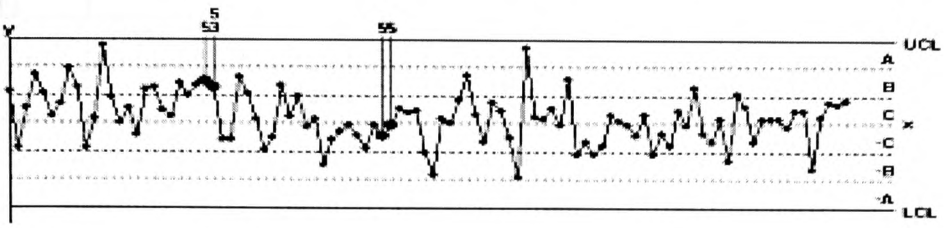


Figure 5.14 Controlled using membership function No.7

In this simulation study, 500 random data were generated by function RAND () for each run where the subgroup size chosen was 5 with the $\overline{X}$ chart plotted for 100 $\overline{X}$ values. The abnormal process is software generated and the system tests and controls it automatically. The simulation was run many times to see the effect of the fuzzy control action on different zone rules. In every SPC zone rule for one adjustment or control action, the abnormal process was controlled successfully by the fuzzy - SPC controller for each of the seven membership functions. Table 5.4 summarises the controlled abnormal process averages (marked by "CAPAverages") for the 7 membership functions. The control errors (marked by "Errors") are calculated from the difference between the normal process averages (marked by "Normal averages") values and CAPAverages values. MF1 ~ MF7 are the seven types of membership functions (see No.(1) ~ (7) in Fig. 5.9). The control errors take the values between |-0.0010|~ |-0.0528|.

Membership Functions (MF)	Item	Zone Rule1	Zone Rule2	Zone Rule3	Zone Rule4	Zone Rule5
MF1	CAPAverages	0.5073	0.4888	0.4582	0.4905	0.5091
	Errors	0.0209	-0.0065	-0.0411	-0.0194	0.0079
MF2	CAPAverages	0.5073	0.4841	0.4532	0.4905	0.5002
	Errors	0.0209	-0.0112	-0.0461	-0.0194	**-0.0010**
MF3	CAPAverages	0.5073	0.4908	0.4632	0.4946	0.5198
	Errors	0.0209	-0.0045	-0.0361	-0.0153	0.0186
MF4	CAPAverages	0.5023	0.4773	0.4465	0.4989	0.5000
	Errors	0.0159	-0.0180	**-0.0528**	-0.0110	-0.0012
MF5	CAPAverages	0.5112	0.4996	0.4687	0.4864	0.5226
	Errors	0.0248	0.0043	-0.0306	-0.0235	0.0214
MF6	CAPAverages	0.5093	0.4891	0.4582	0.4884	0.5093
	Errors	0.0229	-0.0062	-0.0411	-0.0215	0.0081
MF7	CAPAverages	0.5053	0.4886	0.4550	0.4939	0.5127
	Errors	0.0189	-0.0067	-0.0443	-0.0160	0.0115
Normal averages		0.4864	0.4953	0.4993	0.5099	0.5012

Table 5.4 Summary of executed simulation results

The approach of this work is to identify an effective way to change the triangular shape of the membership function only. The control error value and that some abnormal points are still in existence in the chart after the control action should be not used to judge the effectiveness of this controller, since the abnormal processes are controlled by a sample group of membership functions only. These control results are used in section 5.6.3, to investigate the effects of control actions on different membership function shapes.

5.6.3 Comparison of varying the membership function

In this section, the effects of different membership functions are compared using the hypothesis t-test. Before using the t-test, some suppositions need to be discussed:

1. Although the random process generated by the computer in C++ does not belong exactly to a normal distribution, the distribution of the $\overline{X}$ values tends to be close to normal and approach a normal curve (Grant and Leavenworth, 1988), therefore, it is viewed as approximately a normal distribution. Figure 5.15 shows an approximate normal distribution of means $\overline{\overline{X}}$. The means $\overline{\overline{X}}$ can be viewed as random variables. Described in Table 5.2 (section 5.3.3) are averages of $\overline{X}$ for abnormal processes controlled by standard membership function MF1 (No. (1) in Fig. 5.9), and obtained from 50 simulation runs, which includes a total of 25000 random data points and 5000 $\overline{X}$ values.

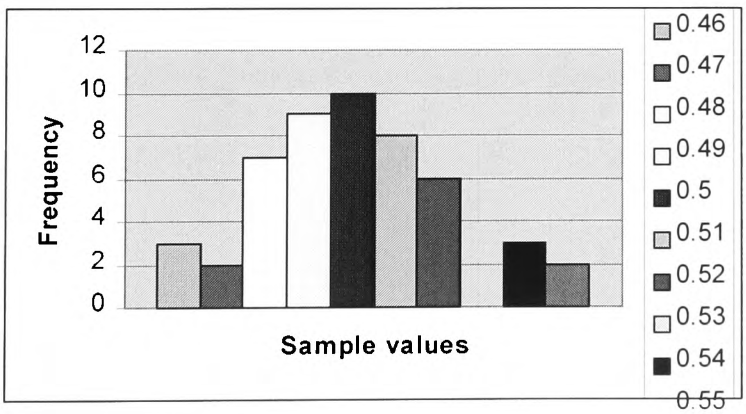


Figure 5.15 The frequent distribution of process averages controlled by MF1

2. The abnormal processes were controlled using the average values only. For the seven different membership functions, the standard deviations in every run were the same.

3. The seven control results in every run were generated from different fuzzy membership functions, and they were assumed to be independent.

The hypothesis *t*-test can be used (Montgomery and Runger, 1994) (Li, 1986) to investigate the effect of the control action of the seven membership functions described. For example, to compare the effects of the control actions using the standard membership function MF1 and sample membership function MF2 (section 5.6.2), the following applies:

Suppose two independent normal populations with unknown means μ_{MF1} and μ_{MF2}, and unknown variances with the same values: $\sigma_{MF1} = \sigma_{MF2}$, the hypothesis H_0 and H_1:

$$H_0 : \mu_{MF1} = \mu_{MF2} \tag{5.5}$$

$$H_1 : \mu_{MF1} \neq \mu_{MF2} \tag{5.6}$$

can be tested by an estimate S_p^2 and test statistic t_0 (Montgomery and Runger, 1994).

where:

$$S_p^2 = \frac{(n_{MF1}-1)S_{MF1}^2 + (n_{MF2}-1)S_{MF2}^2}{n_{MF1}+n_{MF2}-2} \tag{5.7}$$

and

$$t_0 = \frac{\overline{Y}_{MF1} - \overline{Y}_{MF2}}{S_p\sqrt{\frac{1}{n_{MF1}}+\frac{1}{n_{MF2}}}} \tag{5.8}$$

where S, $\overline{Y}$ and n are sample variances, sample mean and sample number ($n = 10$) for the process averages $\overline{\overline{X}}$ of abnormal processes controlled by different membership functions MF1 and MF2 (it is marked by MF1~2 in Table 5.5). The value of t_0 is written in the second column of Table 5.5. Similarly, the above hypothesis t - test and equations can be used to compare MF1 and MF3, MF1 and MF4,..., MF1 and MF7 (they are marked by MF1~3,..., MF1~7 respectively in Table 5.5). The different results for t_0 are given in Table 5.5 (also see Appendix B.6).

	MF1~2	MF1~3	MF1~4	MF1~5	MF1~6	MF1~7
t_0	2.0475	-3.3686	2.3708	-3.9872	-0.3634	-0.1380

Table 5.5 The test statistic t_0 values calculated

If the value 0.10 is chosen as the significance level α then the two sided critical region can be written as $-t_{\alpha/2} \sim t_{\alpha/2}$ = -1.833~1.833. Because the calculated values t_0 of MF1~2, MF1~3, MF1~4 and MF1~5 described in Table 5.5 fall out the significant region –

1.833~+1.833, the related hypothesis H_0 are rejected. That is, the control results which are generated by MF2, MF3, MF4 or MF5 are different to the control results, which are generated by the standard membership function MF1. For MF1~6 and MF1~7, the hypothesis H_0 is not rejected as their t_0 fall inside the significance region. These calculation results imply that, varying the membership functions from a standard one, the tuning of position (MF4 and MF5) and slope (MF2 and MF3) of triangle and trapezoid have a greater effect on the control result than tuning the width of the triangle and trapezoid functions (MF6 and MF7).

5.7 Further approach to SPC zone rules via a set of membership function schemes using visual C++ simulation

A further approach based on the previous section, using a set of nine shapes or structures of consequent membership function schemes with more effective and representative characteristics is designed for control simulation in this section. The control results are summarised and analysed.

5.7.1 A set of membership function schemes

After some discussions in section 5.6.3, several structures of triangular and trapezoidal consequent membership functions with a greater effect on the control results are designed in this section. The main focus is placed on the triangle and trapezoid positions and triangle slopes (section 5.6.3) which are varied to a greater extent than in section 5.6.1, in order to obtain a more effective influence on the control process, and more effective and representative characteristic to varying the triangle and trapezoid functions. The widths of the triangles and trapezoids are also changed depending on their positions and slopes. Figure 5.16 and Figure 5.17 illustrate the set of nine different membership function schemes chosen.

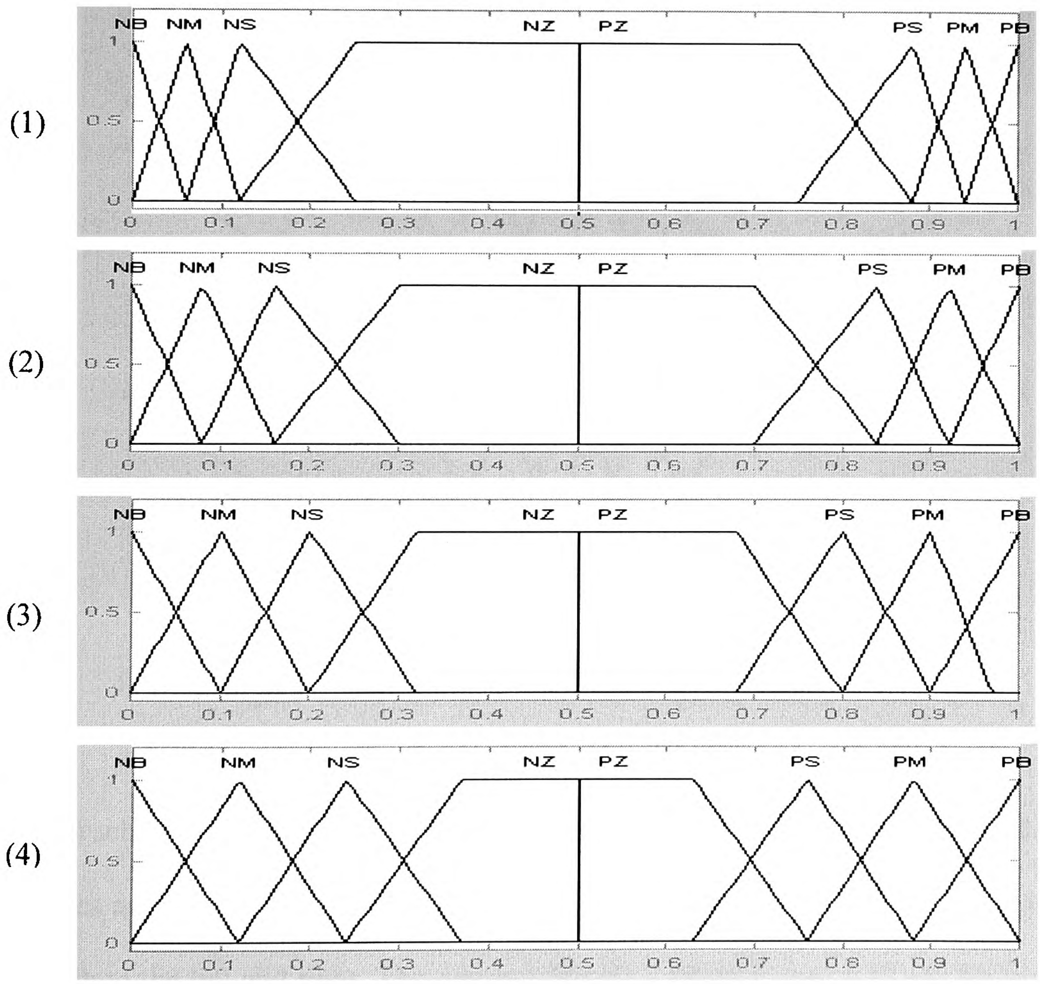


Figure 5.16 No. 1~No. 4 membership functions

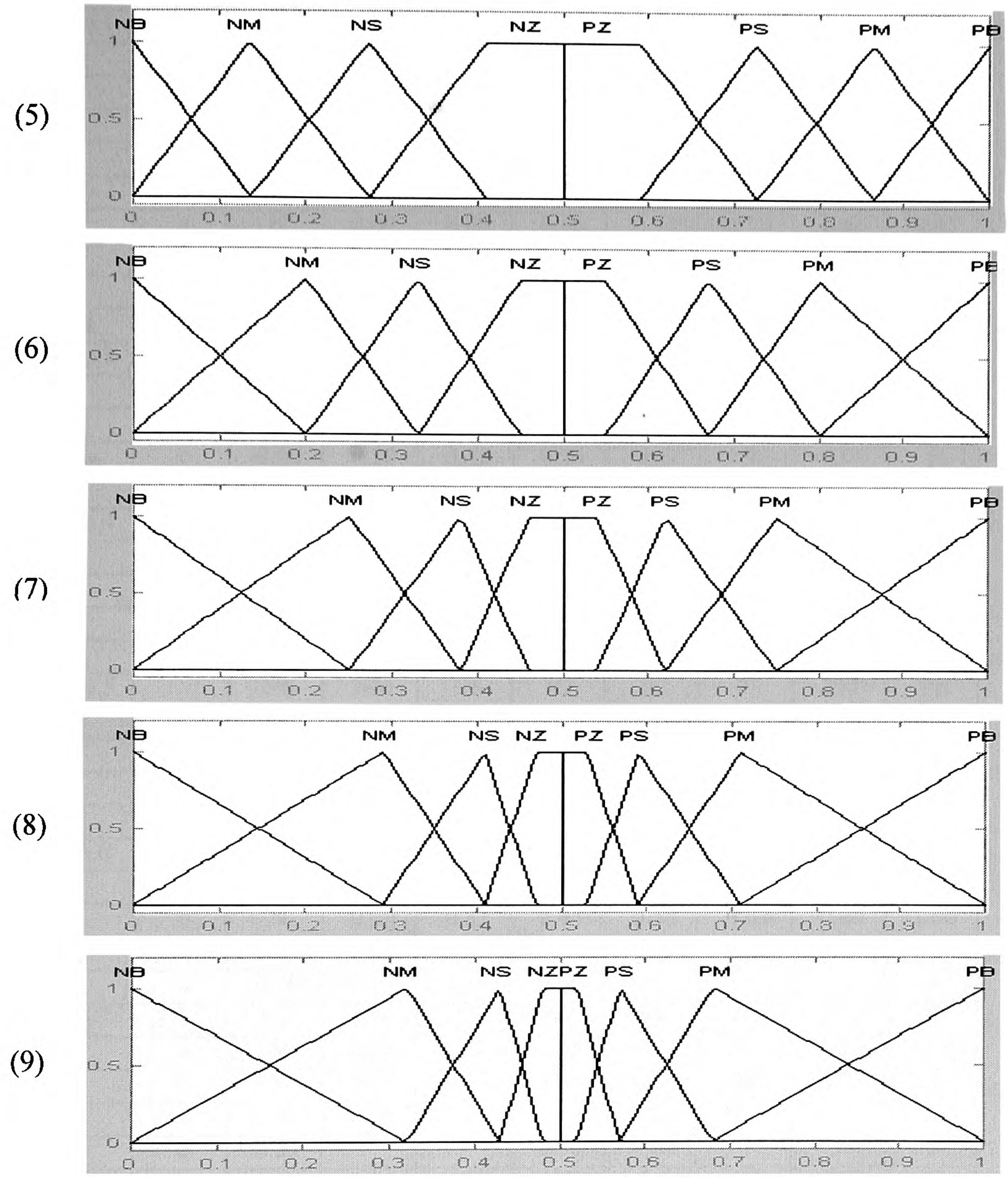


Figure 5.17 No. 5~No. 9 membership functions

The No.(5) scheme of membership function in Figure 5.17 is viewed as the standard type, the triangles and trapezoids of the schemes No.(1) ~ (4) in Figure 5.16 are designed to deviate both to the left and right. The schemes No.(6) ~ (9) in Figure 5.17 are designed to deviate to the centre. Table 5.6 shows the parameters for triangular and trapezoidal membership functions (also see section 3.2.2) which are described in Figs 5.16~5.17.

No.1		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.06	0.12	0.50	0.75	0.88	0.94
	b	N	0.06	0.12	0.25	0.50	0.88	0.94	1.00
	c	0.00	0.12	0.25	0.50	0.75	0.94	1.00	N
	d	0.06			0.50	0.88			N
No.2		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.08	0.16	0.50	0.70	0.84	0.92
	b	N	0.08	0.16	0.30	0.50	0.84	0.92	1.00
	c	0.00	0.16	0.30	0.50	0.70	0.92	1.00	N
	d	0.08			0.50	0.84			N
No.3		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.10	0.20	0.50	0.68	0.80	0.90
	b	N	0.10	0.20	0.32	0.50	0.80	0.90	1.00
	c	0.00	0.20	0.32	0.50	0.68	0.90	0.97	N
	d	0.10			0.50	0.80			N
No.4		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.12	0.24	0.50	0.63	0.76	0.88
	b	N	0.12	0.24	0.37	0.50	0.76	0.88	1.00
	c	0.00	0.24	0.37	0.50	0.63	0.88	1.00	N
	d	0.12			0.50	0.76			N
No.5		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.14	0.29	0.50	0.57	0.71	0.86
	b	N	0.14	0.29	0.43	0.50	0.71	0.86	1.00
	c	0.00	0.29	0.43	0.50	0.57	0.86	1.00	N
	d	0.14			0.50	0.71			N
No.6		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.20	0.33	0.50	0.55	0.67	0.80
	b	N	0.20	0.33	0.45	0.50	0.67	0.80	1.00
	c	0.00	0.33	0.45	0.50	0.55	0.80	1.00	N
	d	0.20			0.50	0.67			N
No.7		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.25	0.38	0.50	0.54	0.62	0.75
	b	N	0.25	0.38	0.46	0.50	0.62	0.75	1.00
	c	0.00	0.38	0.46	0.50	0.54	0.75	1.00	N
	d	0.25			0.50	0.62			N
No.8		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.29	0.41	0.50	0.53	0.59	0.71
	b	N	0.29	0.41	0.47	0.50	0.59	0.71	1.00
	c	0.00	0.41	0.47	0.50	0.53	0.71	1.00	N
	d	0.29			0.50	0.59			N
No.9		NB	NM	NS	NZ	PZ	PS	PM	PB
	a	N	0.00	0.32	0.43	0.50	0.52	0.57	0.68
	b	N	0.32	0.43	0.48	0.50	0.57	0.68	1.00
	c	0.00	0.43	0.48	0.50	0.52	0.68	1.00	N
	d	0.32			0.50	0.57			N

Table 5.6 Summary of parameters for No.1~No.9 functions

In Table 5.6, some values that exceeded the universe of discourse [0,1] are written as "N". The first step of varying from the standard membership functions No.5 to membership functions No.4, the peak position ("b" parameter) of the NS function has been changed by $0.24-0.29=-0.05$. The biggest change in the negative direction for the NS peak is the difference between No.5 and No.1, the value is $0.12-0.29=-0.17$. Similarly in the positive direction, the peak position of the NM function has been changed from No.5 to No.6 ~ No.9 from the difference value 0.06 (0.2-0.14) to 0.18 (0.32-0.14). The maximum value has a deviation of more than 3 times larger than the one mentioned in section 5.6.1.

5.7.2 Simulation and analysis

5.7.2.1 Simulation

A sample of eleven $\overline{X}$ charts shown in (1) ~ (11) of Fig. 5.18 and Fig. 5.19 illustrate the executed outputs in which an abnormal process is simulated, tested and controlled using the SPC zone rule 5 by different consequent membership functions discussed in section 5.7.1. Other experiments that uses zone rule 1 ~ zone rule 4 to test and control the abnormal process are summarised in table 5.7 and discussed in section 5.7.2.2.

In Figure 5.18, chart (1) represent a normal process and chart (2) is an abnormal process, abnormal points are marked by large circle, vertical line and number of zone rule used to detect the abnormality. Charts (3) ~(11) of Figures 5.18 and 5.19 show that the abnormal processes are controlled at the first abnormal point (control point) of chart (2). The processes are tested again after the first control action.

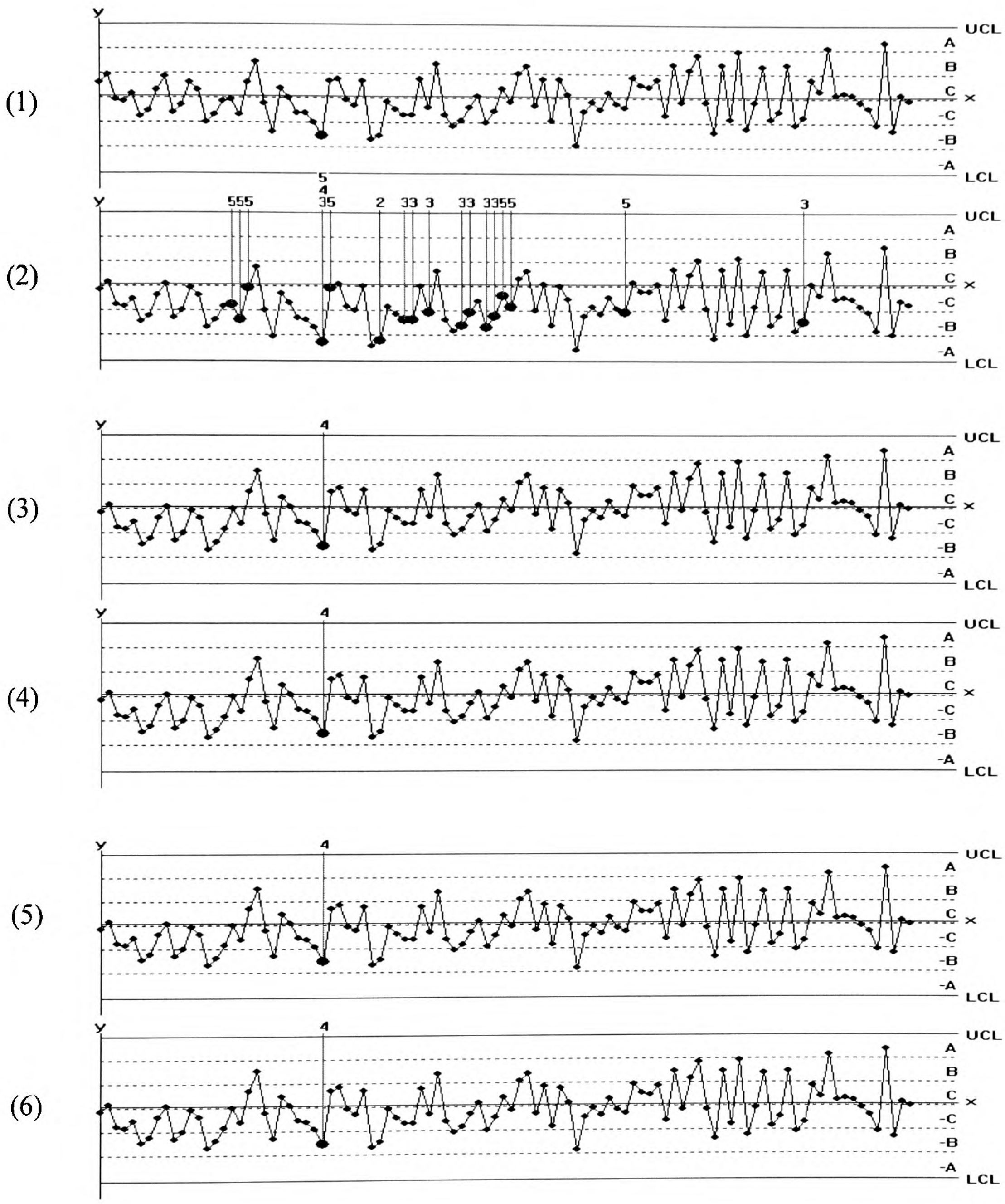


Figure 5.18 Display (1) of output of simulation system (controlled at zone rule 5)

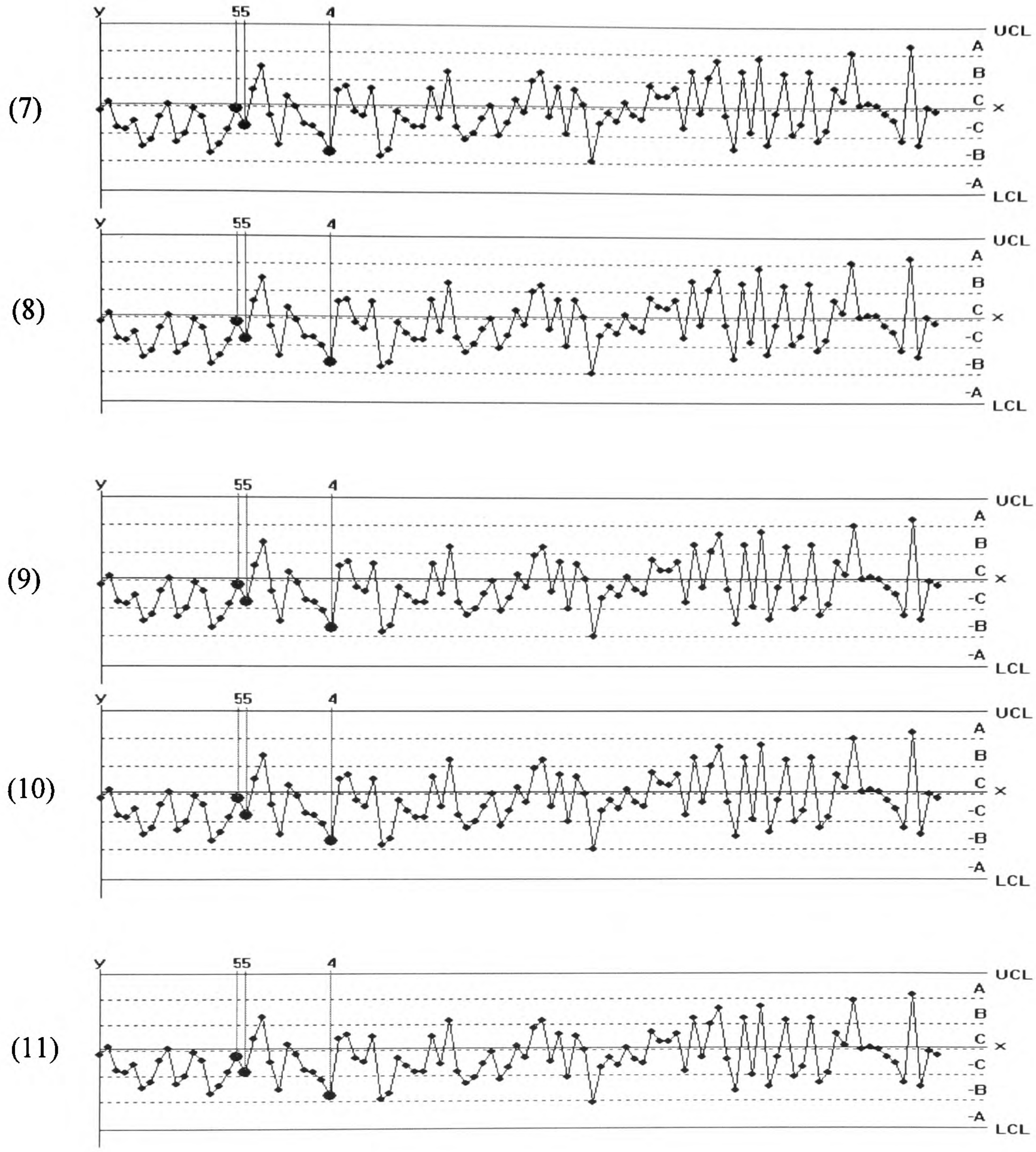


Figure 5.19 Display (2) of output of simulation system (controlled at zone rule 5)

It can be seen that to the right of the control point, subsequent abnormal points in charts (3) ~ (11) are reduced and improved to normal data that is shown on chart (1). The simulation methods are the same as in section 5.6.2. 500 random data were generated by function "RAND ()" for each run and the subgroup size chosen was 5 with the $\overline{X}$ chart plotted for 100 $\overline{X}$ values. The abnormal process is software generated and the system tests and controls it automatically. The table 5.7 summarises 10 abnormal processes averages controlled by 9 different membership functions. The first column describes SPC zone rules 1 to 5 in both of positive and negative areas. The second column shows the related normal processes averages (NPA). The abnormal processes average controlled by nine membership functions are summarised in column 3 ~ column 11. The minimum errors which indicate the best membership functions (MFs) in this table are calculated from the difference between the normal process averages and abnormal values controlled (bold), shown in the errors column.

	NPA	MF1	MF2	MF3	MF4	MF5	MF6	MF7	MF8	MF9	errors
ZR1+	0.5022	0.4979	0.4995	0.4998	0.5011	**0.5029**	0.5073	0.5112	0.5143	0.5166	-0.0007
ZR1-	0.4829	0.4911	0.4898	0.4901	0.4887	0.4871	**0.4830**	0.4796	0.4767	0.4746	-0.0001
ZR2+	0.5035	0.4988	0.5010	0.5013	**0.5033**	0.5108	0.5165	0.5200	0.5228	0.5303	0.0002
ZR2-	0.4974	0.5000	0.4990	0.4987	**0.4967**	0.4892	0.4835	0.4800	0.4770	0.4697	0.0007
ZR3+	0.5186	0.5168	**0.5210**	0.5258	0.5333	0.5465	0.5595	0.5670	0.5718	0.5760	-0.0024
ZR3-	0.4861	**0.4832**	0.4787	0.4742	0.4667	0.4535	0.4405	0.4327	0.4282	0.4240	0.0029
ZR4+	0.5148	0.5000	0.5010	0.5013	0.5035	0.5108	**0.5165**	0.5198	0.5230	0.5303	-0.0017
ZR4-	0.4948	0.5000	0.4990	0.4987	**0.4965**	0.4892	0.4835	0.4800	0.4770	0.4697	-0.0017
ZR5+	0.5073	0.5000	0.5010	0.5013	0.5035	**0.5108**	0.5165	0.5198	0.5230	0.5303	-0.0035
ZR5-	0.4825	0.5000	0.4990	0.4987	0.4965	0.4892	**0.4835**	0.4800	0.4770	0.4697	-0.0010

Table 5.7 Ten abnormal processes controlled by nine membership functions

In table 5.7, the bold values imply that, to obtain the minimum control error, it is better to use the related membership functions (in the bold value column) for the related zone rules (in the same bold value row).

5.7.2.2 *Analysis*

The abnormal process average controlled at every zone rule by the varying membership functions which are represented in Table 5.7 can be plotted on the charts (Figure 5.20 ~ Figure 5.24) for further analysis.

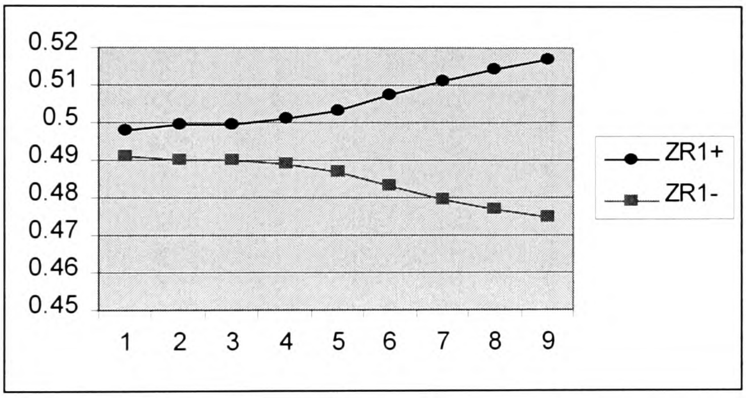


Figure 5.20 Range of abnormal average controlled at ZR1

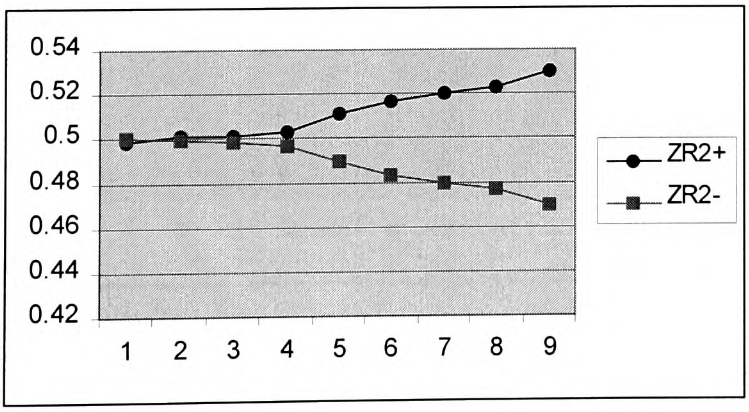


Figure 5.21 Range of abnormal average controlled at ZR2

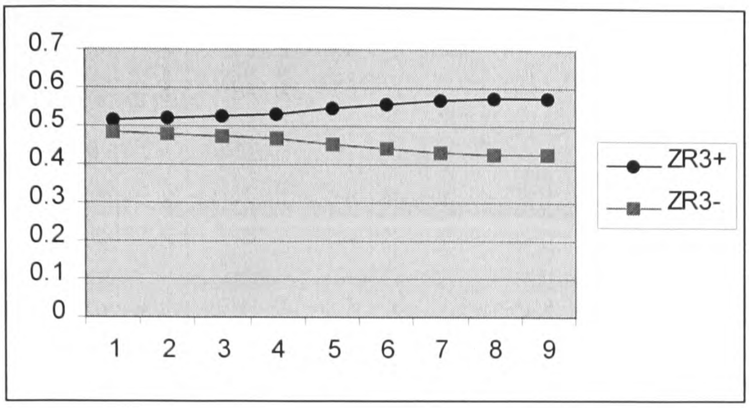


Figure 5.22 Range of abnormal average controlled at ZR3

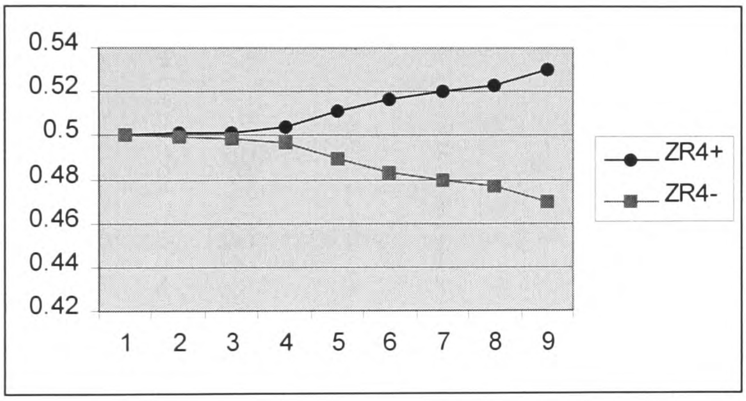


Figure 5.23 Range of abnormal average controlled at ZR4

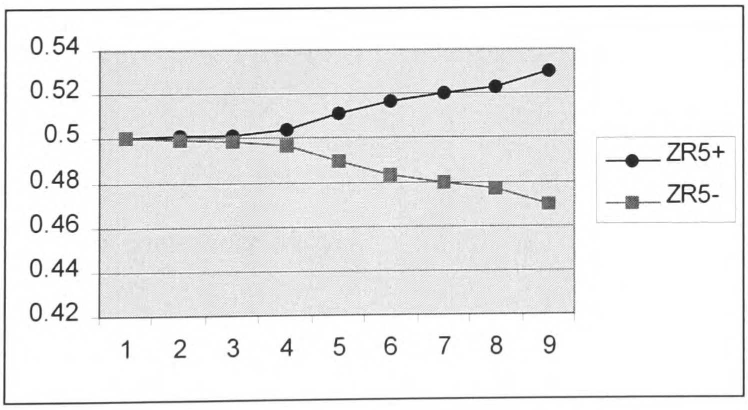


Figure 5.24 *Range of abnormal average controlled at ZR5*

From Table 5.7 and Figs. 5.20~5.24, some phenomena can be identified:

1. The monotone characteristic

The curves in Figure 5.20 ~ 5.24 show a monotonic increase or a monotonic decrease, that is, there are approximate proportional relationships between the control result and the varying of the membership function parameters which is discussed in section 5.7.1. It confirms that one way to investigate the fuzzy controller behaviour and to optimise a fuzzy controller is by varying the membership function parameters. (Ross, 1995) (Li and Li 1996) (Baglio et al, 1994).

2. The normal process average is covered in every curve

Every curve cover the related normal process average (see the second column named "NPA" in table 5.7 and related figure). In other words, the better control results which are closed to the normal process average with the smallest control errors can be obtained from the set of nine membership functions.

3. Controllable area and control accuracy

The abnormal process averages are shifted 0.1 times the universe of discourse (section 5.3.3) in this approach. It can be viewed as the smallest controllable shift or sensitivity of this Fuzzy-SPC control system. Similar to equations 5.3 and 5.4 in section 5.3.3, the control error averages for Table 5.7 are calculated as $RCE_1 = 0.93\%$ and $RCE_2 = 0.76\%$.

Compared to equation 5.3 and 5.4, the control errors are reduced approximately 10 times.

Fig. 5.20~5.24 show that the control results, which are generated by MF1 to MF9 form a controllable area identified by the range of abnormal average results. By varying the membership function further, the controllable area can be increased. For a fixed set of membership functions, a lower control accuracy is achievable due to the resolution step of the membership function being greater and thus increasing the distances between every control point.

5.8 Conclusion

The C++ language, especially the Visual C++ is a powerful tool to build window programs. A Fuzzy-SPC simulation system is designed using AppWizard and MFC in the Visual C++ Developer Studio. The simulation system has a fast performance speed and uses the standard window environment. It is a primary attempt for developing a real time system of the future work.

The abnormal processes, which are shifted by 0.1 times of the universe of discourse, are successfully adjusted or controlled by a standard consequent membership function in a simulation study. The related control errors RCE_1 is 8.7% and RCE_2 is 7.5% (section 5.3).

To further reduce the control errors, the tuning of membership functions is a common and effective method. It is an effective way to tune the position and slope of triangular and trapezoidal membership functions. This idea is verified by a statistical *t*-test. Based on this idea, a set of membership function scheme is designed and used to control the abnormal processes successfully. The best membership functions generate control actions with smaller control errors in the simulation experiments. The range curves of controlled abnormal averages (Fig. 5.20~5.24) show the monotonic characteristic and inclusion (of the normal process average) characteristic. The monotonic increasing or monotonic decreasing of curves indicate that varying of membership function parameters is one feasible way to investigate the fuzzy controller behaviour and to optimise a fuzzy controller. As every curve covers the normal process average, it indicates that the best or more suitable membership function, which causes the reduction of control error ($RCE_1 = 0.93\%$ and $RCE_2 = 0.76\%$), can be selected from the set of membership function schemes.

5.9 Summary

In chapter 5, an overview of the basic concepts of the C++ language and a preliminary simulation study which is written in Borland C++ 5.0 with OOP function are described. The abnormal process is software generated by shifting the process average (Box, 1997) and the system tests and adjusts it automatically. The control results, which are analysed by statistical techniques, show that the Fuzzy-SPC controller successfully adjusts and improves the processes for the simulated data.

The Visual C++ 6.0, Microsoft Foundation Classes (MFC) and its applications of the Fuzzy-SPC simulation system are also introduced. The AppWizard and MFC are used to build the system structure, and a standard window display environment is developed for the Fuzzy-SPC simulation. This system structure also can be further developed as a real time system in the future.

As an initial investigation for the tuning of the Fuzzy-SPC controller, the hypothesis *t*-test is applied to determine the effect of the control action of seven basic forms of membership functions. The analysis results indicates that, to tune the membership function from a standard one for a change in control result, the tuning of position or slope of the triangle and trapezoid functions have a greater effect than tuning of their width. This is an important issue when building a more effective and representative set of membership functions schemes to control abnormal processes.

Based on the analysis result discussed above, a set of membership function scheme is designed and used to control the abnormal processes with a reduction of control errors. Related Table and Figures show that the best membership functions can generate control actions with smaller control errors. The range curves of controlled abnormal average show the monotonic characteristic and inclusion characteristic which indicate the varying of membership function parameters is one feasible way to investigate the fuzzy controller behaviour and to optimise a fuzzy controller. The best or most suitable membership function, which causes the control results which are nearest to the normal process mean can be selected.

However, there are two weakness in the Fuzzy-SPC controller described in this chapter.

1. In the set of nine membership function schemes, the fuzzy subset structures are coarsely tuned. This led to a reduced control accuracy.

2. The simulation is restricted to an abnormal process average shifted level of 0.1 times the universe discourse.

These two weaknesses are addressed and improved in chapter 6.

Chapter 6 Neural network and NN-Fuzzy-SPC

Both neural networks and fuzzy logic are employed to build a NN-Fuzzy-SPC system in this chapter. Fuzzy logic is used to generate the fuzzy control actions, and a neural net is used to optimise the parameters of the fuzzy controller. For different process mean shifts and range spreads, the dynamically optimised $\overline{X}$ and R controllers can adjust the abnormal processes with high accuracy.

6.1 Introduction

This chapter discusses Artificial Neural Network, NeuroFuzzy networks and a NN-Fuzzy-SPC system. Similar to fuzzy logic, the neural network is also a type of artificial intelligence methodology. Neural network is employed in this chapter for the dynamic optimisation of fuzzy membership functions.

An overview of neural network operational principle is given in section 6.2. It contains the general operation of neural networks, typical architecture, Back Propagation algorithm, training and working phases, advantages and disadvantages. Section 6.3 describes the NeuroFuzzy network, which is a combination technique of neural network and fuzzy logic. The advantage of the technique, research background and the operation of a neurofuzzy network for the Takagi-Sugeno model are discussed in this section. As an application of neurofuzzy network, the NN-Fuzzy-SPC system is explained in section 6.4. It involves the design of neurofuzzy controllers, system architecture and performance, simulation and analysis. In section 6.4, the SPC control actions are interpreted by approximate reasoning of fuzzy logic and the dynamic fuzzy membership functions are optimised automatically by the learning capability of a neural network. Finally, the conclusion of this chapter is written in section 6.5.

6.2 Overview of neural networks

6.2.1 The artificial neural networks

The basic computing element in biological system is the neuron. A neuron is a small cell that receive electrochemical stimuli from multiple sources and responds by generating electrical impulses that are transmitted to other neutons. There are more than ten billion neurons, which are used to receive, generate, send and manage the information in the human nervous system (Patterson, 1996) (Li, 1998).

Artificial Neural Networks (NN) are simplified models of the central nervous system or organic brain. The NNs are networks of highly interconnected neural computing elements that have the ability to respond to input stimuli and to learn to adapt to the environment (Patterson, 1996). Neural Networks can also be viewed as the techniques for using computer software to model computational properties, which have storing and learning capabilities (Omidvar and Elliott, 1997). These techniques model the brain with artificial neurons all connected together forming the network. These artificial neurons are made up of simple binary nodes that can have two states of 0 or inactive and 1 or active. The synapses, which cause excitation or inhibition between these neurons, are made up of weighted electrical connections (Fig. 6.1) and the neurons are allowed to interact with each other. With the aid of a suitable mathematical algorithm, the network can learn by adjusting the weight of connections between neurons. A neural network also can efficiently approximate and interpolate multivariate data that might otherwise require huge databases; such techniques are now well accepted for non-linear statistical fitting and prediction (Omidvar and Elliott, 1997).

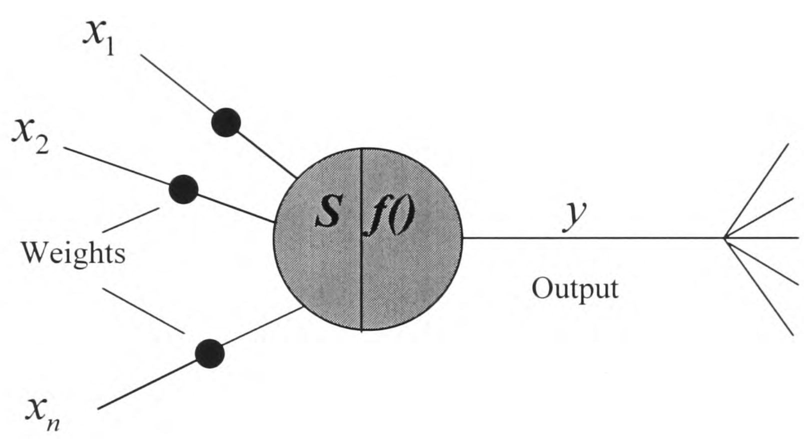


Figure 6.1 The model of an artificial neuron (node)

Neural networks contain a set of processing nodes (artificial neurons) interconnected in parallel. Usually, each node (Fig. 6.1) consists of inputs x_i $(i = 1,...,n)$, weights, a summation function S, an activation function $f()$ and an output y (Oztemel, 1992). The weights determine the influence of inputs or synaptic strength of the neuron (Von Altrock, 1995), the summation function combines all inputs and makes their weighted sum, the activation function computes the output of the neuron by line, step or sigmoid conversion functions in order to simulate the neurons impulsive nature and fatigue performance. The input/output function for j^{th} node in a layer is given by equation (6.1).

$$S_j = \sum_{i=1}^{n} w_{ji} x_i - b_j = \sum_{i=0}^{n} w_{ji} x_i \;\; (x_0 = b_j, w_{j0} = -1) \tag{6.1}$$

$$y_j = f(s) \tag{6.2}$$

where w_{ji} are weights, b_j is called bias which represents the activation threshold of the j^{th} neuron or node, n is the input number for a node.

The activation function can have various shapes depending on the application (Jantzen, 1998). Figure 6.2 illustrate six common activation functions: proportion function (1), sign function (2), saturation function (3), step function (4), hyperbolic function (5) and sigmoid function(6).

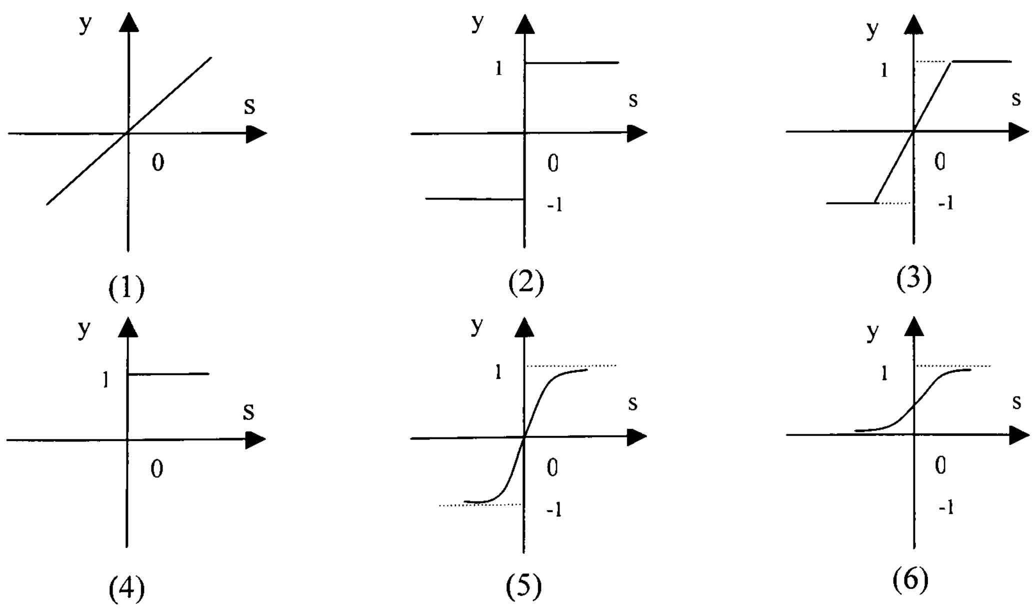


Figure 6.2 Common activation functions

Their equations are given by (6.3) ~ (6.8) respectively.

(1) Proportion function

$$y = f(s) = s \tag{6.3}$$

(2) Sign function

$$y = f(s) = \begin{cases} 1 & s \geq 0 \\ -1 & s < 0 \end{cases} \tag{6.4}$$

(3) Saturation function

$$y = f(s) = \begin{cases} 1 & s \geq \dfrac{1}{k} \\ ks & -\dfrac{1}{k} \leq s < \dfrac{1}{k} \\ -1 & s < -\dfrac{1}{k} \end{cases} \tag{6.5}$$

where k is constant and great than zero.

(4) Step function

$$y = f(s) = \begin{cases} 1 & s \geq 0 \\ 0 & s < 0 \end{cases} \tag{6.6}$$

(5) Hyperbolic function

$$y = f(s) = \frac{e^{s} - e^{-s}}{e^{s} + e^{-s}} \tag{6.7}$$

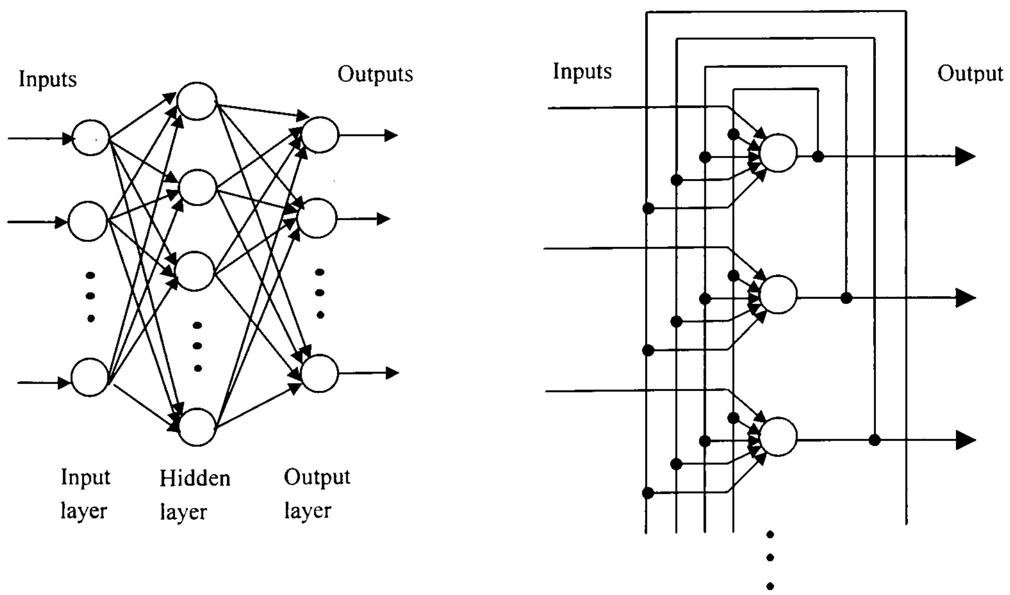
(6) Sigmoid function

$$y = f(s) = \frac{1}{1 + e^{-s}} \tag{6.8}$$

There are multitudes of ways to build a neural network. They differ in their activation function, network topology (architecture) (see section 6.2.2) and learning algorithm (section 6.2.3). For the activation function, the sigmoid function and hyperbolic function are commonly used for the error Back Propagation (BP) algorithm (section 6.2.3) as they satisfy the bounded and differentiable requirements (Patterson, 1996).

6.2.2 *Typical architectures of neural networks*

Figure 6.3 shows more typical models of the neural network architecture classification: (1) feedforward and (2) feedback. They can be used to derive different architectures. In the feedforward model, networks can be divided into input layer, output layer and hidden layer or layers. The nodes are forward connected between adjacent layers, signals propagate only in the direction from the input layer, through intermediate hidden layers nodes, to the output layer. In the feedback model, the signals may propagate in the direction from the output (or output layer) nodes to the input (or input layer) nodes.



(1) (2)

Figure 6.3 Typical architecture of neural networks

In feedforward, a single layer model which is called perceptron can be applied to classify linear separable problems (Rosenblatt, 1958). For linear inseparable problems, it is necessary to use multi-layer perceptron (MLP) which have more layers: input layer, output layer and hidden layers. A popular learning rule which is called error Back-Propagation algorithm (BP) is applied to mapping the non-linear relationships based on the feedforward model (Jantzen, 1998). The feedback model was developed by J. Hopfield in 1982 (Hopfield, 1982). It can be used to generate associative memory system and to solve route optimisation problems.

6.2.3 The error back propagation (BP) algorithm

The learning algorithm conducts a search through the space of parameter values for a set of values with which the network will perform the required function. As discussed in section 6.2.1, neural networks learn or obtain the knowledge via adjusting the connecting weights. The Error Back Propagation (BP) was developed for the multiple layer feedforward network (it is also called BP network) by Rumelhart (Rumelhart and Mcclelland, 1986), it is frequently applied in pattern identification, system identification, forecasting, control and image processing areas (Warwick, 1992), (Sun, 1997).

In BP networks, the sigmoid model (equation 6.8) is used as the activation function. Suppose the node q in the output layer, where $q = 1,2,...,m_Q$, and m_Q is the number of nodes in the current layer (Q^{th} layer) and node p in last layer, $p = 1,2,...,m_{Q-1}$, m_{Q-1} is the number of nodes in last layer ($Q-1^{th}$ layer), y_p is state or output of node *p*. The weighted input s_q and state or output y_q for node q are given by equation 6.9 and equation 6.10.

$$s_q = \sum_p y_p w_{pq} \tag{6.9}$$

$$y_q = \frac{1}{1+e^{-s_q}} \tag{6.10}$$

The system error E is calculated by equation 6.11.

$$E = \frac{1}{2}\sum_q (y_q - d_q)^2 \tag{6.11}$$

where d_q is desired output for node q.

Normally the gradient descent method is used to calculate the minimum objective error function (or system error function) E (Jantzen, 1998),(Sun, 1997). The BP algorithm is summarised in four computing steps:

1. Calculate the error partial derivative of E (equation 6.11) for node state y_q:

$$EA_q = \frac{\partial E}{\partial y_q} = y_q - d_q \tag{6.12}$$

2. Calculate the error partial derivative for node input s_q:

$$EI_q = \frac{\partial E}{\partial s_q} = \frac{\partial E}{\partial y_q} \cdot \frac{\partial y_q}{\partial s_q} = EA_q \cdot y_q(1-y_q) \tag{6.13}$$

3. Calculate the error partial derivative for connecting weight w_{pq}:

$$EW_{pq} = \frac{\partial E}{\partial w_{pq}} = \frac{\partial E}{\partial s_q} \cdot \frac{\partial s_q}{\partial w_{pq}} = EI_q y_p \tag{6.14}$$

4. Calculate the error partial derivative for state y_p of node p in last layer:

$$EA_p = \frac{\partial E}{\partial y_p} = \sum_q \frac{\partial E}{\partial s_q} \cdot \frac{\partial s_q}{\partial y_p} = \sum_q EI_q w_{pq} \tag{6.15}$$

Thus, by iterating steps 2, 3 and 4, all of the previous layers EAs and related EWs can be obtained. Finally, the BP algorithm is summarised by equation 6.16 (Patterson, 1996), (Li, 1998).

$$w_{pq}(t+1) = w_{pq}(t) + \eta \delta_q y_p \tag{6.16}$$

where η is a constant learning coefficient or learning rate, t is the learning epoch, and δ_q is given by equation 6.17.

$$\delta_q = \begin{cases} y_q(1-y_q)(d_q - y_q), & q \textit{ is output layer node} \\ y_q(1-y_q)\sum_k \delta_k w_{qk}, & q \textit{ is hidden layer node} \end{cases} \tag{6.17}$$

where the node k is in the next layer, $k = 1,2,\ldots,m_{Q+1}$ and m_{Q+1} is the number of nodes in next layer ($Q+1^{th}$ layer).

6.2.4 Training phase and working phase

The objective of an artificial neural network is to process information in a way that has been previously trained. Neural networks operate in two phases: the training phase and the working phase.

In the training phase, the network is taught the desired behaviour. Commonly, template sample data sets are used as desired input / output states to train the network. Learning algorithms are used to modify the individual neurons of the network and the weight of their connections in such a way that their behaviour reflects the desired one (Von Altrock, 1995). The training process using BP learning algorithm can be summarised as follows:

1. Initialisation. Assign random small values to weights. Initialise the bias b.
2. Provide training data. Desired input vectors **x** and output vectors **d.**
3. Calculate real output **y** by equation 6.9 and 6.10.
4. Adjust the weights from output layer to hidden layers (in error back propagation direction) by equation 6.16 and 6.17.
5. Return to step 2, to iterate calculation until the error E is satisfactorily small.

In the working phase, the trained neural network is ready to be used to test new inputs. As a result of the training, the neural network can output values similar to those in the template sample data sets if the test input values match one of the training samples. For input values in between, it approximates output values (Von Altrock, 1995). That is, a neural network is able to perform non-linear interpolation (Jantzen, 1998).

6.2.5 *Advantages and disadvantages of neural network*

Neural networks can flexibly and arbitrarily map non-linear functions. They can either be trained off-line and subsequently employed either on- or off-line, or they can be trained on-line as part of an adaptive control scheme or simply as a real-time system identifier. Neural networks are also particularly well suited to multivariable applications due to their ability to map interactions and cross-couplings readily whilst incorporating many inputs and outputs. Neural networks are also inherently parallel processing devices. Fast data processing is therefore achievable in a framework of graceful degradation (Hunt et al, 1992). Such networks are best suited to the control of non-linear systems. Neural networks are especially used for "black box" control systems with unknown models (Sun, 1997). Based on these advantages and the inherent characteristics mentioned above, neural networks are also frequently applied in diagnostics, forecasting, optimisation and pattern recognition areas (Patterson, 1996).

As a negative aspect, it is often difficult to decide on the network structure and to explain the training results, although those results are available (Psichogios and Ungar, 1994). The selection of the appropriate net model and setting the parameters of the learning algorithm is still a "black art" and requires much experience (Von Altrock, 1995). Neural networks range widely in type, the selection of any particular network being dependent on the characteristics of the intended application, e.g. necessary accuracy required and overall problem complexity (Zalzala and Morris, 1996). If the training data are not sufficient, neural network can not provide satisfactory solutions.

6.3 NeuroFuzzy network

6.3.1 The combination of neural networks and fuzzy logic

As discussed previously, fuzzy logic can be used to describe desired system behaviour with simple "if-then" relations (Von Altrock, 1995). That is, fuzzy logic theory provides a formal framework to abstract the approximate reasoning characteristics of human decision-making, and offers an excellent mode of knowledge representation. Nevertheless, it can be also viewed a bottleneck, the fuzzy inference depends on the specification of good rules which are generated from human experts. That is, fuzzy systems lacks learning and adaptive capabilities (Sun,1997). Neural network can simulate biological learning capabilities, but they can not be extracted to explain the learned knowledge, and it requires much experience and a large number of data to be designed and trained. Combining the explicit knowledge representation of fuzzy logic with the learning power of neural networks, the NeuroFuzzy model, which is a more powerful network or model, is engendered.

6.3.2 Research background in the integration of neural network and fuzzy logic system

Many alternative ways of integrating neural network and fuzzy logic have been proposed for the control area in the literature. These approaches integrate neural and fuzzy techniques to optimise control through tuning a fuzzy controller's parameters using a NN. Horikawa et al presented a fuzzy modelling method using fuzzy neural networks with the back propagation algorithm. This method can automatically identify the fuzzy model of a non-linear system. The feasibility of the method is also examined using simple numerical data (Horikawa et al, 1990). Khalid, et al proposed an adaptive fuzzy-neural control scheme by integrating two neural models with a basic fuzzy logic controller. The first neural network is trained as a plant emulator and second neural network is used as a compensator for the basic fuzzy controller to improve its performance on-line. The function of the emulator is to provide the correct error signal at the output of the compensator without the need for any mathematical modelling of the plant. The difficulty of fine-tuning the scaling factors and formulating the correct control rules in a basic fuzzy controller may be reduced using the proposed scheme. The experimental results show that the fuzzy-neural controller in this approach is superior to the conventional fuzzy controller and PI controller (Khalid et al, 1994).

The most common trend has been to apply NN to tune the membership functions for defined rule sets. A typical approach is to assume a particular shape of membership functions and define its characteristics, which can be learned by a neural network (Nicholas, 1999). For example, Horikawa et al (Horikawa et al, 1990) used a fixed number of if-then rules, and the membership functions were subsequently adjusted by NN through BP algorithm until they fitted the data. However, these adapting membership functions tend to lose meaning, when they are changed from the original models chosen by the designer. Shi and Shimizu presented a NeuroFuzzy controller in a bioreactor system. The neural network is used to recognise the patterns of changes in the ethanol concentration in baker's yeast fed-batch cultivation. The membership functions are adjusted on-line(Shi and Shimizu, 1992). Geisler et al presented a NeuroFuzzy approach as a general tool for modelling chemical vapor deposition (CVD) phenomenon in semiconductor manufacturing processes. A five-layer feedforward neural network is proposed to model the input-output relationships of a plasma-enhanced CVD deposition of a SiN film. The fuzzy membership functions of the input and output variables are optimally adjusted using the BP learning algorithm. The input-output relationship can be described linguistically in terms of fuzzy if-then rules (Geisler et al, 2000).

Neural networks have also been designed to generate or adjust fuzzy rules. Cha and Cho proposed a neurofuzzy compliance model (NFCM) which can be used to design a control scheme to determine automatically suitable compliance for a given task. An NFCM, composed of a fuzzy logic controller and a rule-learning mechanism, is used as a compliance controller. The fuzzy logic controller receives contact forces as inputs and generates corresponding corrective motions as outputs. The rule learning mechanism, composed of two neurons, trains the rule base of the fuzzy logic controller until the given task is successfully performed (Cha and Cho, 1996). Piramuthu discovered a drawback of using neural networks for credit-risk evaluation decision. That is, it is extremely difficult to explain the rationale behind that decision of NN. Researchers have developed methods using neural network to extract rules, which are then used to explain the reasoning behind a given neural network output. These rules do not capture the learned knowledge well enough. Piramuthu developed a neurofuzzy system utilising the desirable properties of both fuzzy systems and neural networks, and generated fuzzy rules naturally (Piramuthu, 1999). Meesad and Yen applied neurofuzzy for pattern classification in vibration monitoring. A fuzzy interpretation is incorporated into the network design to handle imprecise information. A neural network architecture is used to automatically deduce fuzzy if-then rules based on a supervised learning scheme. This network can be considered a self-organised classifier with the ability to adaptively learn new information without forgetting old knowledge, and can achieve 97.33% correct classification (Meesad and Yen, 2000).

6.3.3 *NeuroFuzzy network for T-S model*

6.3.3.1 *An overview of research background*

The Takagi-Sugeno (T-S) model has the advantages of computational tractability, continuous solution outputs and adaptive capability (section 3.4.1). It is frequently used in the NeuroFuzzy system. Tanaka discussed stability analysis of fuzzy-neural-linear (FNL) control systems, which consist of fuzzy models, neural network (NN) models, and linear models. It is pointed out that the dynamics of linear models and NN models can be perfectly represented by Takagi-Sugeno (T-S) fuzzy models whose consequent parts are described by linear equations. The stability conditions are employed to analyse the stability of four types of FNL control systems (Tanaka, 1995). Theocharis and Vachtsevanos proposed an adaptive fuzzy neural network, which uses a BP algorithm to learn the structure and parameters of a T-S fuzzy model to identify the discrete-time non-linear dynamic system (Theocharis and Vachtsevanos, 1996). Juang and Lin discussed a self-constructing neural fuzzy inference network with on-line learning ability based on the T-S model. Initially there are no rules in this network, they are created and adapted as an on-line learning process via structure and parameter identification (Juang and Lin, 1998). Ying presented a simplified T-S fuzzy controller, which requires that all the rule consequents employ a common linear function and are proportional to one another. This scheme drastically reduces the number of adjustable parameters required by the original T-S rule scheme (Ying, 1998). Chen et al presented a fault detection and isolation scheme for non-linear dynamic systems. This scheme utilises a fuzzy observer to generate the diagnostic residual signal for fault detection and isolation. The fuzzy observer based on the T-S model, comprises a number of locally linear observers and the final state estimate is a fuzzy fusion of all local observer outputs (Chen et al, 1999).

6.3.3.2 MISO Takagi-Sugeno (T-S) model

Because the Multiple Inputs-Multiple Outputs (MIMO) system can be decomposed to Multiple Inputs-Single output (MISO) system (Tzafestas et al, 1996), only the MISO is explained in this section as it is also used directly to build the NN-Fuzzy-SPC application in section 6.4. As discussed in section 3.4.1 of chapter 3, the MISO Takagi-Sugeno rule can be described as:

Suppose input $\mathbf{x} = [x_1 \; x_2 \; \cdots x_n]^T$, every x_i is the linguistic variable, and its linguistic term is given by:

$$T(x_i) = \{A_i^1, A_i^2, ..., A_i^{m_i}\},\ i=1,2,...,n \tag{6.18}$$

where $A_i^{k_i}\,(k_i = 1,2,...,m_i)$ is a fuzzy subset which is the k_i^{th} linguistic variable value for x_i $(i = 1,2,...n)$, the related membership function is $\mu_{A_i^{k_i}}(x_i)$ or $\mu_i^{k_i}$ in short, and n is the number of input variables, m_i is the fuzzy partition number for x_i.

The T-S model if-then rule is:

R_j: if inputs x_1 is A_1^j and x_2 is A_2^j ... x_n is A_n^j, then outputs z_j are given by:

$$z_j = f_j(x_1, x_2, ... x_n) = p_{j0} + p_{j1}x_1 + \cdots + p_{jn}x_n, \quad (j = 1,2,...,m) \tag{6.19}$$

where m is the total number of rules, $m \le \prod_{i=1}^{n} m_i = m_1 m_2 \cdots m_n$.

If the sample data are fuzzified to singleton fuzzy sets (section 3.4.2.2), the matching degree for linguistic input $\boldsymbol{x}$ (section 3.3.3.3) can be calculated by equation 6.20 (R_C) or equation 6.21 (R_p):

$$\alpha_j = \mu_{A_1^j}(x_1) \wedge \mu_{A_2^j}(x_2) \wedge ... \wedge \mu_{A_n^j}(x_n) \tag{6.20}$$

$$\alpha_j = \mu_{A_1^j}(x_1)\mu_{A_2^j}(x_2)...\mu_{A_n^j}(x_n) \tag{6.21}$$

The system output z is a weighted sum and given by:

$$z = \frac{\sum_{j=1}^{m} \alpha_j z_j}{\sum_{j=1}^{m} \alpha_j} = \sum_{j=1}^{m} \overline{\alpha}_j z_j \tag{6.22}$$

where $\overline{\alpha}_j$ is normalised matching degree:

$$\overline{\alpha}_j = \frac{\alpha_j}{\sum_{j=1}^{m} \alpha_j} \tag{6.23}$$

6.3.3.3 NeuroFuzzy network for MISO T-S model

6.3.3.3.1 Architecture

The NeuroFuzzy networks are special neural networks, which are designed for fuzzy system. Every node in the network has a physical significance. Figure 6.4 shows a NeuroFuzzy network, which is applied to describe the MISO T-S model. It consists of two sub nets: antecedent network and consequent network.

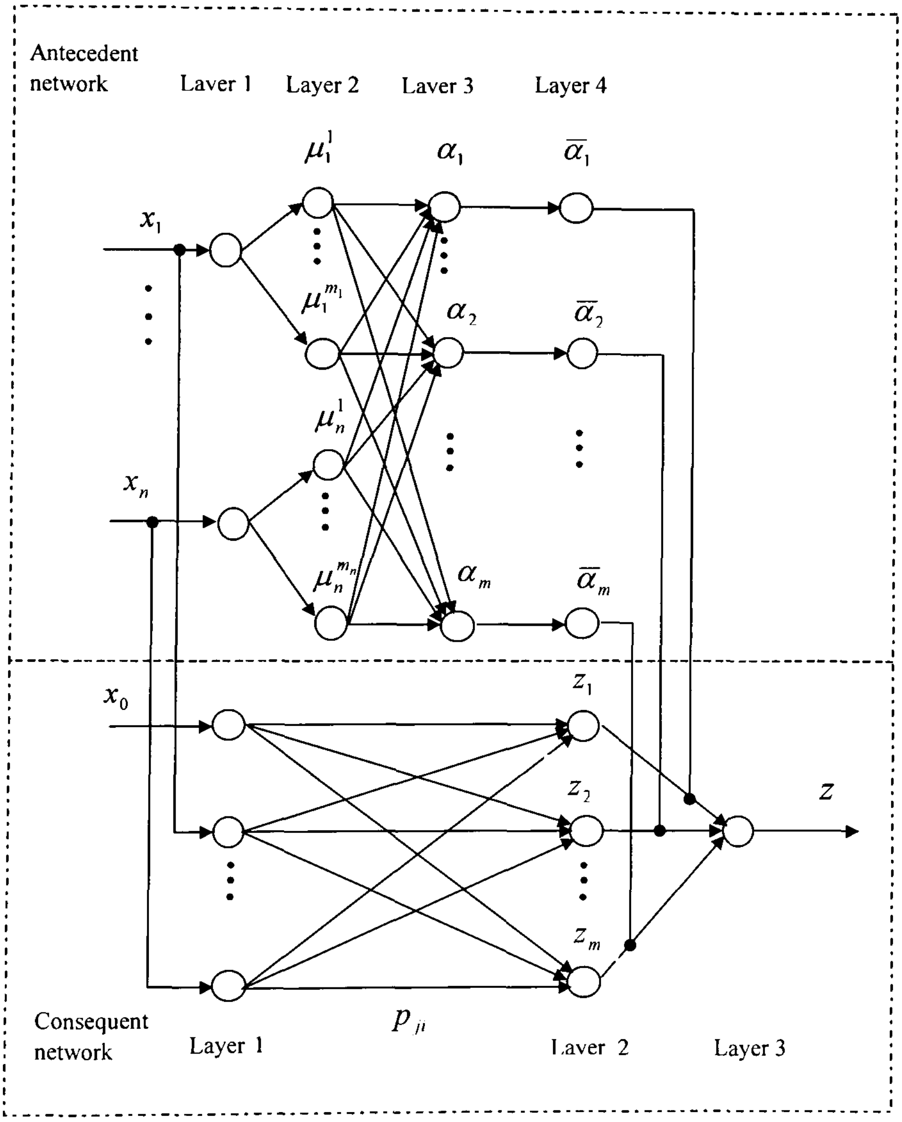


Figure 6.4 NeuroFuzzy network for MISO T-S model

The antecedent network contains four layers. Layer 1 is the input layer, the number of nodes is *n*, which is the same as the number of inputs variables **x**. The nodes in layer 2 represent the linguistic variables by membership functions. The number of nodes in layer 2 is $\sum_{i=1}^{n} m_i$.

As mentioned in section 6.3.3.2, m_i is the fuzzy partition number for x_i. In layer 3, every node expresses a fuzzy rule. Layer 3 forms the antecedent through the calculation of the matching degree α_j (equation 6.24 for R_C or equation 6.25 for R_P) for every rule. The node number is *m*.

$$\alpha_j = \min[\mu_1^{k_1}, \mu_2^{k_2}, \ldots, \mu_n^{k_n}] \tag{6.24}$$

or

$$\alpha_j = \mu_1^{k_1} \mu_2^{k_2} \cdots \mu_n^{k_n} \tag{6.25}$$

where $k_1 = 1,2,\ldots,m_1$, …, $k_n = 1,2,\ldots,m_n$, $j = 1,2,\ldots,m$, $m = \prod_{i=1}^{n} m_i$.

Layer 4 calculates normalised matching degree $\overline{\alpha}_j$ by equation 6.23. The node number is *m*.

There are three layers in the consequent network. Layer 1 is the input layer. Input $x_0 = 1$, is used to calculate the constant component in the consequent linear function. Layer 2 has

m nodes, which are used to express consequent solution (equation 6.19) for m rules. Layer 3 provides the system output, which is described by equation 6.22.

6.3.3.3.2 Learning algorithm

Figure 6.4 shows that the neurofuzzy network is also a feedforward model, hence the BP algorithm can be used to learn inputs/outputs relations through adjusting its parameters. Suppose the fuzzy partition number for inputs is determined, the connecting weights p_{ji} ($i = 1,2,...,n$; $j = 1,2,...,m$) in the consequent network and the central value $c_{k_i i}$ and width $\sigma_{k_i i}$ ($i = 1,2,...n$; $k_i = 1,2,...,m_i$) of the membership function in the antecedent network can be adjusted (Sun, 1997).

Suppose z_d is the desired output and z is the actual output, then the error function E can be given by equation 6.26.

$$E = \frac{1}{2}(z_d - z)^2 \tag{6.26}$$

The learning algorithm for connecting weights p_{ji} is obtained (equation 6.28) through calculating the partial derivative of E for p_{ji} (equation 6.27), and using simultaneous equations 6.22 and 6.19.

$$\frac{\partial E}{\partial p_{ji}} = \frac{\partial E}{\partial z_j}\frac{\partial z_j}{\partial p_{ji}} = -(z_d - z)\bar{\alpha}_j x_i \tag{6.27}$$

$$p_{ji}(t+1) = p_{ji}(t) - \eta \frac{\partial E}{\partial p_{ji}} = p_{ji}(t) + \eta(z_d - z)\overline{\alpha}_j x_i \tag{6.28}$$

where t is the learning epoch and η is the learning rate, $i = 1,2,...,n$, $j = 1,2,...,m$. In the same method, the learning algorithms for central value $c_{k,i}$ and width $\sigma_{k,i}$ of the membership function can be obtained by equation 6.29 and 6.30.

$$c_{k,i}(t+1) = c_{k,i}(t) - \eta \frac{\partial E}{\partial c_{k,i}} \tag{6.29}$$

$$\sigma_{k,i}(t+1) = \sigma_{k,i}(t) - \eta \frac{\partial E}{\partial \sigma_{k,i}} \tag{6.30}$$

where $k_i = 1,2,...,m_i$ and $i = 1,2,...n$.

6.4 NN-Fuzzy-SPC system design and simulation

As discussed in chapter 5, the Fuzzy-SPC system has been successfully realised in a C++ simulation study. However, two weaknesses of the Fuzzy-SPC controller need to be overcome:

1. The fuzzy subset structures are not consecutively tuned. This led to a reduced control accuracy.
2. Different shift levels from the 0.1 time universe of discourse need to be made for further testing and control in the simulation processes.

In this section, the Takagi-Sugeno model, which has advantages of mathematical tractability, continuous solution outputs and adaptive capability is used to achieve the fuzzy inference. The T-S model can describe a highly non-linear system using a small number of rules, and it is convenient to identify its parameters using some learning algorithms (Yen and Langari, 1999). The neurofuzzy network, which has both approximate reasoning characteristic of fuzzy logic and learning and adaptive capability of neural networks is applied to build the NN-Fuzzy-SPC control system, in order to overcome the above three weaknesses and to obtain the best controlled result after one implementation step (or one control action). The SPC control actions are interpreted by the approximate reasoning of fuzzy logic and the dynamic fuzzy membership functions are optimised automatically by the learning capability of neural networks.

6.4.1 Design of NeuroFuzzy controllers

In this NN-Fuzzy-SPC system, a NeuroFuzzy controller consists of a fuzzy controller and its NeuroFuzzy network. That is, two NeuroFuzzy models are designed for the $\overline{X}$-fuzzy controller and R-fuzzy controller respectively. Before the design work, several selection criteria are discussed below.

The conclusion of chapter 5 shows that, to tune the consequent membership function from a standard one for a change in control output, the tuning of position or slope of the triangle and trapezoid have a greater effect than tuning of the width of the triangle and trapezoid. In this section, the singleton consequent membership function is chosen to replace the triangle and trapezoid, as the position (central value) is a major factor which has greatest effect in the triangular and trapezoid shapes. This is done in order to simplify the computing procedure and reduce computation time. Therefore, the NeuroFuzzy controllers can be represented by the Zero-Order T-S model (Hines, 1997).

Based on the conclusiòns from chapters 2 and 4, to observe the randomness of the sample data, only two inputs are used to the fuzzy system. These are the current sample data and the average of 5 data points after the FAP. Using the current sample data is considered simpler than the use of several sample data points used in section 4.2.2 (also see Appendix B.5). It can be considered that the current sample data value (data point position) has a distinct characteristic, which can indicate the pattern classification for SPC zone rule 1, 2 and 3. For example, for zone rule2 ("The existence of two of any three successive points in zone A or beyond..."), if the current sample abnormal point is beyond zone A, it can be tested by zone rule 1; for zone rule 3 ("The existence of four of any five successive points in zone B or beyond..."), if the current sample abnormal point is beyond zone B, this current data point position can be determined as the upper limit for zone B. For other situations, the current data point position must fall in the related zone. Therefore, in the NN-Fuzzy-SPC system, the abnormal patterns are tested as alarms by SPC zone rules in the program, and related pattern classification is represented by a single input variable x_1, which is the current sample data point position for the fuzzy inference system. The position x_1 is also used for zone rules 4 and 5, however, the control error can be reduced quickly by training and optimisation. In addition, more importantly, the average of five data point after the first abnormal point (FAP) which is discussed in section 2.7 of chapter 2 is applied as input x_2 and the *min* operator is used for antecedent

AND, in order to improve the control accuracy and robustness into the NN-Fuzzy-SPC system.

6.4.1.1 $\overline{X}$ -fuzzy controller and its NeuroFuzzy (2) network

Suppose input $\mathbf{x} = [x_1\ x_2\ ]^T$, and $x_1 \in [0,100]$ is the current sample data point position, $x_2 \in [0,100]$ is the average of five data after FAP and every x_i is a linguistic variable, and their linguistic terms are given by:

$$T(x_1) = \{NOUT, NA, NB, NC, C, B, A, OUT\} \tag{6.31}$$

where A, B, C and OUT are SPC zones and N indicates negative.

$$T(x_2) = \{Low, High\} \tag{6.32}$$

where *Low* indicates negative area, and *High* indicates positive area in control charts.

Therefore, the T-S model if-then rules for the $\overline{X}$ -fuzzy controller are summarised below.

R_1: if x_1 is NOUT and x_2 is Low then output z_1 is p_1.

R_2: if x_1 is NOUT and x_2 is High then output z_2 is p_2.

R_3: if x_1 is NA and x_2 is Low then output z_3 is p_3.

R_4: if x_1 is NA and x_2 is High then output z_4 is p_4.

R_5: if x_1 is NB and x_2 is Low then output z_5 is p_5.

R_6: if x_1 is NB and x_2 is High then output z_6 is p_6.

R_7: if x_1 is NC and x_2 is Low then output z_7 is p_7.

R_8: if x_1 is NC and x_2 is High then output z_8 is p_8.

R_9: if x_1 is C and x_2 is Low then output z_9 is p_9.

R_{10}: if x_1 is C and x_2 is High then output z_{10} is p_{10}.

R_{11}: if x_1 is B and x_2 is Low then output z_{11} is p_{11}.

R_{12}: if x_1 is B and x_2 is High then output z_{12} is p_{12}.

R_{13}: if x_1 is A and x_2 is Low then output z_{13} is p_{13}.

R_{14}: if x_1 is A and x_2 is High then output z_{14} is p_{14}.

R_{15}: if x_1 is OUT and x_2 is Low then output z_{15} is p_{15}.

R_{16}: if x_1 is OUT and x_2 is High then output z_{16} is p_{16}.

Figure 6.5 illustrates the MISO neurofuzzy network for the Zero-Order T-S model which was applied as a NeuroFuzzy(2) for $\overline{X}$-fuzzy controller (see also figure 6.6). Where **X** is the input vector, $\mu_i^{k_i}$ is antecedent membership function ($i = 1,2$; $k_1 = 1,2,...,8$; $k_2 = 1,2$), z_j is the output for the j^{th} rule and z is the system output.

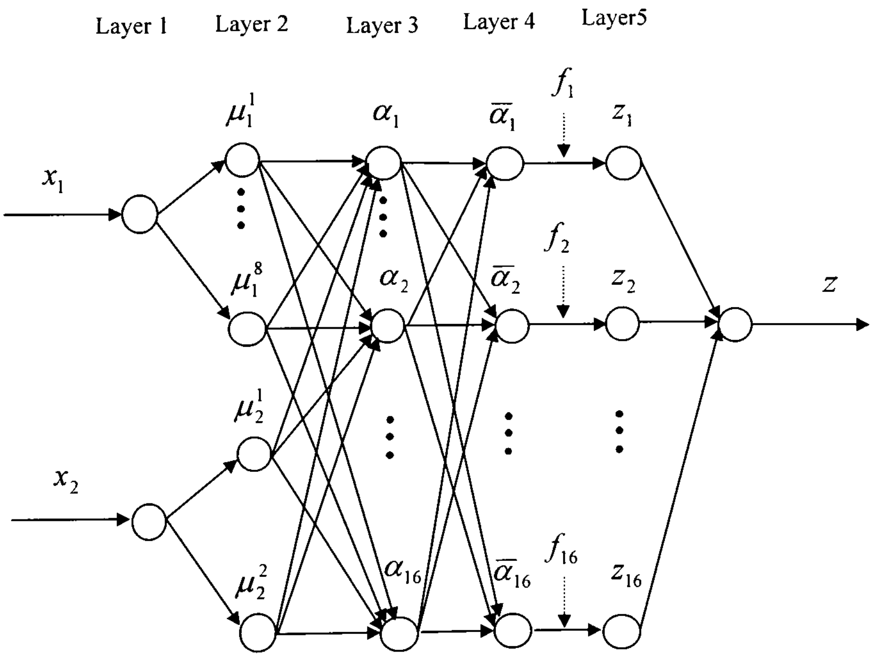


Figure 6.5 NeuroFuzzy(2) network for MISO Zero-Order T-S model

Figure 6.5 is a simplified model of figure 6.4, as the consequent outputs are constant components of the T-S linear function f_j *(j=1,2,...,16)* which are described as the connecting weights in layer 5.

Layer 1: input layer, translate inputs to next layers.

Layer 2: calculates the membership $\mu_i^{k_i}$, $k_i = 1,2,...,m_i$, $i = 1,2$, $m_1 = 8$, $m_2 = 2$.

Layer 3: calculates j^{th} matching degree:

$$\alpha_j = \min[\mu_1^{k_1}, \mu_2^{k_2}] \tag{6.33}$$

where $k_1 = 1,2,...,8$, $k_2 = 1,2$, $j = 1,2,...,16$.

Layer 4: calculates j^{th} normalised matching degree:

$$\overline{\alpha}_j = \frac{\alpha_j}{\sum_{j=1}^{m} \alpha_j} \tag{6.34}$$

Layer 5: calculates the output of the j^{th} rule:

$$z_j = \overline{\alpha}_j f_j \tag{6.35}$$

where f_j is the linear equation mentioned previously, which is considered as a weight in the neural network. In this zero-order T-S model, the singleton membership function p_j is defined for output:

$$f_j = p_j \tag{6.36}$$

$$z_j = \overline{\alpha}_j f_j = \overline{\alpha}_j p_j \tag{6.37}$$

Layer 6: calculates the system output:

$$z = \sum_{j=1}^{m} z_j \tag{6.38}$$

The NeuroFuzzy network in figure 6.5 is a multiple feed forward network, and the Back Propagation algorithm (equation 6.39) is used to train the input / output relation via optimisation of the connecting weight $f_j = p_j$ which is a consequent singleton membership function or zero-order T-S linear function.

$$p_j(t+1) = p_j(t) - \eta \frac{\partial E}{\partial p_j} = p_j(t) + \eta(z_d - z)\overline{\alpha}_j \tag{6.39}$$

Equation 6.39 is a simplified version of equation 6.28, since the consequent linear function $z_j = p_j$ (compare to equation 6.19) which is used to describe the singleton functions.

6.4.1.2 *R-fuzzy controller and its NeuroFuzzy (1) network*

The design procedure is similar to section 6.4.1.1, but the number of antecedent fuzzy sets and related rules are different to the $\overline{X}$-fuzzy controller. Suppose input $\mathbf{x} = [x_1 \, x_2]^T$, $x_1 \in [0,100]$ is the current sample data point position, $x_2 \in [0,100]$ is the average of previous sample data and every x_i is a linguistic variable, their linguistic terms are given by:

$$T(x_1) = \{NOUT, C, OUT\} \tag{6.40}$$

where *C* and *OUT* represent SPC zones and *N* indicates negative.

$$T(x_2) = \{Low, High\} \tag{6.41}$$

where *Low* indicates negative area, and *High* indicates positive area in the control charts.

The T-S model if-then rules for the *R*-fuzzy controller are summarised below.

R_1: if x_1 is NOUT and x_2 is Low then output z_1 is p_1.
R_2: if x_1 is NOUT and x_2 is High then output z_2 is p_2.
R_3: if x_1 is C and x_2 is Low then output z_3 is p_3.
R_4: if x_1 is C and x_2 is High then output z_4 is p_4.
R_5: if x_1 is OUT and x_2 is Low then output z_5 is p_5.
R_6: if x_1 is OUT and x_2 is High then output z_6 is p_6.

For the *R*-fuzzy controller, the NeuroFuzzy (1) structure and related calculations are the same as figure 6.5 of the NeuroFuzzy (2) and equations 6.33~6.39, if $m_1 = 3$, $k_1 = 1,2,3$ and $j=1,2,...,6$.

6.4.2 Architecture of NN-Fuzzy-SPC system

Figure 6.6 illustrates the structure of the NN-Fuzzy-SPC system. The inputs are process settings or targets (process average μ and standard deviation *SD*) and outputs are abnormal process average μ' and standard deviation SD' to be controlled.

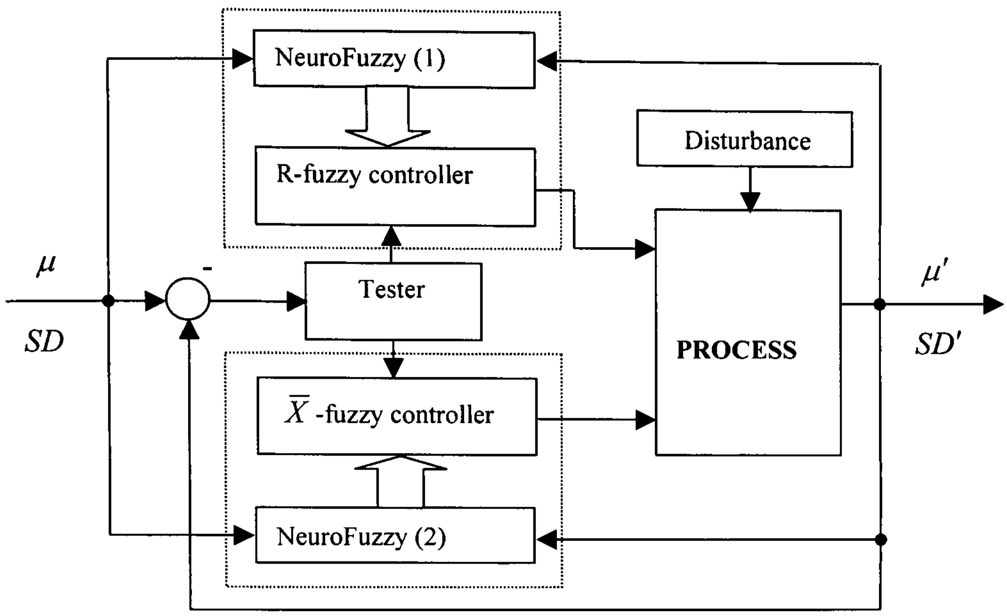


Figure 6.6 NN-Fuzzy-SPC system architecture

The disturbance includes changing the process average and changing the standard deviation. Process signals can be tested, classified and transferred to the R - Fuzzy controller and the $\overline{X}$ - Fuzzy controller separately, in order to generate control actions to adjust the abnormal process. Every controller (R or $\overline{X}$) has two inputs: current sample value and average of 5 points after FAP. The controlled result is tested in process standard deviation and average by the NeuroFuzzy (1) and (2), if the errors are larger than predefined limits, the neural networks will optimise the consequent membership functions automatically in the R - Fuzzy controller and the $\overline{X}$-Fuzzy controller until the best control results are obtained.

The NN-Fuzzy-SPC system is simulated by an m-file in MATLAB. Figure 6.7~6.11 illustrate the performance of the control-training-control process. In figure 6.7, the upper two pictures describe a normal process in $\overline{X}$ and R charts, where as the lower two pictures describe an abnormal process (process average shifted by a population standard deviation σ value and range spread 4 times of σ). Large circles mark abnormal data.

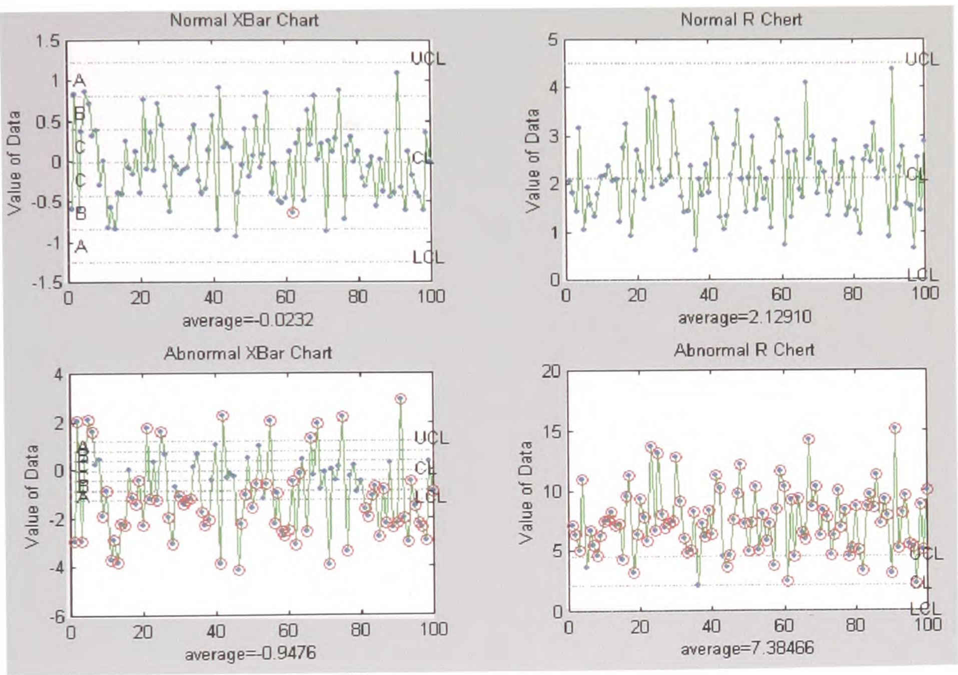


Figure 6.7 Normal and abnormal processes

The abnormal process is initially controlled in a one step control action by the non-optimised controllers and the process still shows many abnormal data (figure 6.8). The control errors are still large. In figure 6.8, the left two charts are $\overline{X}$ charts, the right two are R charts; upper two charts describe an abnormal process controlled by the R-fuzzy controller, where the lower two charts describe the abnormal process controlled by the $\overline{X}$-fuzzy controller. The two charts to the right are the same. That is, when the $\overline{X}$-fuzzy controller adjust the abnormal shift, it does not affect the process behaviour in the R chart if the calculation error is ignored. By contrast, when the R-fuzzy controller works, the control action adjust both the $\overline{X}$ and R charts: the upper two charts are improved from their last states which are shown on the lower two related pictures in figure 6.7.

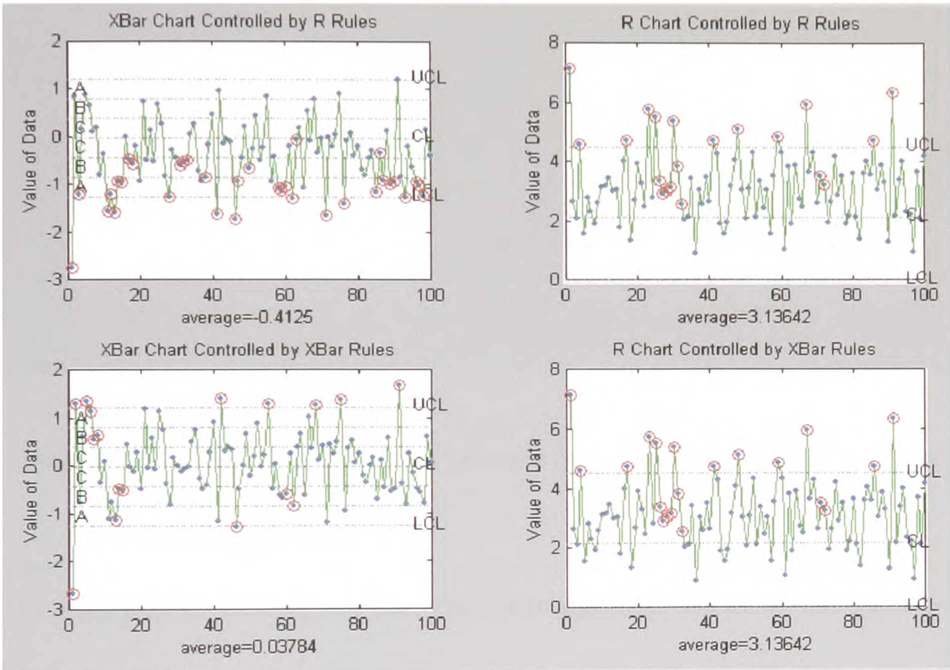


Figure 6.8 A one step controlled by non-optimised fuzzy controllers

In the figure 6.9, the charts layout is same as figure 6.8, but the charts shows the control results in a one step control action using optimised R-fuzzy and $\overline{X}$-fuzzy controllers. The abnormal process controlled average $\overline{\overline{X}}$ values and average range $\overline{R}$ values are shown on the bottom of the $\overline{X}$ and R charts respectively. The lower two charts are the final results. They are much closer to a normal process, which is shown on the upper charts of figure 6.7.

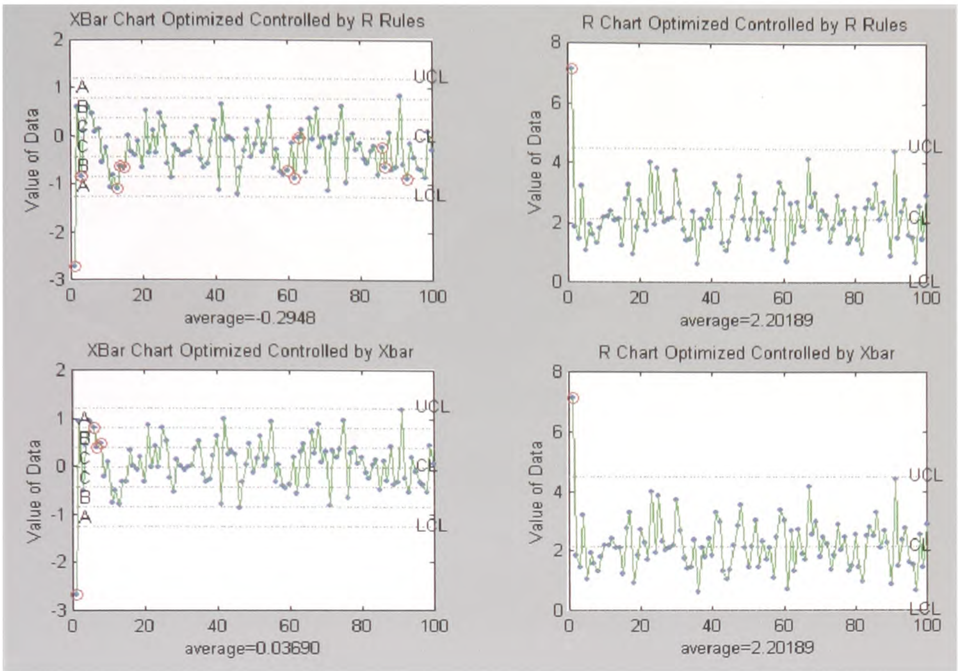


Figure 6.9 Controlled abnormal process by optimised fuzzy controllers

In this NN-Fuzzy-SPC system, the triangular membership functions are used to describe the antecedent, and the consequent membership function are simplified to crisp singletons which correspond to the zero-order T-S model mentioned in section 6.4.1. Figure 6.10 and 6.11 illustrate the network training results for the R and $\overline{X}$ controllers. The consequent (output) membership function are simplified to crisp singletons to increase the speed of implementation. The sum of squared error (SSE) describes the training curve. The training epochs are less than 20, and every final training SSE is less than 0.01. There are two charts shown in the bottom of figures 6.10 and 6.11. They are results of abnormal $\overline{X}$ and R charts, which are adjusted by the R-fuzzy controller (Fig. 6.10) and the $\overline{X}$-fuzzy controller (Fig. 6.11) respectively in the training processes.

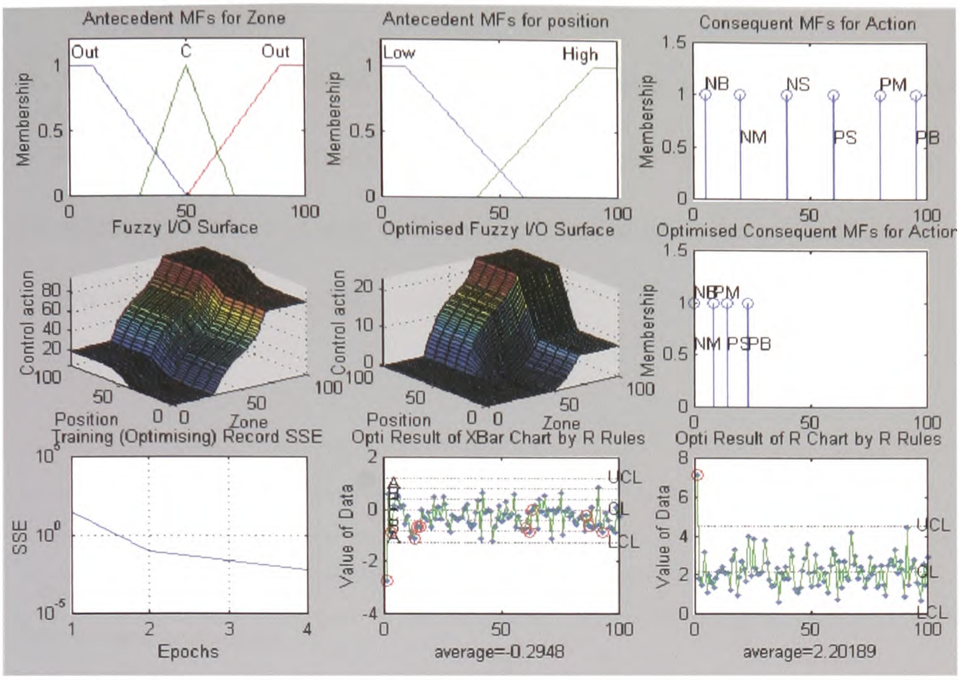


Figure 6.10 Optimisation and results for R-fuzzy controller

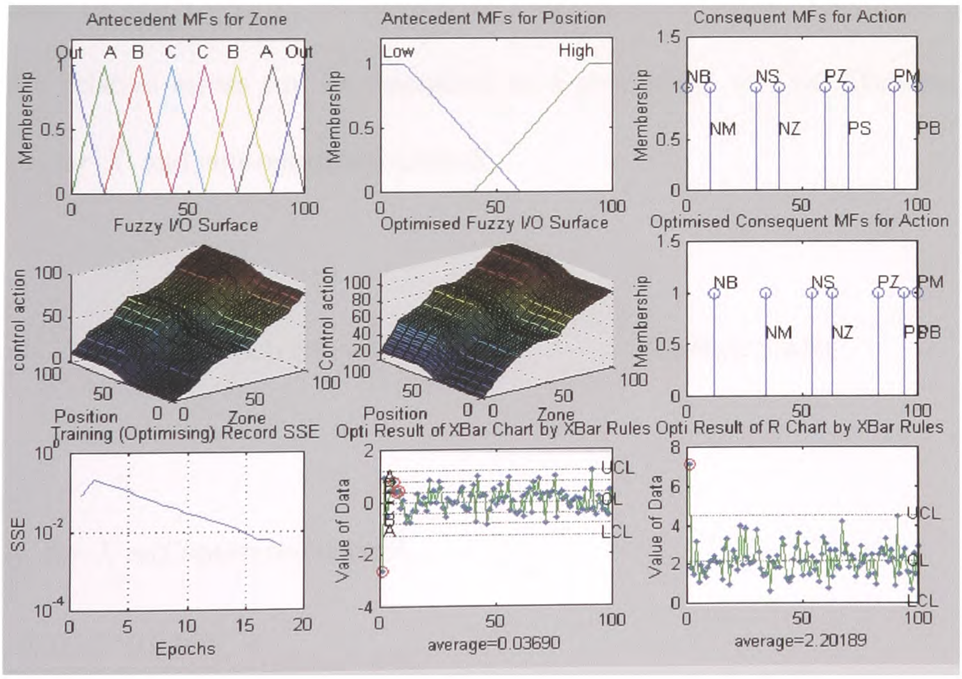


Figure 6.11 Optimisation and results for $\overline{X}$ -fuzzy controller

Table 6.1 summarises the control results, which are shown in figure 6.7 ~ figure 6.9. In Table 6.1, the absolute values of $\overline{\overline{X}}$ and $\overline{R}$ (average of R values) are very much improved after a one step control action which is non-optimised. However, the best results are obtained after a one step optimised control action, the controlled values shown on bottom row are very close to the normal values.

	$\overline{\overline{X}}$	$\overline{R}$
Normal	-0.0232	2.1291
Abnormal	-0.9476	7.3847
After one control action	0.0378	3.1364
After one optimised control Action	0.0369	2.2019

Table 6.1 Summary of control results

If the value of $(UCL - LCL)$ is viewed as a full-scale deflection (f.s.d.) in $\overline{X}$ and R charts, the relative errors can be calculated as a percentage of f.s.d. (Bentley, 1995). Therefore, for $\overline{X}$ and non-optimised control,

$$e_{\overline{X}N} = \left|\frac{-0.0232 - 0.0378}{UCL - LCL}\right| \times 100\% = \left|\frac{-0.0232 - 0.0378}{1.2493 - (-1.2502)}\right| \times 100\% = 2.44\% \qquad (6.42)$$

Similarly, for $\overline{X}$ and optimised control,

$$e_{\overline{X}O} = \left|\frac{-0.0232 - 0.0369}{1.2493 - (-1.2502)}\right| \times 100\% = 2.40\% \qquad (6.43)$$

For R and non-optimised control,

$$e_{RN} = \left|\frac{2.1291 - 3.1364}{UCL - LCL}\right| \times 100\% = \left|\frac{2.1291 - 3.1364}{4.5101}\right| \times 100\% = 22.33\% \tag{6.44}$$

For R and optimised control,

$$e_{RO} = \left|\frac{2.1291 - 2.2019}{4.5101}\right| \times 100\% = 1.61\% \tag{6.45}$$

These results show that when the average shift (or spread) is confined to the control limits (e.g. 1σ shift in $\overline{X}$ chart), the non-optimised Fuzzy-SPC controller can adjust it to a satisfactory accuracy (equation 6.42). However, the optimised control results are more accurate (equation 6.43). This is specially so when the average spread (or shift) is beyond the control limits, the optimised control accuracy is much increased (equation 6.45) from non-optimised control results (equation 6.44).

6.4.3 Statistical analysis of simulation results

In the 40 experiments implemented, the abnormal processes are given different shift levels ($\sigma/3 \sim \sigma$,) and different spread levels ($2\sigma \sim 4\sigma$). The process standard deviation SD is used as the estimated population standard deviation σ in the simulation, and the average of SD value takes 0.94 in 40 experiments. The NN-Fuzzy-SPC system provided fully satisfactory optimisation procedures, the error curves are decayed and the final training sum of squared errors (SSE) are less than 0.01. The ideal consequent membership functions are also obtained.

Table 6.2 summarises the results of 20 abnormal processes (processes averages have been shifted by $\sigma/3 \sim \sigma$), which are controlled by the optimised $\overline{X}$-fuzzy controller. The first column indicates the experiment numbers, the second and third columns list normal process averages and abnormal process averages respectively.

i	Normal	Abnormal	Shifted	Controlled	Ae_{1i}	Ae_{2i}	UCL-LCL	$R\overline{X}e_i$ %
1	0.02361	0.69737	σ	0.01160	0.01201	0.01201	2.70898	0.4433
2	-0.02320	-0.63880	σ	-0.03110	0.00790	0.00790	2.47518	0.3192
3	0.06463	0.74923	σ	0.02730	0.03733	0.03733	2.75236	1.3563
4	-0.02300	-0.63880	σ	-0.03110	0.00810	0.00810	2.47518	0.3272
5	0.01733	0.34281	$\sigma/2$	0.01369	0.00364	0.00364	2.61706	0.1391
6	0.02361	0.36049	$\sigma/2$	0.01456	0.00905	0.00905	2.70898	0.3341
7	-0.02322	0.18199	$\sigma/3$	-0.03917	0.01595	0.01595	2.48146	0.6429
8	0.06460	0.40693	$\sigma/2$	0.03944	0.02516	0.02516	2.74530	0.9602
9	-0.02320	0.18199	$\sigma/3$	-0.03980	0.01660	0.01660	2.47690	0.6702
10	-0.03680	0.18186	$\sigma/3$	-0.04910	0.01230	0.01230	2.63846	0.4662
11	0.06461	0.29283	$\sigma/3$	0.03690	0.02771	0.02771	2.75236	1.0068
12	0.02645	0.70449	σ	0.05639	-0.02994	0.02994	2.47462	1.2100
13	-0.02312	0.59242	σ	-0.03750	0.01438	0.01438	2.47804	0.5803
14	-0.06294	0.25569	$\sigma/2$	-0.03239	-0.03055	0.03055	2.56224	1.1920
15	-0.02331	0.28460	$\sigma/2$	-0.03256	0.00925	0.00925	2.62048	0.3530
16	-0.06294	0.14948	$\sigma/3$	-0.03315	-0.02979	0.02979	2.56224	1.1460
17	0.02645	0.25246	$\sigma/3$	-0.00135	0.02780	0.02780	2.72610	1.0150
18	-0.00780	0.33351	$\sigma/2$	0.00401	-0.01181	0.01181	2.80020	0.4220
19	-0.03680	-0.69310	σ	-0.04260	0.00580	0.00580	2.63846	0.2539
20	0.06352	0.38604	$\sigma/2$	0.041	0.02252	0.02252	2.70460	0.8327
Average					0.00767	0.01788		0.6835
Standard Deviation					0.01937	0.01003		0.3751
Maximum Value					0.03733	0.03733		1.3563
Minimum Value					-0.03055	0.00364		0.1391

Table 6.2 Summary of control results of optimised $\overline{X}$-fuzzy controller

The abnormal process averages shifted levels and optimal control results are described in the 4^{th} and 5^{th} columns. The Ae_{1i}, Ae_{2i} and $R\overline{X}e_i$ which indicate the errors between normal values and controlled values are given by equation 6.46~equation 6.48.

$$Ae_{1i} = i^{th}\text{"}normal\text{"} - i^{th}\text{"}controlled\text{"} \tag{6.46}$$

$$Ae_{2i} = |Ae_{1i}| \tag{6.47}$$

$$R\overline{X}e_i = \frac{Ae_{2i}}{UCL - LCL} \times 100\% \tag{6.48}$$

The abnormal process averages are controlled by the optimised $\overline{X}$-fuzzy controller, the relative control errors $R\overline{X}e_i$ are distributed in the interval 0.1391%~1.3563%, their average is 0.6835%, standard deviation is 0.3751%. Figure 6.12 shows a chart for the control behaviour. After a control action, the abnormal (process) line is improved to the opti-controlled line, which is very close to the normal line.

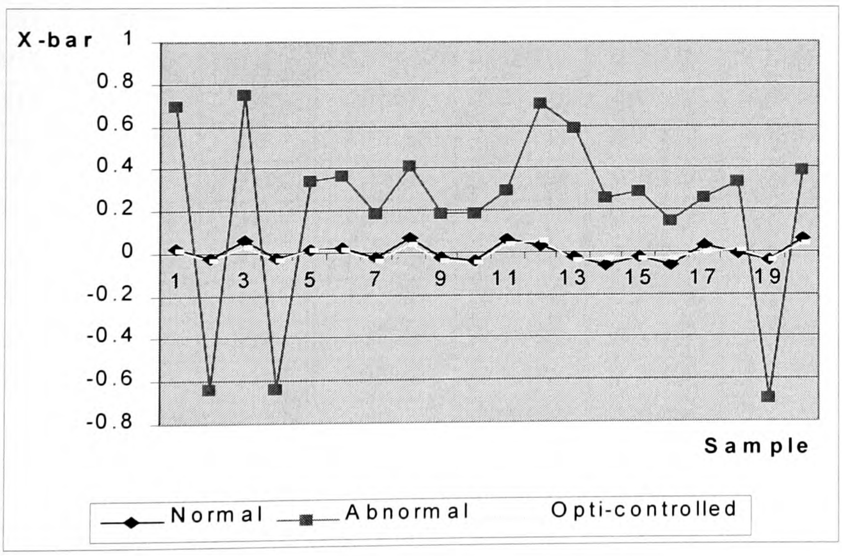


Figure 6.12 Abnormal processes controlled by optimised $\overline{X}$-fuzzy controller

In a similar way, the control results of the R-fuzzy controller are described in table 6.3 and figure 6.13. The 20 abnormal processes (range averages have been spread by 2σ~4σ), which are controlled by the optimised R-fuzzy controller. The first column indicates the experiment numbers, the second and third columns list normal range averages and abnormal range averages respectively. The abnormal range averages spread levels and optimal control results are described in 4^{th} and 5^{th} columns. The Ae_{3i}, Ae_{4i} and RRe_i which indicate the errors between normal values and controlled values are given by equation 6.49~6.51.

i	Normal	Abnormal	spread	Controlled	Ae_{3i}	Ae_{4i}	UCL-LCL	RRe_i%
1	2.35660	4.71320	2σ	2.31028	0.04632	0.04632	4.98421	0.9293
2	2.35756	4.71513	2σ	2.25638	0.10118	0.10118	4.98624	2.0292
3	2.33201	6.99604	3σ	2.42196	-0.08995	0.08995	4.93220	1.8237
4	2.28612	6.85837	3σ	2.12011	0.16601	0.16601	4.83514	3.4334
5	2.28601	6.85803	3σ	2.23025	0.05576	0.05576	4.83491	1.1533
6	2.36479	9.45916	4σ	2.41187	-0.04708	0.04708	5.00153	0.9413
7	2.21094	8.84376	4σ	2.35776	-0.14682	0.14682	4.67614	3.1398
8	2.12910	4.25820	2σ	2.17500	-0.04590	0.04590	4.50305	1.0193
9	2.36210	4.72958	2σ	2.29449	0.06761	0.06761	4.99584	1.3533
10	2.12910	6.38730	3σ	2.05246	0.07664	0.07664	4.50305	1.7020
11	2.31301	9.32806	4σ	2.28196	0.03105	0.03105	4.89202	0.6347
12	2.28915	4.57225	2σ	2.38134	-0.09219	0.09219	4.84155	1.9041
13	2.31911	6.97923	3σ	2.25833	0.06078	0.06078	4.90492	1.2392
14	2.11999	8.51640	4σ	2.19712	-0.07713	0.07713	4.48378	1.7202
15	2.32561	7.01527	3σ	2.25833	0.06728	0.06728	4.91867	1.3679
16	2.26620	6.69239	3σ	2.22843	0.03777	0.03777	4.79301	0.7880
17	2.21035	8.80996	4σ	2.12067	0.08968	0.08968	4.67489	1.9183
18	2.35529	9.29962	4σ	2.27993	0.07536	0.07536	4.98144	1.5128
19	2.45645	5.12366	2σ	2.37086	0.08559	0.08559	5.19539	1.6474
20	2.46702	4.93405	2σ	2.36300	0.10402	0.10402	5.21775	1.9936
Average					0.02830	0.07821		1.6125
Standard Deviation					0.08220	0.03408		0.7109
Maximum Value					0.16601	0.16601		3.4334
Minimum Value					-0.14682	0.03105		0.6347

Table 6.3 Summary of control results by optimised R-fuzzy controller

$$Ae_{3i} = i^{th}\,"normal"-i^{th}\,"controlled" \tag{6.49}$$

$$Ae_{4i} = |Ae_{3i}| \tag{6.50}$$

$$RRe_i = \frac{Ae_{4i}}{UCL - LCL} \times 100\% \tag{6.51}$$

Table 6.3 shows that the abnormal range averages are controlled by optimised *R*-fuzzy controller, the relative control errors RRe_i are distributed in the interval 0.6347%~3.4334%, their average is 1.6125%, standard deviation is 0.7109%. Figure 6.13 shows that after a control action, the abnormal range line is improved to the "opti (optimised)-controlled" line, which is much close to the normal line.

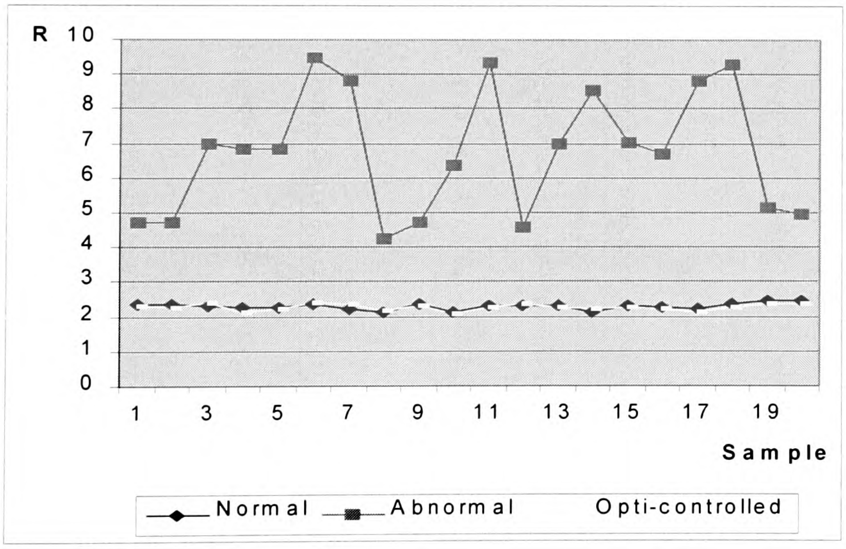


Figure 6.13 Abnormal processes controlled by optimised R-fuzzy controller

6.5 Conclusion

Neural networks can flexibly and arbitrarily map non-linear functions, and their learning capability can be used to achieve adaptive control. Fuzzy logic provides a framework to abstract the approximate reasoning of human decision-marking. The NeuroFuzzy network contains advantages of both neural networks and fuzzy logic. The NeuroFuzzy network for zero-order T-S model have been successfully applied though the use of simulated examples. The SPC control actions are interpreted by approximate reasoning of fuzzy logic and the dynamic fuzzy membership functions are optimised automatically by the learning capability of neural networks. The results are obtained with short (less than 20) epoch training times (Figure 6.10 and 6.11). Optimised consequent membership functions are used to provide the ideal control actions in the control of abnormal process. After a one step adjustment by the tuned R-fuzzy controller and $\overline{X}$-fuzzy controller, spread deviation and shifted average can be returned to a normal situation with satisfactorily small errors.

In the 40 experiments implemented, the abnormal processes are given using different shift levels ($\sigma/3 \sim \sigma$) and different spread levels ($2\sigma \sim 4\sigma$). The NeuroFuzzy network provided a satisfactory optimisation procedure (error curve decay) and ideal consequent membership function. The control errors range for the R-fuzzy controller is 0.6347%~3.4334%, their average is 1.6125% (table 6.3); for the $\overline{X}$-fuzzy controller the error range is 0.1391%~1.3563%, the average is 0.6835% (table 6.2).

$\overline{X}$ and R control charts are commonly used in industry. It is normal to investigate the behaviour of process average in the $\overline{X}$ chart and check the variation of process deviation in the R chart. It can be understood from the definition of $\overline{X}$ and R, Fig. 6.8 and Fig.6.9: that when the averages $\overline{X}$ and $\overline{\overline{X}}$ are shifted as translation only, the R and $\overline{R}$ values are not changed at the same time. By contrast, when the spread of the process deviation is increased, it will effect the $\overline{X}$ value. It is necessary to develop the capabilities for measuring and controlling both shift and spread. The NN-Fuzzy-SPC system involves a R-fuzzy controller and a $\overline{X}$-fuzzy controller which are used to control the spread process deviations and shifted process averages respectively. The R-fuzzy controller works first if both abnormal patterns are synchronised.

6.6 Summary

This chapter presents the neural network and its application in a NN-Fuzzy-SPC system. The operational principles of neural networks have been briefly introduced. It involves the general operation of neural networks, typical architecture, the BP algorithm, training and working phases, advantages and disadvantages. The NeuroFuzzy network is a combination technique of common neural network and fuzzy logic. The NeuroFuzzy technique and its advantages, related research backgrounds and the operation of neurofuzzy network for Takagi-Sugeno model are discussed. The NN-Fuzzy-SPC system is an application of a neurofuzzy network. The design of neurofuzzy controllers, system architecture and performance, simulation and analysis also are discussed. In the NN-Fuzzy-SPC system designed, the spread and/or shifted abnormal patterns are controlled by R-fuzzy controller and/or $\overline{X}$-fuzzy controller respectively. The control actions are interpreted by approximate reasoning of fuzzy logic. When the control errors are greater than the predefined limits, the system trains the neural network automatically through optimisation of fuzzy consequent membership functions until the control errors are smaller than the predefined limits. The simulations show that the control results are satisfactory, even though the spreads and shifts take various different values in the abnormal processes tested.

It should be noted that for the research work in this chapter, the actual system outputs, which are used to train the neural network in the NN-Fuzzy-SPC system, are obtained from the next sample phase with about a 30 sample points delay. As such this system can be used in automatic process control, as its sample frequency is normally higher. For quality control, the long delay should be reduced. Therefore, based on chapter 6, chapter 7 further hybridises EWMA control charts and digital filter techniques to build a combined forecaster for the NN-Fuzzy-SPC control system, in order to reduce the long delay.

Chapter 7 Forecast function and filtering

The forecasting technique can reduce the measure and control delay. In this chapter, the EWMA forecast function is applied to the NN-Fuzzy-SPC system to generate the estimated system outputs to train the neural network with a shorter control delay. The forecast errors are reduced by the optimising smoothing constant procedure and a finite impulse response (FIR) filter.

7.1 Introduction

In chapter 6, the NN-Fuzzy-SPC as a fundamental control system is shown to perform well using the NN-Fuzzy-SPC model for different process average shifts and process range spread levels. The actual system outputs, which are used to train the neural network, are obtained from the next phase sample with around a 30 sample points delay. As such, this system can be used in automatic process control, since its sample frequency is normally high. For quality control, the 30 sample points delay is viewed as too long. In this chapter, the Exponentially Weighted Moving Average (EWMA) forecast function and filtering techniques are used to build a combined forecaster for the NN-Fuzzy-SPC system. The estimated system outputs which reflect the actual system outputs can be obtained from the forecast function, instead of the actual outputs, and used in the training process of the NN. This is done in order to reduce the control delay and improve the NN-Fuzzy-SPC system practicability in quality control.

Section 7.2 describes the EWMA forecast method, its behaviour, and its related research background. Section 7.3 explains the design for a FIR filter. Section 7.3.1 introduces the basic notion for a digital filter. Section 7.3.2 describes the details of the lowpass filter design. The filter realisation structure and the filtering effects with EWMA forecast are described in section 7.3.3. Section 7.4 shows the system performance results.

7.2 EWMA forecast

7.2.1 The EWMA forecast formula

As mentioned in chapter 2, the forecast function is the basis of the EWMA chart. The capability of the EWMA to predict future values of a time series can be used for its application in automatic adaptive controllers (Wiklund, 1995). The forecast method in EWMA is similar to its application in the monitoring process. The use of exponential smoothing for forecasting was first arrived at empirically on the grounds that it was a weighted average with the sensible property of giving most weight to the last observation and less to the next-but-last and so on (Box and Luceno, 1997).

Suppose $\tilde{x}_t$ and x_t are current EWMA value and sample value respectively at time or sample t. The EWMA forecast one step ahead is given by equation 7.1.

$$\hat{x}_{t+1} = \tilde{x}_t = \lambda(x_t + \theta x_{t-1} + \theta^2 x_{t-2} + \theta^3 x_{t-3} + \cdots)$$
$$= \lambda x_t + \theta \hat{x}_t \tag{7.1}$$

where symbols λ and θ are smoothing parameters, and $\lambda = 1 - \theta$.

Equation 7.1 is equivalent to the formulation of the EWMA model, which is represented by equation 2.25 in chapter 2.

7.2.2 Research background and research scheme in EWMA forecast

Exponentially weighted moving average (EWMA) is frequently used in process control applications (Luceno, 1995). The EWMA forecasts are also central to many commercial systems (Johnston and Boylan, 1996). It is well known that the existence of uncertainty is a feature of the business world. Johnston and Boylan suggested the decomposing of the forecast error into three components, which are unexplained error, the error resulting from the estimation procedures of the model or uncertainty about the parameter values, and the errors due to the approximate nature of the model in the business area. A quantification of the forecast errors based on EWMA is represented. It confirms that the EWMA forecasting does not remove the existence of uncertainty in the business system but sets out to measure and minimise it (Johnston and Boylan, 1994).

Wold applied the EWMA technique to principal components analysis (PCA) and projections to latent structures (PLS) for modelling processes with memory and drift in chemical processes. These models are based on exponentially weighted observations, and are formulated as multivariate generalisations of the EWMA. Principles and estimation algorithms for exponentially weighted moving (EWM)-PCA and EWM-PLS are presented, and the predictive control schemes based on these models are discussed (Wold, 1994).

In the engineering process control field, the mean level of the quality characteristic normally can be assumed to wander over time. If the process disturbance is represented by the integrated moving average (IMA) model, the EWMA of the past data has optimal properties as a forecast of the next observation (Box and Luceno, 1997). Unfortunately, it is sometimes difficult to estimate the smoothing constant needed to update the EWMA of past data. Luceno suggested a simple method for the maximum likelihood estimate of smoothing constant with a confidence interval. An accurate and efficient computer routine is provided for performing the computations (Luceno, 1995).

In the EWMA chart, the most recent data is determined by the choice of the smoothing constant usually denoted by the symbol θ, and it is always assumed that the average is updated at regular review intervals. However in practice, it may be necessary to amalgamate the data from several periods, which are split by holidays or some malfunctions and the smoothing constant should be increased to give more weight to the combined data. Johnston described the relationship to compute the necessary adjustment to the smoothing constant with simulation results. It will enable the correct weight to be applied to data collected for only part of a normal forecast review interval, and thus prevent over-reaction to very short-term events (Johnston, 1993).

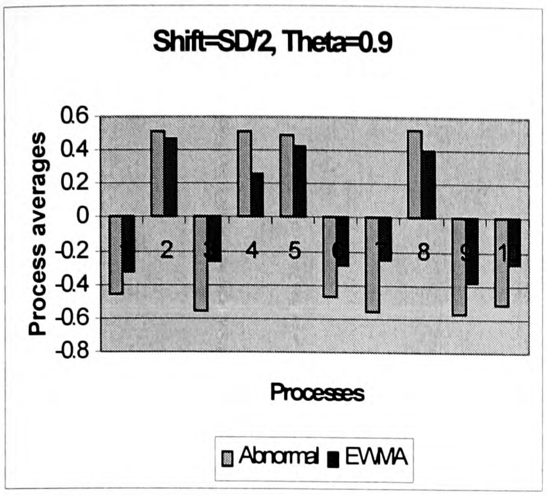
Although the EWMA forecasting can not remove the existence of uncertainty in a monitoring system (Johnston and Boylan, 1994) or the forecasting error (Tseng and Adams, 1994), it is appropriate to apply the EWMA forecast method in this research work as it has a smoothing function and can minimise the uncertainty. In this chapter, EWMA forecast behaviours are investigated by different smoothing constant in different shifted levels of $\overline{X}$ control charts. The ideal smoothing constant or parameter is obtained and used to build the forecaster which involves a typical structure of finite impulse response (FIR) filter, which further reduces the forecasting error. Finally, the NN-Fuzzy-SPC system with combined forecaster is performed and analysed in section 7.4.

7.2.3 The behaviour of the EWMA forecast

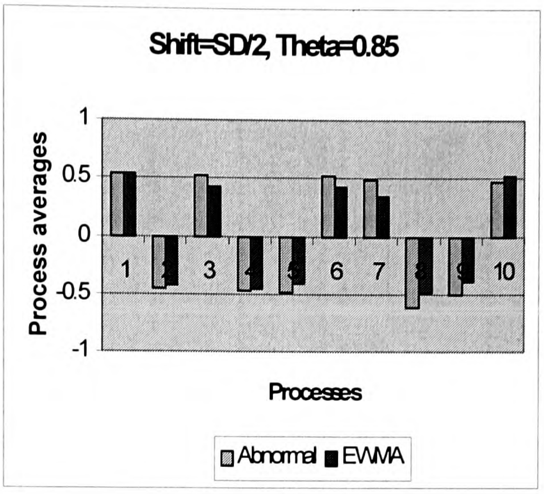
As discussed in section 2.4.3 of chapter 2, the different values of smoothing parameters λ and θ result in different effects on the weights. Large λ and small θ produce more emphasis on recent samples; small λ and large θ result in a chart that averages the effects of the weight. The purpose of this section is to find the ideal smoothing parameter θ, which brings minimum errors between the EWMA forecasting values at the first abnormal points (FAP) tested and the abnormal process average through the simulations of different shifted abnormal processes in EWMA charts.

The simulations were performed ten times for every different smoothing parameter θ using MATLAB. Simulation results are summarised in tables and plotted on the charts. For example, figure 7.1 and figure 7.2 illustrate the comparison for abnormal process averages (shift level is SD/2) and related EWMA values at FAP for different smoothing parameter θ (theta). They are quantified in the tables in the Appendix C.1.1. The EWMA values follow the varying of abnormal process averages in every chart (1) ~ (9) in figure 7.1 and figure 7.2, which indicates the behaviour of the EWMA forecast. The differences or forecast errors are also expressed simultaneously.

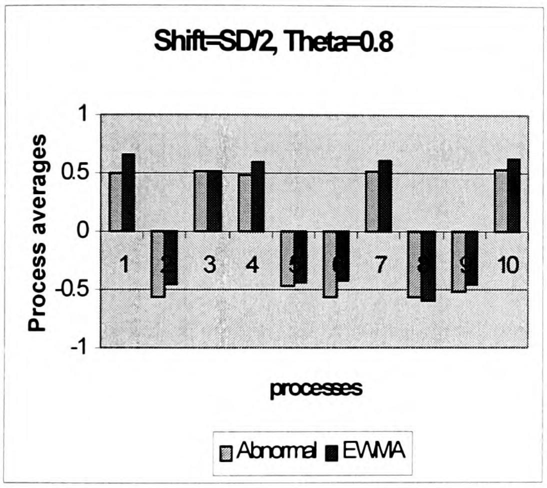
The results of the average of the forecast error for every chart are summarised in table 7.1 and plotted in chart (10) of Fig.7.2. The minimum forecast error average is obtained as 0.0745 with the smoothing parameter $\theta = 0.85$. The forecast results for different θ value in shift levels SD and SD/3 are also represented by tables in Appendix C.1.2 and Appendix C.1.3. Their final results, the averages of forecast errors are described in figure 7.3 and table 7.2.



(1)



(2)



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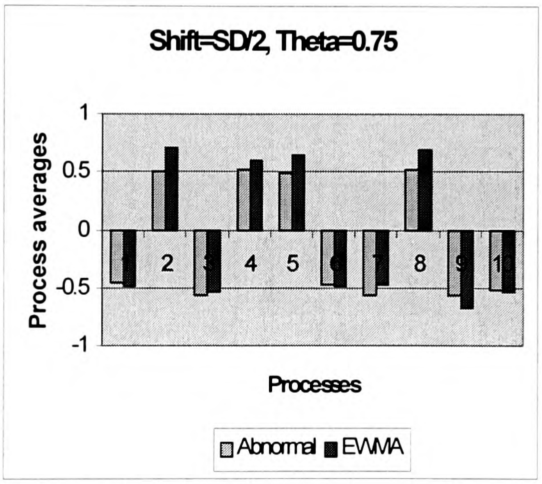

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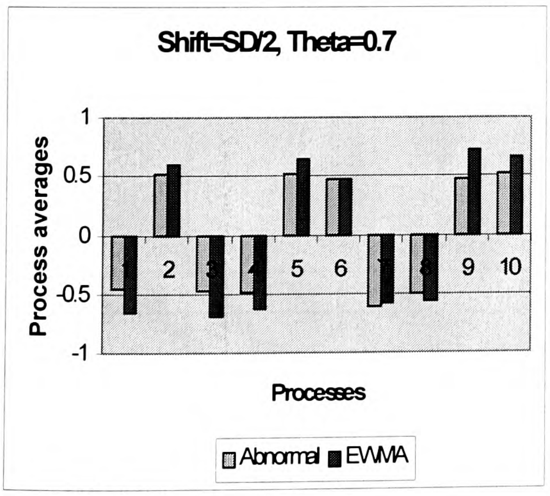

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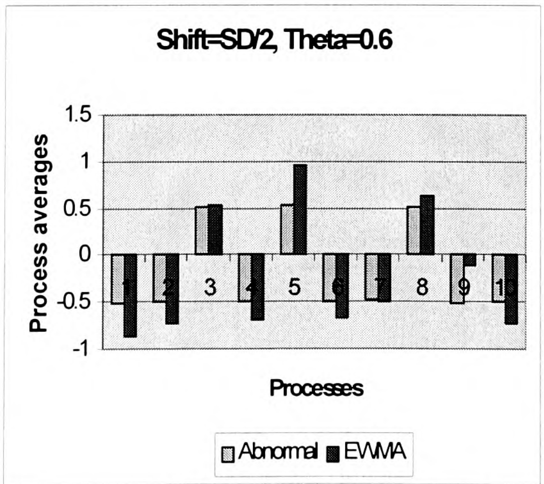
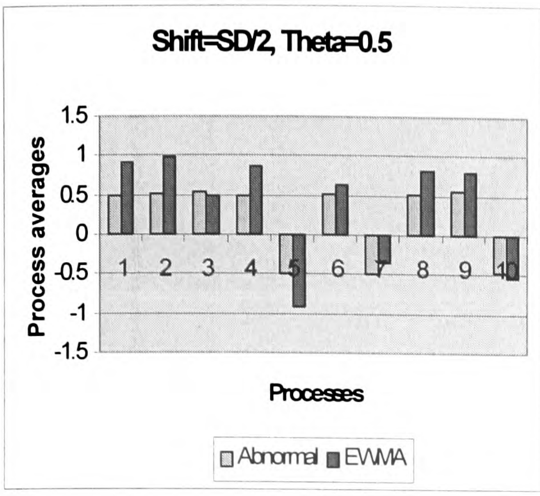


(6)

Figure 7.1 Forecast results (1)



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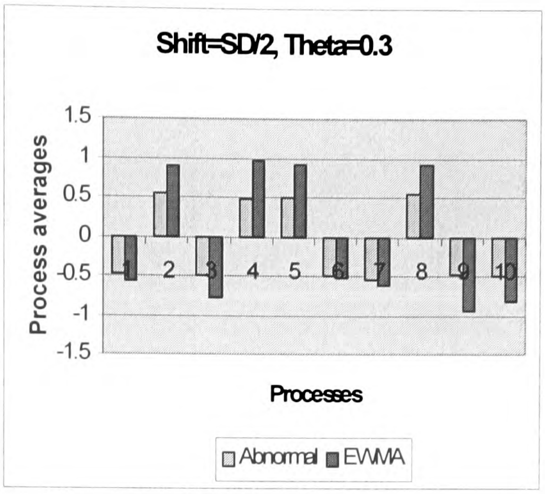

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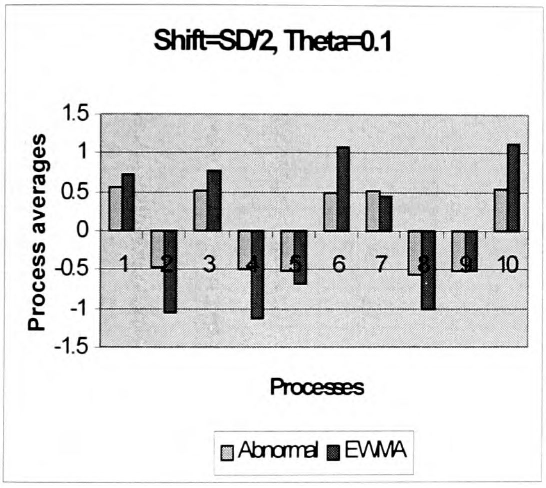
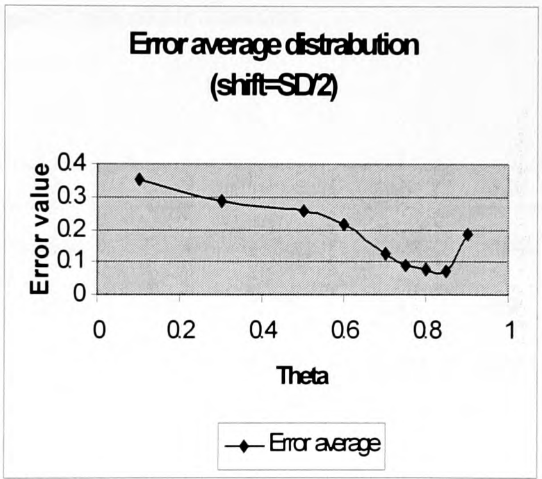


(9)

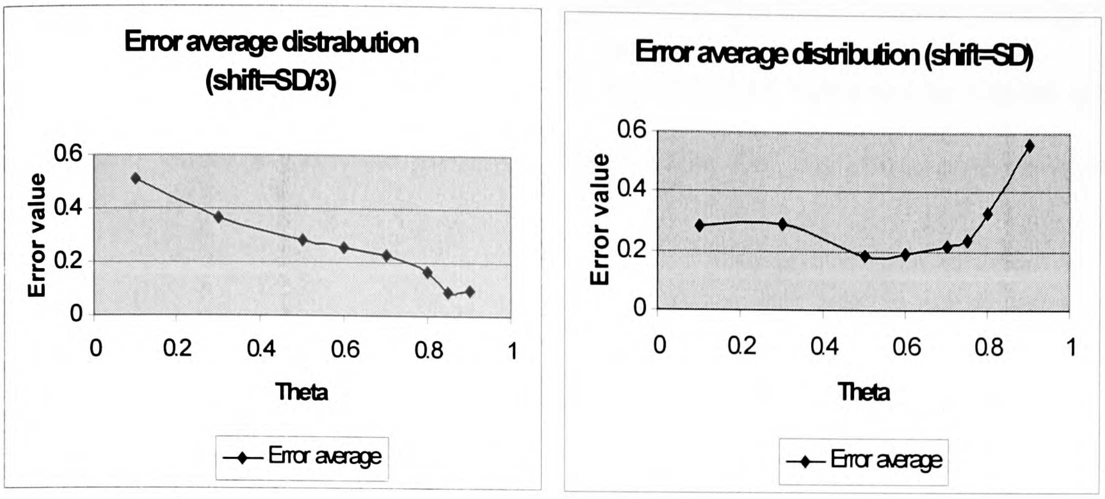


(10)

Figure 7.2 Forecast results (2)

Theta	0.1	0.3	0.5	0.6	0.7	0.75	0.8	0.85	0.9
Error average	0.3498	0.2849	0.2558	0.2137	0.1265	0.0894	0.0791	0.0745	0.1850

Table 7.1 Summary of average of forecast error for SD/2 shift



(1) (2)

Figure 7.3 Forecast error average distributions

Shift	Theta	0.1	0.3	0.5	0.6	0.7	0.8	0.85	0.9
SD/3	Error average	0.5088	0.3649	0.2815	0.2544	0.2260	0.1653	0.0926	0.0946
SD	Theta	0.1	0.3	0.5	0.6	0.7	0.75	0.8	0.9
	Error average	0.2846	0.2903	0.1859	0.194	0.2179	0.237	0.3308	0.5631

Table 7.2 Summary of average of forecast error for SD/3 and SD shifts

Figure 7.3 and table 7.2 show that for the abnormal process average shifted SD/3, the minimum forecast error average obtained is 0.0926 where the smoothing parameter $\theta = 0.85$, which is the same for shift level SD/2. For shift level SD, the minimum error is found when the parameter $\theta = 0.5$. When the shift value changes from SD/3 and SD/2 to SD, the minimum error point (the concave point of the curve) is moved from the right side ($\theta = 0.85$ which is shown in No.(1) of Fig.7.3 and No.10 of Figure 7.2) to the left side ($\theta = 0.5$) and the curve concave area becomes wider (No.(2) in Figure 7.3). Figure

7.4 shows an abnormal process sequence X_2 shifted SD/2 at No. 55 sample point on the $\bar{X}$-EWMA chart using $\theta = 0.85$. In this chart, every EWMA value can be viewed as an interpolating value between the previous EWMA value and last observation (Box and Luceno, 1997).

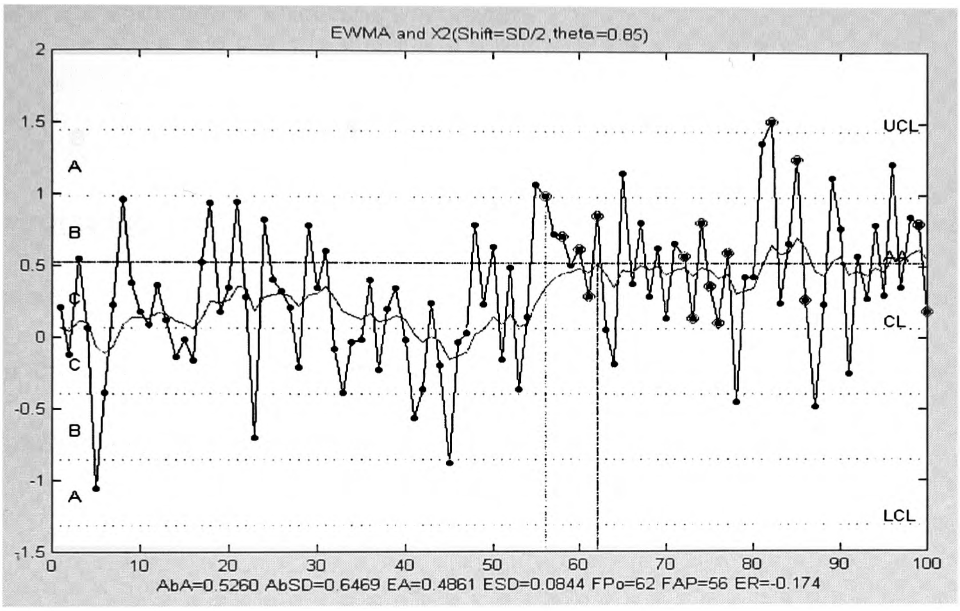


Figure 7.4 EWMA forecast curve in $\bar{X}$ chart

In figure 7.4, the solid horizontal line indicates the abnormal process average, which is the forecast target. AbA and AbSD are abnormal process average and SD respectively, EA and ESD are EWMA average and SD. FAP is the first $\bar{X}$ abnormal point observed, and FPo is the first EWMA point with a forecast error less than 2% of f.s.d. (Bentley, 1995). They are marked by broken vertical line and solid vertical line respectively in Fig. 7.4. The ER indicates the forecast error between EWMA value and X_2 at the FAP, which is used as the forecast point or location for the current shift. To compare EA and AbA, the absolute value of the difference is 0.0399, which is satisfactorily small. The distance between FPo and FAP is 62-56=6. After a delayed interval of 6 samples, the error is less than 2%. Although the EWMA curve has been much smoothed from the $\overline{X}$ chart, it still has quite larger fluctuations (ESD=0.0844). The ER has a large value of –0.174. These data indicate that the EWMA can not be directly used for indicating or forecasting the process average with higher forecasting accuracy.

To remove or reduce the signal fluctuations or randomness, digital filtering is a frequently used technique in engineering process monitoring and control. Digital filtering can provide a more precise design and analysis method. The forecast effects using filter+ EWMA method are further discussed in section 7.3.

7.3 The digital filter and EWMA forecast

Based on section 7.2.3, this section employs a lowpass filter technique to smooth and reduce the signal fluctuations in order to achieve a comparative steady EWMA forecast with minimum delay.

7.3.1 Overview of digital filters

The filter is a device (electric network or algorithm) that transforms an input signal in some specified way to yield a desired output signal (Johnson, 1976). A lowpass filtering objective is to remove or reduce the high frequency noise, extract or enhance the useful information from a mix of conflicting signal or information (Ingle and Proakis, 1997). In engineering, the filter can be classified as two types: analogue filter and digital filter (Willianms and Taylor, 1988). Since the advent of Very Large Scale Integration (VLSI) and computer technologies, digital filters are widely applied in control, image processing, speech/audio and telecommunications. It is quite possible that many established continuous-time filter systems will be replaced by equivalent digital filter systems as they have many inherent advantages: no drift, approximate ideal frequency response, simple to achieve the adaptive function and low cost etc. (Terrell, 1988).

To design a digital filter, convolution must first be addressed. Convolution is one of the most frequently used operations in digital signal processing (DSP). It describes how the input to a system interacts with the system to produce the output. Consider two finite and causal sequences $x(n)$ and $h(n)$, of lengths N_1 and N_2 respectively, their convolution is defined by equation 7.2.

$$y(n) = h(n) \otimes x(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{k=0}^{\infty} h(k)x(n-k) \tag{7.2}$$

where the $\otimes$ is convolution symbol, $n = 0,1,\ldots,(M-1)$, $M = N_1 + N_2 - 1$.

Digital filters are broadly divided into two classes of filters, infinite impulse response (IIR) and finite impulse response (FIR). They are represented by their impulse response sequence $h(k)$ $(k = 0,1,\ldots)$. For IIR filters (equation 7.3), the impulse is of infinite duration, whereas for FIR (equation 7.4) it is of finite duration, since $h(k)$ has only N

values. For a given filter, $h(k)$ is a unique coefficient, which determines the filter's characteristics (Ifeachor and Jervis, 1993).

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) \tag{7.3}$$

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \tag{7.4}$$

In equation 7.3 and 7.4, the input sequence $x(n)$ and the output sequence $y(n)$ are related by a convolution sum, which is discussed above.

Compared to the IIR filter, the FIR filters have some advantages. For example, (1) an exact linear phase response is achievable and is always stable by the evaluation of equation 7.4. (2) The coefficient quantization errors are much less severe. If the number of filter coefficients is not too large and if little or no phase distortion is desired, the FIR is recommended (Ifeachor and Jervis, 1993). Therefor, the FIR filter is chosen for the NN-Fuzzy-SPC system. The z-transform of equation 7.4 $Y(z)$ and the transfer function $H(z)$ for the FIR is given by equation 7.5 and 7.6 respectively. Equation 7.6 will be used in section 7.3.3.

$$Y(z) = \sum_{k=0}^{N-1} h(k)X(z)z^{-k} \tag{7.5}$$

$$H(z)=\frac{Y(z)}{X(z)}=\sum_{k=0}^{N-1}h(k)z^{-k} \tag{7.6}$$

where $X(z)$ is the z-transform of the input sequence, and complex variable $z=e^{(\sigma+j\varpi)T}$, σ and ω are real part and imaginary part respectively, T is the sample cycle.

7.3.2 Coefficient calculation

The three most commonly used methods for obtaining the impulse response $h(n)$ are window, optimal and frequency sampling methods. Their choice depends on the filter's specifications. The window method is used in this section as it has a flexible and simple way to compute the FIR filter coefficients.

For the lowpass filter, if $H_D(\omega)$ is the theoretical frequency response and is represented by:

$$H_D(\omega)=\begin{cases}1, & -\omega_c+2\pi n\le\omega\le\omega_c+2\pi n\\ 0, & \omega=other\end{cases} \tag{7.7}$$

then the related theoretical impulse response $h_D(n)$ is calculated from the inverse Fourier transform:

$$
\begin{aligned}
h_D(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_D(\omega)e^{j\omega n}d\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} 1\times e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1\times e^{j\omega n}d\omega \\
&= \begin{cases} \dfrac{2f_c \sin(n\omega_c)}{n\omega_c}, & n \neq 0, -\infty \leq n \leq \infty \\ 2f_{c,} & n = 0 \end{cases}
\end{aligned}
\tag{7.8}
$$

where ω is the angular frequency, ω_c is the cutoff angular frequency, f_c is the cutoff frequency and subscript D is used to distinguish the ideal or theoretical response.

The curve of $h_D(n)$ is given by the top left chart in Figure 7.5. The curve tails in the two sides carry on to $n = \pm\infty$. The window method mentioned above is used to truncate the

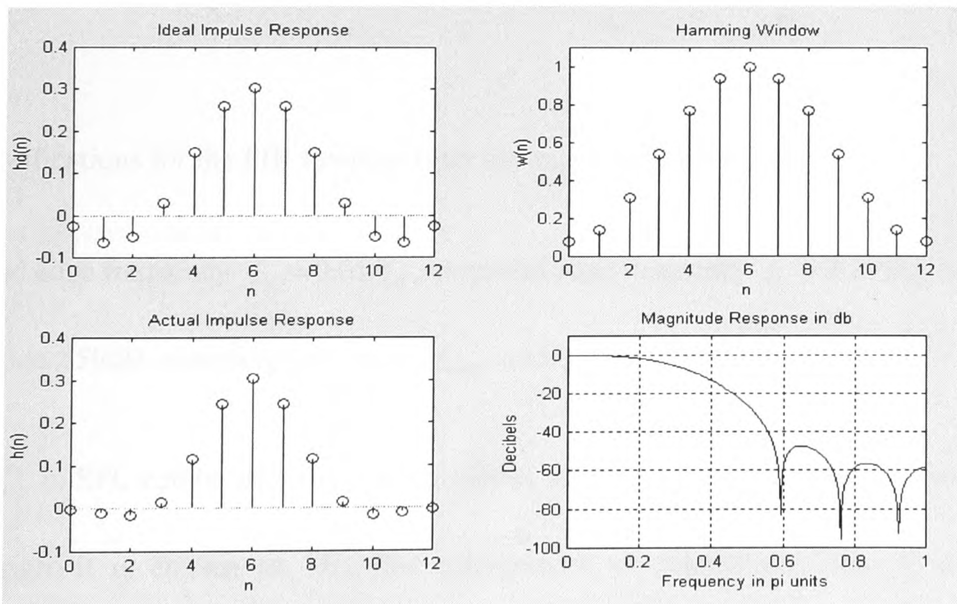


Figure 7.5 Impulse response and Hamming window

theoretical curve by setting $h_D(n) = 0$, in order to make the actual impulse response $h(n)$ shown on the bottom left chart in Figure 7.5, which is used for the FIR filter. However, the more coefficients that are retained in $h(n)$, the closer the filter spectrum is to the ideal response (Ifeachor and Jervis, 1993).

The Hamming window that is the most widely used window function is applied in this section as it has good response in both time domain and frequency domain. The top right chart of Figure 7.5 shows the Hamming function (also see equation 7.12), which decreases gently towards zero on either side in the time domain, and its frequency response is described in the bottom right chart on the same Figure. The main lobe is wider and side lobes are smaller 50dB down on the main lobes. That is, most components or coefficients of $h_D(n)$ are retained in $h(n)$.

The specifications for the FIR lowpass filter are explained as follows:

Passband edge frequency $f_p = 0.01H_Z$, stopband edge frequency $f_s = 0.27H_Z$, stopband attenuation >50dB, sampling frequency $f_{samp} = 1H_Z$.

The f_{samp} in SPC can be taken as various values, which depend on different processes. In this design, it is chosen as $1H_Z$ for convenience of calculation. The $f_p = 0.01H_Z$ indicates the passband width is 0.01 H_Z (0~0.01 H_Z). That is, in an $\overline{X}$ chart which involves a hundred samples, the signal after filtering has a single cycle (

cycle=1/0.01=100 seconds). For $f_s = 0.27H_Z$, this indicates the signal after filtering most of the cycle components in the data (100/(1/0.27)=27 times on an $\overline{X}$ chart). This will cause a long delay to further reduce the f_s. As for the "stopband attenuation >50dB" in the specification, it indicates that when a signal sequence wave frequency exceed f_s, it is attenuated by a more severe criteria than a suggested minimum standard of 30 dB (Ifeachor and Jervis, 1993).

Based on the specifications discussed above, the related design procedure is given below.

The transition band $\Delta f = f_s - f_p = 0.27 - 0.01 = 0.26$

The length of filter $N = \frac{c}{\Delta f} = \frac{3.3}{0.26} \approx 13$, where constant c is 3.3 for the Hamming window.

The filter coefficients $h(n)$ are given by equation 7.9 and plotted on the bottom left chart in Figure 7.5.

$$h(n) = h_D(n)w(n), \quad -6 \le n \le 6 \tag{7.9}$$

where

$$h_D(n) = \frac{2f_c \sin(n\omega_c)}{n\omega_c}, \quad n \ne 0 \tag{7.10}$$

$$h_D(n) = 2f_c, \quad n = 0 \tag{7.11}$$

Hamming function $w(n) = 0.54 + 0.46\cos(2\pi n / 53)$ (7.12)

The calculation result is:

$h(n)$=[-0.0022, -0.0090, -0.0157, 0.0151, 0.1153, 0.2444, 0.3050, 0.2444, 0.1153, 0.0151, -0.0157, -0.0090, -0.0022].

The filter designed magnitude responses are shown in the upper two figures in Figure 7.6. The filter response is attenuated to zero at a frequency of less than 0.6π (top left in Figure 7.6) equivalent to 0.3 H_Z. After frequency point 0.6π, the side lobes are smaller than –100db (top right in Figure 7.6). They satisfy the filter specifications of stopband edge frequency $f_s = 0.27 H_Z$ and stopband attenuation >50dB.

In the calculation results, $h(n)$ has 13 components which cause longer delays in SPC control. The components that are negative and nearest zero are truncated (Hines, 1997) to yield a $h_t(n)$ in this application, in order to reduce the delay to 4 samples.

$h_t(n)$=[0.1153, 0.2444, 0.3050, 0.2444, 0.1153]

This truncated filter spectrum is shown in the bottom left and bottom right in Figure 7.6. Compared to $h(n)$, higher harmonic ripples appear but can be ignored as their |H| responses are very small. The attenuation curves still satisfy the specifications.

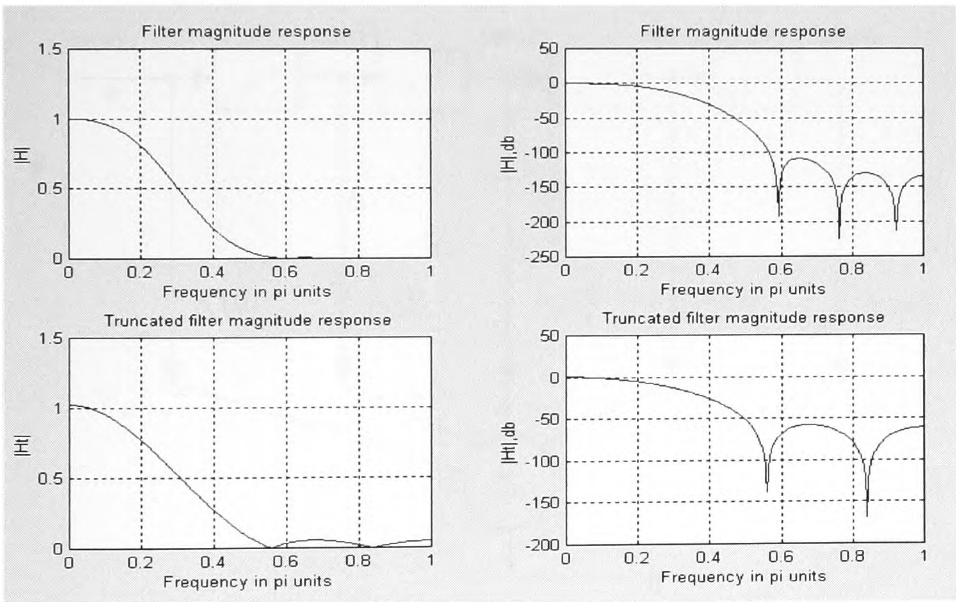


Figure 7.6 Filter magnitude responses

7.3.3 The filter realisation structure and filtering effects for EWMA forecast

The realisation structures are essentially block diagram representations of the different theoretically equivalent ways the transfer function which characterises the FIR filter (equation 7.6) can be arranged (Ifeachor and Jervis, 1993). As an implementation, the output of the filter is obtained through these structures. For the realisation of the filter, the transversal structure (Figure 7.7) is used in this application which is a commonly used structure. The output $y(n)$ for the transversal structure is given by equation 7.4, and it is used to filter and smooth the abnormal process sequence X_2 before the EWMA forecasting.

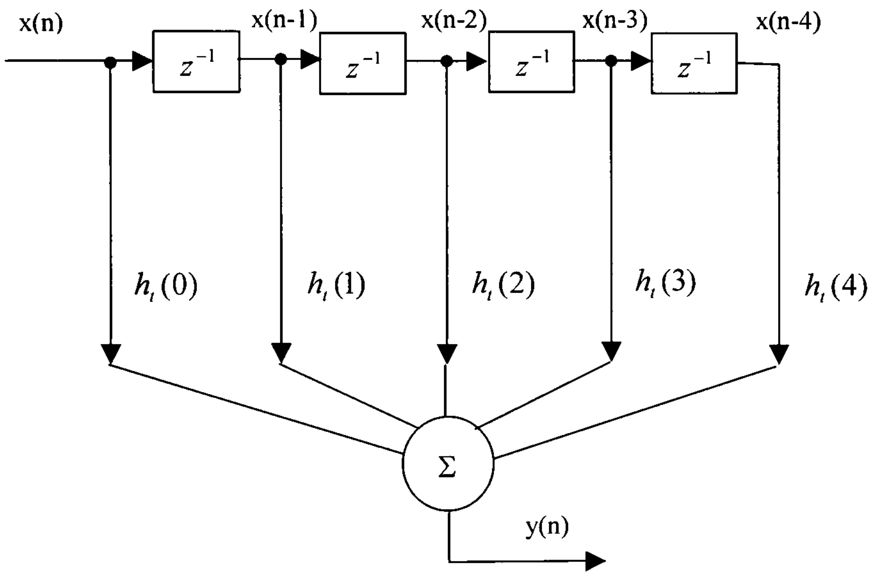


Figure 7.7 Transversal structure

An example of the filtering + forecasting result for abnormal process sequence X_2 which is the same as Figure 7.4 is shown in Figure 7.8. After the filter action and twice EWMA forecast, the smoothness of forecast curve is improved from Figure 7.4, but the forecast error ER value at first abnormal point (FAP) location is –0.262 which is not satisfactorily small. In other words, the smooth curve can not follow closely the shifted X_2 sequence or abnormal process average (AbA) at point FAP. The smooth curve moves gradually closer to AbA in an approaching phase (from FAP to Fpo). This phenomenon is similar to the step response curve of the first-order system in automatic control, and results in some forecast delay.

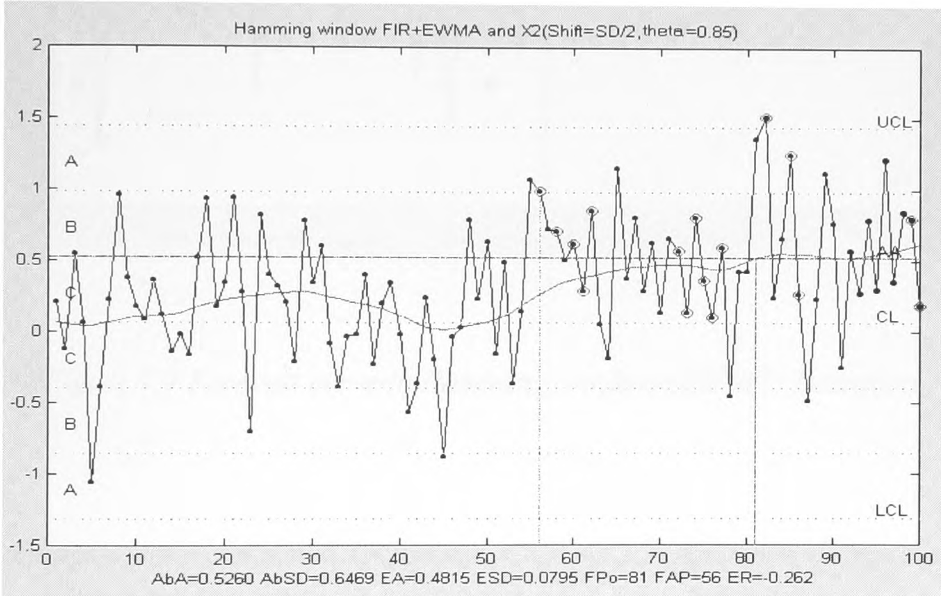


Figure 7.8 Forecast curve in Hamming window and EWMA

Figure 7.9 shows an example of the forecast curve using the same Hamming window + EWMA forecaster with a derivative block that is represented by $K(x_2(i)-x_2(i-1))/T$, (K is a derivative coefficient and T is the sample cycle). The derivative block adds the capability of prediction of the future tendency or error (Astrom and Hagglind, 1995) and raises the response speed in the transient process (Shi, 1985) in order to reduce the forecast error ER |-0.262| to |0.0094| at location FAP in the Hamming window + EWMA forecaster.

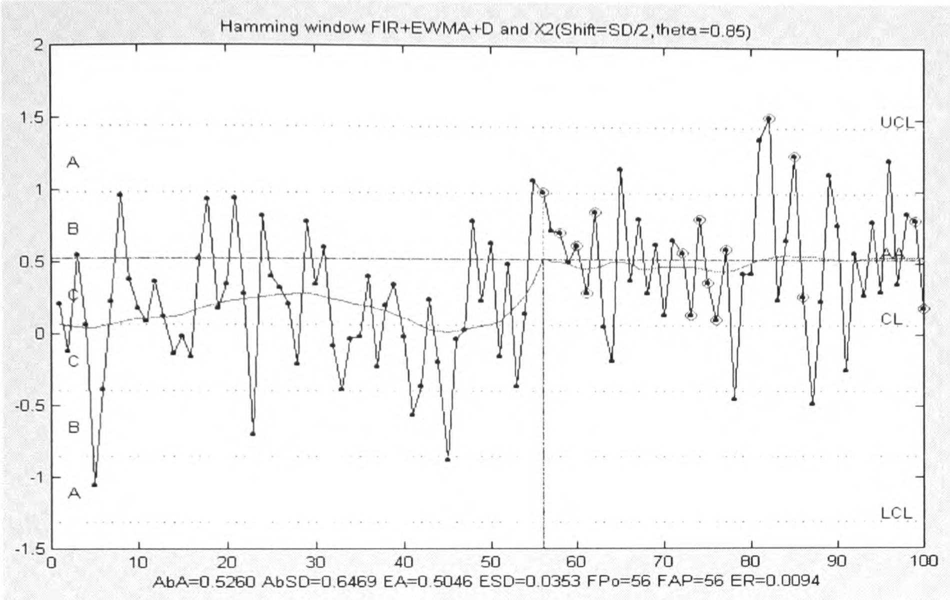


Figure 7.9 Forecast curve in Hamming window+EWMA+Derivative

30 experiments were conducted (Appendix C.2.1~C.2.3) and their forecast errors are summarised by absolute values in table 7.3. They are satisfactorily small if the hybrid method of Hamming window filter and EWMA is used to forecast the abnormal process average.

Item	SD/3	SD/2	SD
Average	0.0342	0.0404	0.0375
SD	0.0196	0.0149	0.0208

Table 7.3 The absolute value of forecast error average and SD for $\overline{X}$ chart

A similar method is used for the R chart, the smoothing constant θ is chosen as 0.85 (Appendix C.3) for the spread levels which is distributed by $1.2\sigma \sim 1.5\sigma$, and the forecast results are summarised in table 7.4.

spread	1.2σ	1.3σ	1.4σ	1.5σ
average	0.1113	0.1246	0.1034	0.0829
SD	0.0661	0.0824	0.0673	0.0229

Table 7.4 The absolute value of forecast error average and SD for R chart

7.4 Simulation of NN-Fuzzy-SPC system

The forecast or estimate abnormal process average which is discussed in section 7.3.3 is used in the NN-Fuzzy-SPC system as the control system output in a forecast or prediction horizon. In the engineering predictive control field, the prediction horizon is chosen according to the settling time of the step response of the processes (Soeterboek, 1992). In this NN-Fuzzy-SPC system, it is chosen as 30 samples. After this prediction horizon, if the control error is still not small enough, the estimated system output will be modified by the actual system output, which is calculated using the last 30 samples. Furthermore, an optimisation process is used in the simulation of the EWMA constant θ, as it has the error standard deviation which can be not ignored (Appendix C1.1~C1.3 for section 7.2.3). Compared to chapter 6, the NN-Fuzzy-SPC system is improved with the optimal forecaster and minimum SSE in the training procedure (Figure 7.10).

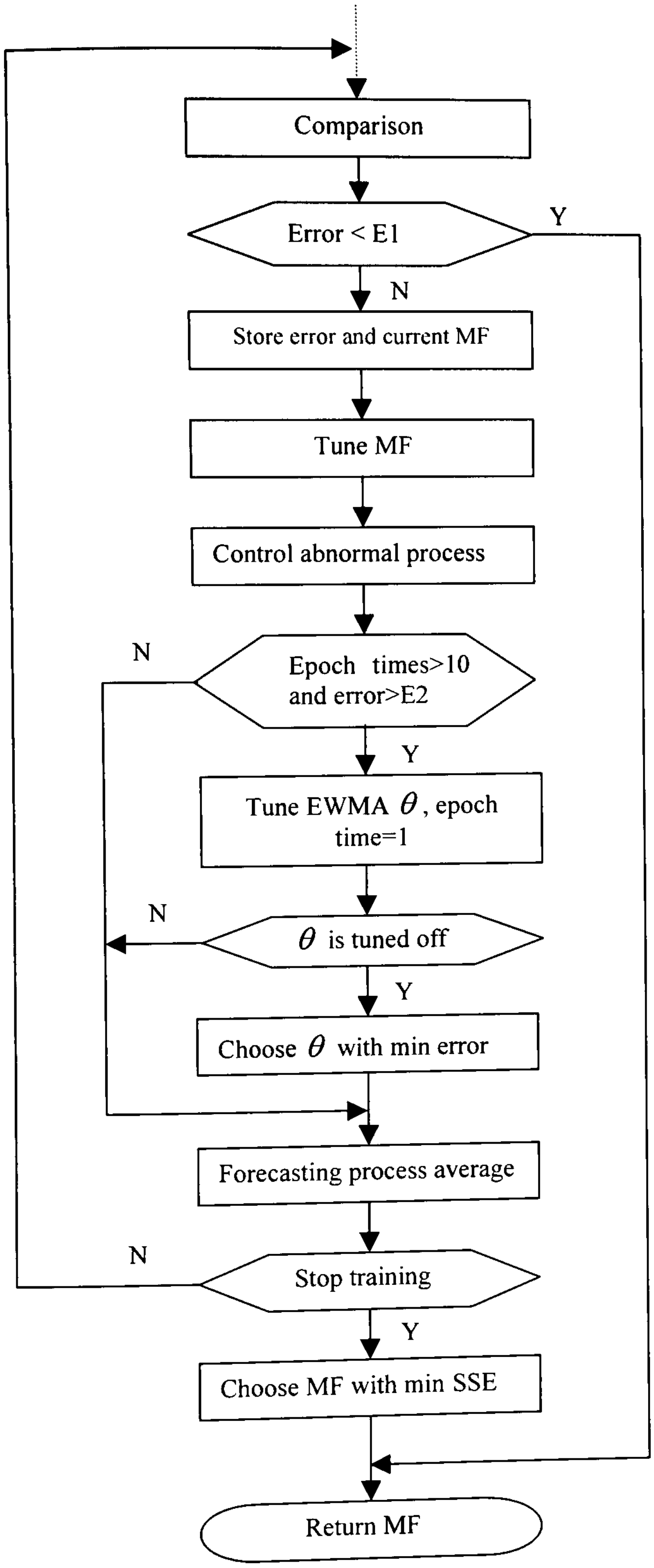


Figure 7.10 The training process with optimal θ

The training process starts with the comparison of target output and controlled output forecast value. If the "Error" is larger than a predefined limit E1, the system tunes the membership function (MF) and adjusts the abnormal process. When the epoch times are more than ten and the control "error" is still larger than a predefined limit E2, the system tunes the EWMA θ in the interval 0.5~0.95 which is discussed in section 7.2.3 to reduce the forecast errors. When the 45 values of θ from 0.5 to 0.95 have been accessed, and the control error is still great than the condition E1, the system selects the best θ value from the θ set with minimum forecast error and optimal membership function with minimum SSE. As the operation continues, the forecast monitoring and control actions in the NN-Fuzzy-SPC system are verified and modified by the subsequent sampled data in the next phase, in order to further reduce the forecast error.

Figure 7.11~7.14 illustrates 40 executed simulations of range averages, which are spread in 1.2σ, 1.3σ, 1.4σ and 1.5σ respectively and are controlled by optimised R-controller with the forecast function. Table 7.5 shows the related control error averages and standard deviations for Figure 7.11~7.14. In table 7.5, |N-C| indicates the absolute value of difference between the normal range average and the controlled abnormal range average. UCL is upper control limit in the R chart, |N-C|*2/UCL describes the relative control errors calculated as f.s.d. (Bentley, 1995). The details of the calculations for control errors are summarised in Appendix C.4, their averages and standard deviations show that the control errors are small enough and control effects are stable.

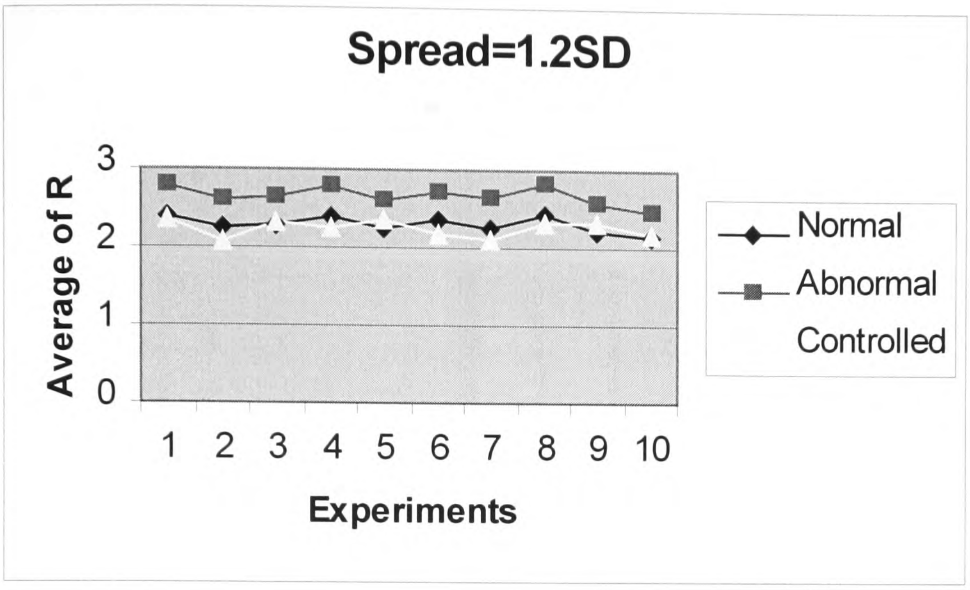


Figure 7.11 Control results (1) of R-fuzzy controller

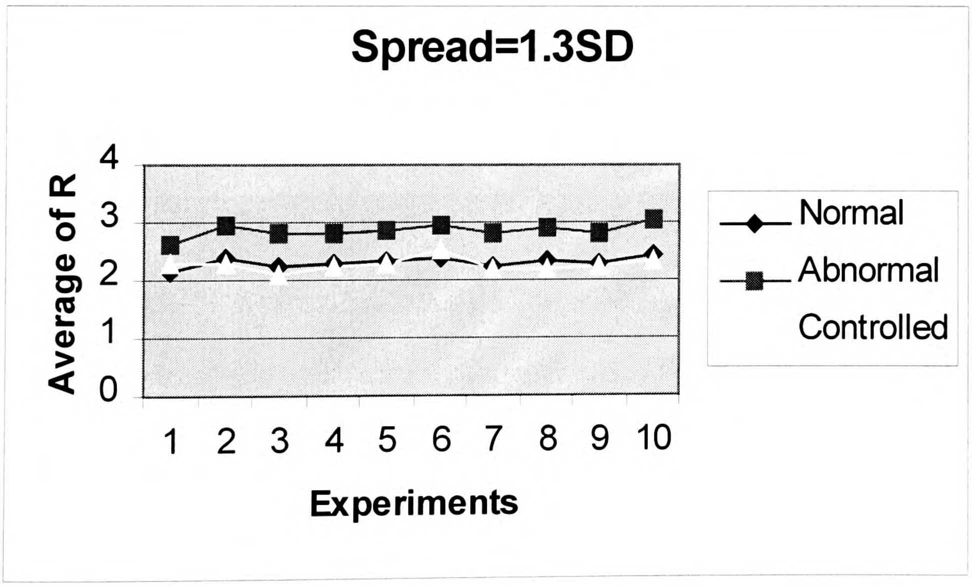


Figure 7.12 Control results (2) of R-fuzzy controller

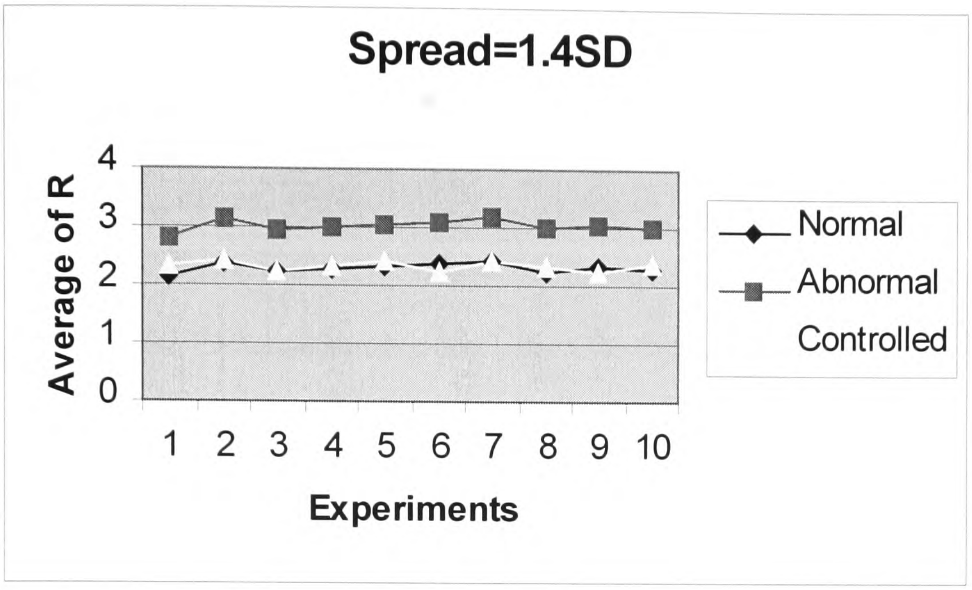


Figure 7.13 Control results (3) of R-fuzzy controller

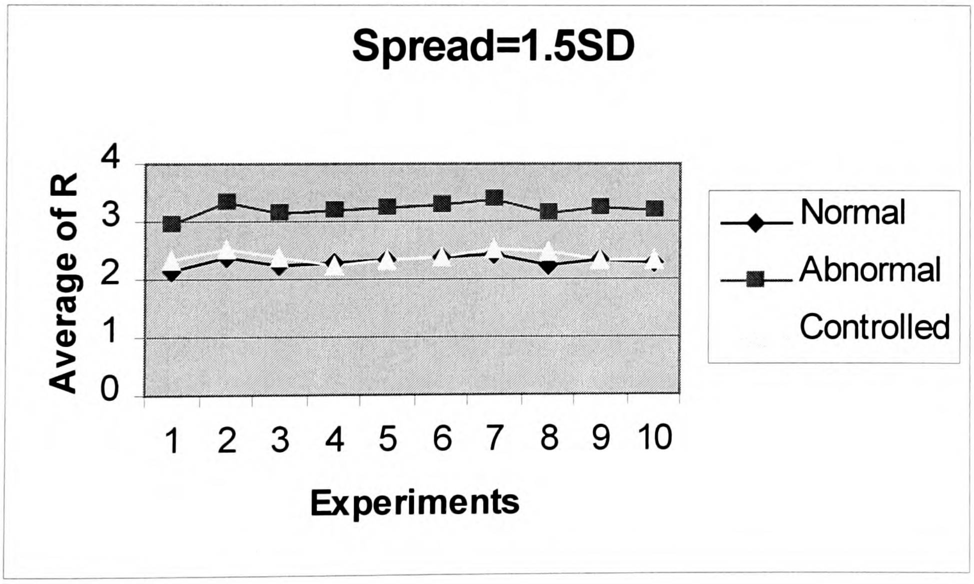


Figure 7.14 Control results (4) of R-fuzzy controller

Spread	1.2 σ		1.3 σ		1.4 σ		1.5 σ	
Items	|N-C|	|N-C|*2/Ucl	|N-C|	|N-C|*2/Ucl	|N-C|	|N-C|*2/Ucl	|N-C|	|N-C|*2/Ucl
Average	0.1137	0.0468	0.0903	0.0371	0.0929	0.0388	0.1155	0.0400
SD	0.0554	0.0226	0.0559	0.0237	0.0706	0.0307	0.0956	0.0357

Table 7.5 Summary of control errors for R-fuzzy controller

In the same way, Figure 7.15~7.18 illustrate 40 executed simulations of process averages, which are shifted by SD/3, SD/2, SD/1.5 and SD respectively and are controlled by an optimised $\overline{X}$-controller with the forecast function. Table 7.6 shows the related control errors averages and standard deviations for Figure 7.15~7.18. In table 7.6, |N-C| indicates the absolute value of difference between normal process average and controlled abnormal process average. U-L is the difference between the upper control limit and the lower control limit in the $\overline{X}$ chart. The |N-C|*2/(U-L) describes the relative control errors. The details of calculations for control errors are summarised in Appendix C.5.

Figure 7.15~7.18 and table 7.6 indicate that the optimised $\overline{X}$-fuzzy controller with forecast function has satisfied the control accuracy and stable control effect.

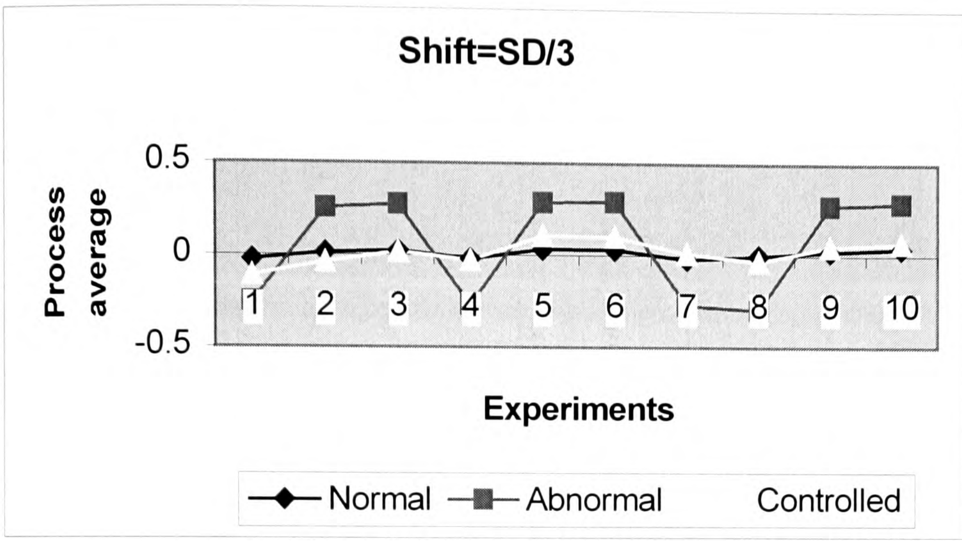


Figure 7.15 Control results (1) of $\overline{X}$-fuzzy controller

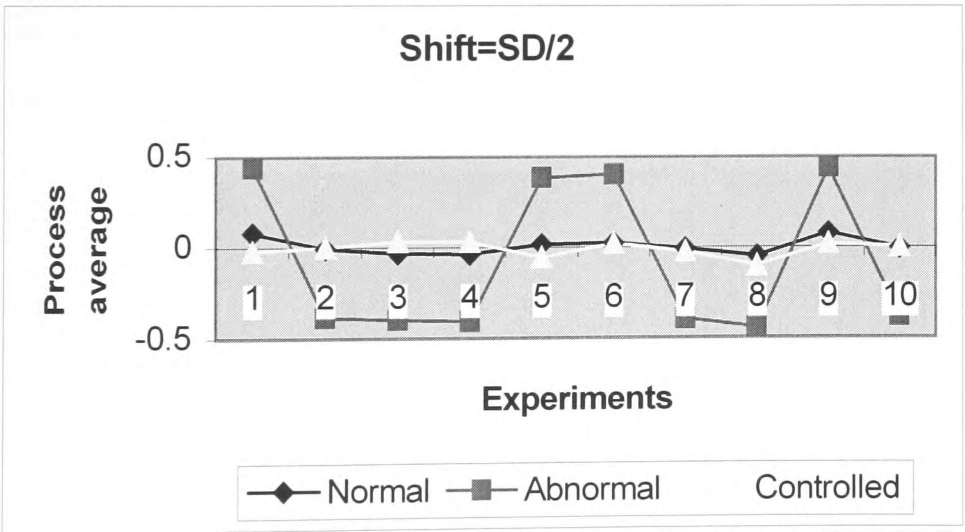


Figure 7.16 Control results (2) of $\overline{X}$-fuzzy controller

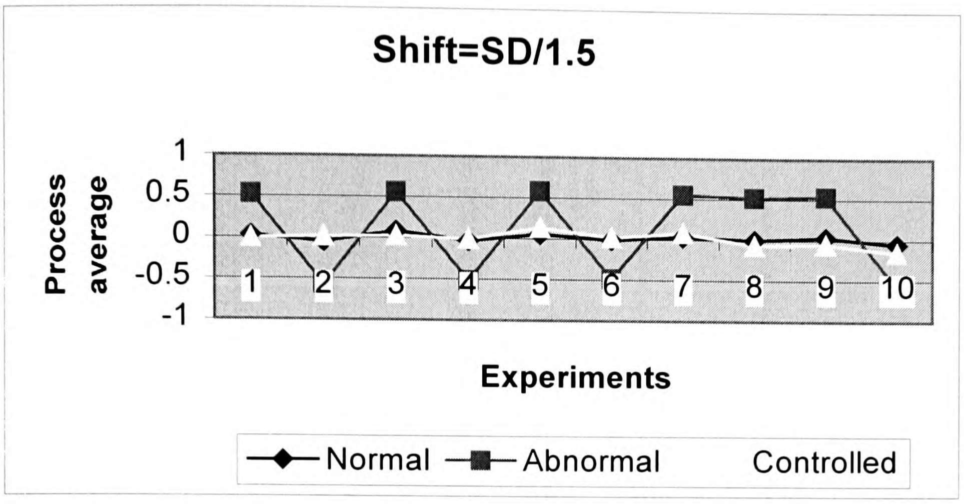


Figure 7.17 Control results (3) of $\overline{X}$-fuzzy controller

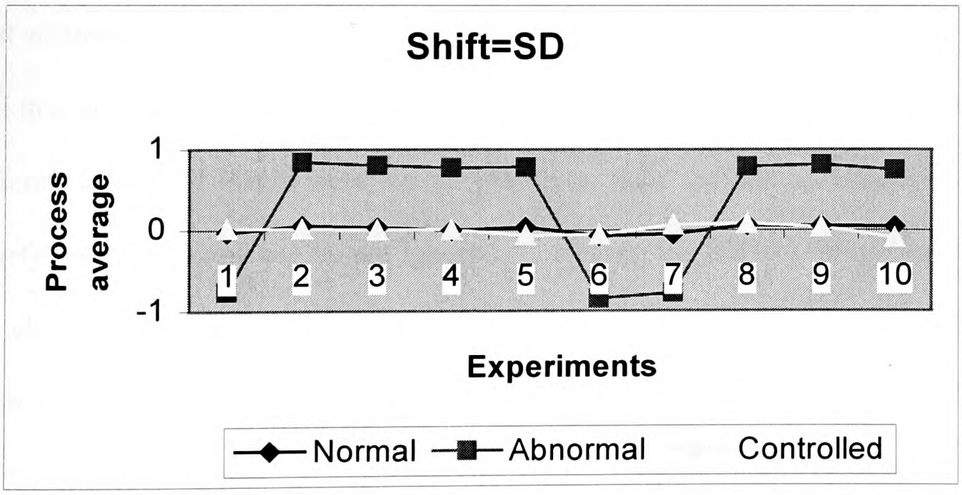


Figure 7.18 Control results (4) of $\overline{X}$-fuzzy controller

Shift	SD/3		SD/2		SD/1.5		SD	
Items	|A-C|	|A-C|*2/(U-L)	|A-C|	|A-C|*2/(U-L)	|A-C|	|A-C|*2/(U-L)	|A-C|	|A-C|*2/(U-L)
average	0.0409	0.0310	0.0450	0.0343	0.0574	0.0420	0.0508	0.0388
SD	0.0284	0.0219	0.0342	0.0266	0.0264	0.0184	0.0485	0.0381

Table 7.6 Summary of control errors for $\overline{X}$-fuzzy controller

7.5 Conclusion

The EWMA forecast function is very useful in business and management areas. Its forecast error depends on the selection of the smoothing constant θ. A calculation method suggested by Johnston (1993) is not suitable for this research work. In monitoring and control applications, it is advisable to combine some optimal methods to keep the forecast accuracy. In this chapter, after investigating the behaviour of the EWMA forecast method in a number of experiments, the ideal θ values with minimum errors in averages for different abnormal levels were found. However, their standard deviation values did not satisfy the criteria and can be not ignored. Therefore, the EWMA smoothing constant θ was chosen initially based on statistical experiment calculations and subsequently, it was finally decided by an on-line optimising operation in the NN-Fuzzy-SPC system. In addition, a FIR filter was also designed for the forecast process. It is used to smooth abnormal process sequences, in order to increase the forecast accuracy. In this chapter, the forecast and control approach with a small (four samples) delay is undertaken with the basic shift and spread level of process mean and variability. The simulation results show control error averages of less than 5%, which satisfy the criteria for this type of controller.

7.6 Summary

This chapter represented an improved NN-Fuzzy-SPC system. The EWMA forecast and FIR digital filter were employed to reduce the control delay. The forecast function was used to obtain the forecast or estimate system outputs with a 4 sample delay only, instead of actual system outputs, which were used to train neural network in NN-Fuzzy-SPC system with a 30 sample delay. The EWMA forecast method and its smoothing constant θ are described and analysed using many statistical experiments. For further smoothing the data or signal fluctuation, an FIR filter is designed. The design details and the realisation structure are explained and the effect of the filter+EWMA forecast is simulated and analysed by experimental data. The system performed with satisfactory results. For the R-fuzzy controller, the related control error averages are 3.71%~4.68%, for the $\bar{X}$-fuzzy controller, the related control error averages are 3.10%~4.20%.

Chapter 8 Conclusion, discussion and future work

This thesis is based on SPC zone rule characteristics and weaknesses, resulting in the development of a NN-Fuzzy-SPC system. This chapter summarises its conclusions, details the contributions made and outlines relevant future work.

8.1 Introduction

This thesis is based on the discussion and analysis of SPC pattern characteristics and weakness, in order to identify methods which can be used to overcome zone rule problems, realise and improve automatic SPC feedback control. Section 8.2 lists the main contributions. The details of the contributions and conclusions of the thesis are explained in sections 8.3~8.7. Section 8.3 discusses the uncertainty of zone rule indication. As the preliminary approach in this thesis, the number of zone rules that are required for feedback control and the frequency distribution of the process average shift levels (FDPASL) are investigated. Section 8.4 describes the design of a Fuzzy-SPC system. The system's frame and design procedure using both manual calculation and MATLAB calculation are summarised. A C++ simulation study is explained in section 8.5 where a Fuzzy-SPC system is successfully simulated, and an attempt to tune fuzzy membership functions is explained. Section 8.6 applies neural networks to build a NN-Fuzzy-SPC system to reduce control errors by optimisation of the membership functions. Optimised controllers show their high control accuracy. Section 8.7 explains the design of an EWMA forecast and digital filter. This combined forecaster which involves the optimal smoothing constant θ and filtering function, is successfully used in a NN-Fuzzy-SPC system to reduce the control delay. A discussion on the feasibility of implementation and the application of NN-Fuzzy-SPC system is described in section 8.8. Some future work relevant to this thesis is discussed in section 8.9.

8.2 Main contributions in this thesis

- Experiments and analysis for frequency distribution of process average shift levels (FDPASL).

- The design and analysis of a Fuzzy-SPC controller.

- Visual C++ simulation and investigation of effect of varying structure of membership function.

- The design and analysis of a NN-Fuzzy-SPC system.

- Investigation of smoothing constant of EWMA forecast, design of FIR lowpass filter and analysis of EWMA+FIR forecaster.

- The design and analysis of NN-Fuzzy-SPC+forecast system

8.3 A discussion of SPC automation and preliminary approach

Statistical process control is a powerful and effective technique to maintain and improve products quality. It can be applied in many industries and in a wide application area. The $\bar{X}$ and R control charts as simple and effective tools in SPC and are frequently used in many applications to identify assignable causes. As an SPC pattern recognition tool, zone rules are developed to increase detection sensitivity of control charts. Through the use of control charts and zone rules, some uncertainty indications such as "out-of-control", "shifted from the centre line" or "systematic trend" for describing abnormal patterns can be obtained. These uncertainty indications can not be directly used to automatically adjust related abnormal processes. The neural network-SPC pattern recognition approach is a very significant attempt to implement SPC for on-line measurement and automation. However, weaknesses of the availability of plenty of training data and relative rough results should be improved. Some papers have been published in the SPC feedback control research section (section 2.6 of chapter 2). What magnitude of adjustment or control action should be taken still is a big challenge. As discussed in chapter 2, it is easy to cause control vibration and incorrect control action for random processes, as the control actions are generated only from the difference between the target and every sample data.

The aim of this research work is to give more accurate and robust control action. Based on basic SPC zone rules, the fuzzy inference is designed to generate the control actions (chapter 4). Before the building this Fuzzy-SPC system, two problems are discussed. Firstly, how many zone rules should be selected is a design issue. An analysis indicates that too many rules will cause an increase of type I error. In a control system, similar to the control method mentioned in chapter 2, this will cause some wrong control actions and control vibration. Another problem is that, based on zone rules, how is SPC's control feasibility? That is, which shift level can be indicated by which zone rule, and how often this level shift is occurs (frequency distribution of process average shift levels (FDPASL)). Through the outcomes of sufficient experiments and related analysis, the particular control action ranges, which are indicated by FDPASL at the first abnormal points (FAP), are obtained for zone rules 1, 2, 3 and 5. That is, when the first abnormal point is detected by a zone rule, the control action can be given from related control action range or fuzzy set. Because the occurrence frequency of zone rule 4 is too small, its control action can be taken to depend on other zone rules.

8.4 Development of the Fuzzy-SPC system: an application of fuzzy logic

Fuzzy logic is applied to generate SPC feedback control action. As discussed in section 8.2, SPC zone rules can give some inaccurate or uncertainty indications for assignable causes in abnormal processes. Therefore, fuzzy set theory and fuzzy logic control which achieves process control with uncertainty and vague relationships between inputs and outputs are applied in this research work. Control chart patterns, which are discussed in chapter 2, are interpreted in quantified expressions by fuzzy sets and the related control actions are generated by fuzzy approximate reasoning in the fuzzy inference system which is designed in chapter 4.

As discussed in chapter 3, the Mamdani model, which is a general application frame is used for Fuzzy-SPC inference. In the approximate reasoning, the antecedent part, the numerical expression (membership function) of control chart patterns and the Mamdani implication are used to calculate the consequent part by max-min composition, as they (Mamdani implication and max-min composition) are frequently used algorithms. Connective "and" and union calculations are also used for multiple input and several if-then rules applied in the Fuzzy-SPC system. The COA method of defuzzification is chosen in the fuzzy inference, in order to obtain the crisp output as the numerical control action.

Based on the analysis results of chapter 2, the triangular and trapezoidal membership functions are used in the design of a Fuzzy-SPC system, which is described in chapter 4. The simple calculation is also an advantage for these two types of membership functions. Based on SPC zone rules 1 to 5, related if-then rules are determined to represent the input/output relationship in this approach. Among the Fuzzy-SPC if-then rules design, the simplification of rules and reduction of rules numbers are required, in order to increase the system's working speed. Some experiments and analysis for this simplification are also discussed. In the Fuzzy-SPC system design, the design calculations confirm that even though the fuzzy logic technique is used to describe the uncertainty event, it is based on certain mathematical computations. However, MATLAB is power tool to design many engineering systems including fuzzy schemes and without multitudes of manual calculations. A Fuzzy-SPC system is successfully designed in MATLAB which confirm the manual calculations and results. The input-output Surface Viewer shows that the control scheme is preliminarily satisfied.

8.5 The C++ simulation study and an approach to tune the membership functions

A simulation study is designed and performed to assess the effectiveness of the Fuzzy-SPC control. Using the C++ language can be viewed as design of an application system for the future work. The C++ language especially Visual C++ is a powerful tool to build windows programs. Fuzzy-SPC simulation system is designed using Borland C++ with in the DOS environment, and using Visual C++ AppWizard and MFC within the standard windows environment. The simulation system has a fast performance speed in both applications.

In the Borland C++ simulation system, the abnormal patterns, which are shifted by 0.1 times the universe of discourse, are successfully adjusted and controlled. 0.1 shift being the smallest controllable shift or control sensitivity in the Fuzzy-SPC system using the standard consequent membership functions. The related control error (average) RCE_1 is 8.7% and RCE_2 is 7.5% (chapter 5, section 5.3). In the Visual C++ simulation system, the consequent membership functions are tuned to further reduce the control errors. The experimental results and the statistical t-test verify that, it is an effective way to tune the position and slope of triangular and trapezoidal membership functions. Based on this idea, a set of membership function scheme is designed and used to successfully control abnormal processes. In the simulation experiments, the best membership functions give control actions with smaller control errors. The range curves of controlled abnormal averages show their monotone characteristic and inclusion characteristic. The monotonic increasing or monotonic decreasing curves indicate that the variation of membership function parameters is one feasible way to investigate the fuzzy controller behaviour and to optimise the fuzzy controller. As every curve covers the normal process average, it indicates the best or most suitable membership function, which causes the reduction of control error ($RCE_1 = 0.93\%$ and $RCE_2 = 0.76\%$), can be selected from the set of membership function schemes.

8.6 NN-Fuzzy-SPC system design and performance

The Neural network technique is used in a Fuzzy-SPC system to increase the control accuracy. Neural networks can flexibly and arbitrarily map non-linear functions via their learning capability. They can be used to achieve adaptive control. Fuzzy logic can abstract the approximate reasoning of human decision-marking to achieve intelligent control. The NeuroFuzzy network, which contains advantages of both neural networks and fuzzy logic, is used to build a NN-Fuzzy-SPC system. The Takagi-Sugeno (T-S) model and the Back Propagation (BP) algorithm are applied in this system since the T-S model has the adaptive capability and mathematical tractability and the BP algorithm can achieve non-linear mapping. Because the $\overline{X}$ chart and the R chart are commonly used in industry processes, both of them are used in the NN-Fuzzy-SPC system. It is suitable to investigate the behaviour of the process average by the $\overline{X}$ chart and to check the variation of process deviation by the R chart. The NN-Fuzzy-SPC system involves an R-fuzzy controller and an $\overline{X}$-fuzzy controller which are used to control the spread of process deviations and shift of process averages respectively.

The SPC control actions are generated by the approximate reasoning of the fuzzy controllers. When the control error is larger than a predefined limit, the dynamic fuzzy membership functions are optimised automatically by the learning capability of a neural network. The training results are obtained with short (less than 20) epoch training times. The ideal control actions for the control process are provided by optimised consequent membership functions. After a one step control by the optimised R-fuzzy controller and $\overline{X}$-fuzzy controller, spread deviation and shifted average can be returned to a normal situation with satisfactorily small errors.

An example of performance (chapter 6, Fig. 6.8 and Fig.6.9) show that, when the averages $\overline{X}$ and $\overline{\overline{X}}$ are shifted as translation only, the R and $\overline{R}$ values are not changed at the same time. By contrast, when the spread of the process deviation is increased, it will affect the $\overline{X}$ value. This is characterised by the definitions of $\overline{X}$ and R. To prevent of interaction in the NN-Fuzzy-SPC system, the R-fuzzy controller works first if both abnormal patterns are synchronised.

The abnormal processes are simulated by different shift levels ($\sigma/3 \sim \sigma$) and different spread levels (2σ~4σ) in the 40 experiments implemented. The NeuroFuzzy network provided satisfactory error curve decay in the optimisation procedure and ideal consequent membership functions. The control error ranges are small enough. For the R-fuzzy controller, the error range is 0.6347%~3.4334%, their average is 1.6125% (chapter 6, table 6.3); For the $\overline{X}$-fuzzy controller, it is 0.1391%~1.3563%, and the average is 0.6835% (chapter 6, table 6.2).

8.7 A development of combined SPC forecast using EWMA and finite impulse response (FIR) filter

A combined forecaster using EWMA and FIR filter is designed and used to estimate the abnormal process averages in the NN-Fuzzy-SPC system, in order to reduce the control delay. EWMA forecast function is a very useful tool in business and management applications. Its forecast error depends on the selection of the smoothing constant θ. Some experiments show that the experiential calculation method suggested by Johnston (1993) is not suitable for this research work. Therefore, a number of experiments are undertaken to investigating the behaviour of the EWMA forecast method. The ideal θ values with minimum errors in averages for different abnormal levels respectively are found. Their standard deviations of forecast error do not satisfy the criteria and can be not ignored. Therefore in the NN-Fuzzy-SPC system, the EWMA smoothing constant θ is initially chosen based on statistical experiment calculations and subsequently, it is finally decided by an on-line optimising operation. The simulation results confirm that, it is advisable to use some optimal method for EWMA to keep the forecast accuracy. In addition, a lowpass FIR filter is designed for the forecast process to smooth the abnormal process sequence in order to increase the forecast accuracy. Based on basic shift and spread levels of process mean and variability, forecast and control processes of NN-Fuzzy-SPC +forecast system are successfully performed. The simulation results show a small (four samples) delay and control error averages of less than 5% (for the R-fuzzy controller, the related control error averages are 3.71%~4.68%, for the $\overline{X}$-fuzzy controller, the related control error averages are 3.10%~4.20%), which satisfy the criteria for this type of controller.

8.8 System implementation and application

8.8.1 An overview of assignable causes and some implementation aspects in process control

As discussed in chapter 2, high quality products are made based on stable processes under statistical control. Production exhibits variability in the quality characteristic of interest due to a constant set of common causes. Over time, a process can be subject to several kinds of disturbances (assignable or special causes) that can produce a variety of unstable behaviour with respect to either its mean level (shift) or variability level (spread) or both (Devor et al, 1992).

The assignable causes or fault diagram can be classified into five components (Muhlemann et al. 1992) (Oakland, 1996):

- Product, including services, materials and any intermediates.
- Plant, the building, equipment or machine.
- Processes or methods of transformation.
- Programmes or timetables for operations.
- People, operators, staff and managers.

Based on an on-line and automatic process control application, the first and second components are considered relevant for this research work. Specifically, the material and the machine condition are two major fields for monitoring and control. For example, in the material side, if the chemical material quality (e.g. pH value or concentration) is changed or fluctuates, the chemical product quality will be changed in mean level (shift) or variability level. If the cotton quality (e.g. fibre length and thickness) declines in a textile plant, this causes a reduction of strength and a fluctuation of yarn thickness. If the metallic material qualities (e.g. heat treatment, hardness, strength and dimensional variations) are changed in the manufacturing plant, it leads to the varying of the accuracy and smoothness for the machine work. It is similar in the machine condition side. When the water vapor increases in the drying machine, or environment humidity-temperature are changed, it causes the product to have extra water or not to have necessary moisture; if the cutting tool is worn, the accuracy of product will deteriorate; and if the hydraulic pressure or pneumatic pressure is changed, the pressure effect and pressed product quality will be decreased.

In general in process control, many disturbances for the various qualities of products mentioned above can be compensated or controlled by the regulating valves or other control actuator. For the chemical process, when the pH is changed, the acid or base injection flow can be adjusted to maintain the ideal pH neutralisation (Narayanan et al, 1997). In the manufacturing process, the cutter location and feed rate being the most important factors that can be adjusted to machine high efficiency and quality products (Lo, 1998). In the textile process, when the material or environmental humidity-temperature is changed, the cylinder dryer surface temperature can be adjusted to keep the target moisture of the yarn and save energy, in order to reduce the product costs (Radu et al, 1988).

8.8.2 The application of the NN-Fuzzy-SPC system based on a common physical model

The NN-Fuzzy-SPC system can be applied in the general field of process control. Based on the discussion in section 8.8.1, various processes can be extracted to the models, which indicates the relation between the inputs and outputs. As discussed in section 3.4.2.1 of chapter 3 and section 4.2.1 of chapter 4, the real data sampled from processes may have different physical backgrounds and quality value eg: moisture, pH or dimension, etc.

They are converted to the universe of discourse in the fuzzy controllers to generate the inferences, then through inverse conversion to yield the control actions based on the specific processes. Figure 8.1 illustrates the NN-Fuzzy-SPC system flow diagram.

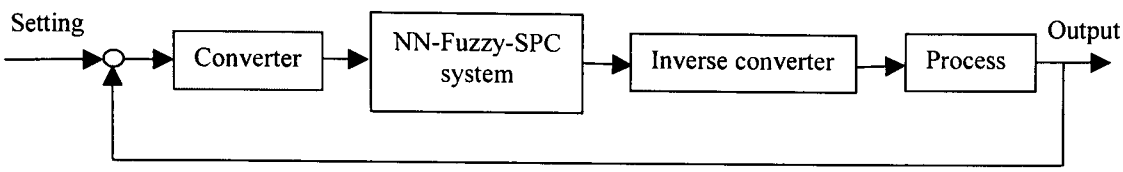


Figure 8.1 NN-Fuzzy-SPC system flow diagram

The details of the NN-Fuzzy-SPC system are shown in Figure 6.6 in chapter 6. The conversion formula is described by equation 3.46 in chapter 3. The inverse-conversion formula is the inverse operation of equation 3.46.

A discussion of the implementation and the application of NN-Fuzzy-SPC system were described in this section. This feasibility discussion indicates that the improved NN-Fuzzy-SPC system can be applied in both automatic control and on-line quality control.

8.9 Future work

$\bar{X}$ and R charts and zone rules 1~5 as basic tools are used in this research work. As for other charts and zone rules, these can be applied in future work. For example, the

CUSUM chart can be used as a detector or alarm for small shift. It is also necessary to use attribute charts to approach feedback control for attribute data, which describe those quality characteristics which cannot be conveniently represented numerically.

In this research work, the abnormal processes are given by basic and frequently used methods: i.e. shift of process averages and spread of the process variation. Other abnormal types of data, such as trend, cycle, systematic variation and mixture can occur and these can be simulated and related to the zone rules and should be investigated as future work to extend the capabilities of the new approach developed.

Visual C++ is a powerful language to design the system for real time applications. In this thesis, the Visual C++ simulation study achieves the implementation of a Fuzzy-SPC controller only. As a further step of the research work, it can be used as the basis to design further systems to be implemented in real time processes.

Nowadays, powerful Digital Signal Processing (DSP) techniques are used to analyse process signals and data arising in many areas of engineering and science, medicine, economics and the social sciences (Lynn and Fuerst, 1994). It is very important in future work to further apply DSP techniques such as filter design or spectrum analysis to SPC or combine DSP and SPC, to develop new methods and results.

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Appendix A: Frequency distribution of process averages shift levels (FDPASL) experiments data

As discussed in section 2.7 of chapter 2, for an investigation of frequency distribution of process average shift levels (FDPASL) at the first abnormal point (FAP) tested by SPC zone rules 1~5, a MATLAB simulation program is designed and performed to obtain sufficient information for 4500 FAPs in order to calculate the frequency distribution. Total data are classified in two sections: original data and modified data. Shift levels from 0.1 SD to 1.5 SD in steps of 0.1 SD were conducted. This appendix shows 6 tables only for shift levels 0.1 SD, 0.5 SD and 1.0 SD in No.1 group.

The notations in the tables are explained below.

No. – Experiments number

Average – Normal process average

AbA – Abnormal process average

AA5 – The average of 5 data after FAP

Zone – Number of Zone rule which is used to test the related FAP

Fq – Occurrence frequency

A.1 Original FDPASL data

No.1 group, shift=0.1SD													
No.	Average	AbA	AA5	Zone	FAP	Fq	No.	Average	AbA	AA5	Zone	FAP	Fq
1	-0.083	-0.184	0.2037	1	22		51	0.053	0.1211	0.1865	4	26	
2	-0.047	-0.138	0.0012	1	24		52	-0.08	-0.184	-0.172	4	31	
3	0.0328	0.1655	0.4018	1	24		53	-0.061	-0.208	-0.26	4	38	
4	-0.071	-0.195	-0.316	1	34		54	0.0529	0.1284	0.016	4	48	
5	0.0122	0.1343	-0.168	1	35		55	-0.004	-0.053	-0.32	4	54	
6	-0.037	-0.11	-0.083	1	35		56	0.0436	0.241	-0.332	4	64	
7	0.0046	0.1132	-0.029	1	35		57	0.0772	0.2395	-0.042	4	70	
8	-0.065	-0.156	-0.129	1	37		58	0.0198	0.1134	0.106	4	78	
9	0.0173	0.1066	0.4209	1	37		59	-0.057	-0.121	-0.341	4	79	
10	0.0185	0.1271	0.0444	1	42		60	-0.014	-0.148	0.1524	4	87	
11	-0.018	-0.15	-0.17	1	47		61	-0.012	-0.058	-0.109	4	89	0.11
12	0.0176	0.1887	0.3013	1	57		62	-0.035	-0.092	-0.08	5	23	
13	0.0067	0.1467	0.2275	1	64		63	-0.055	-0.092	-0.012	5	25	
14	-0.089	-0.114	-0.288	1	67		64	0.0398	0.1034	0.1483	5	25	
15	0.0107	0.1484	-0.062	1	69		65	-0.002	-0.075	-0.08	5	26	
16	0.0068	0.227	0.088	1	78		66	-0.046	-0.11	-0.204	5	26	
17	0.0246	0.1414	-0.075	1	89		67	-0.06	-0.134	0.1018	5	26	
18	0.0817	0.3341	0.2721	1	89	0.18	68	-0.049	-0.109	0.1189	5	27	
19	0.03	0.0882	0.0327	2	21		69	0.0157	0.1282	0.0027	5	28	
20	-0.004	-0.102	-0.432	2	31		70	0.0235	0.1047	0.4187	5	29	
21	0.0499	0.1428	-0.09	2	32		71	0.0614	0.1853	0.5773	5	30	
22	0.0736	0.1314	0.374	2	50		72	-0.021	-0.129	0.1271	5	31	
23	-0.021	-0.103	-0.092	2	52		73	0.0468	0.155	-0.032	5	32	
24	0.0134	0.1244	-0.003	2	52		74	0.1238	0.2315	0.3497	5	35	
25	-0.032	-0.173	-0.331	2	56		75	0.0339	0.116	0.687	5	35	
26	0.0832	0.1657	0.2313	2	63		76	-0.054	-0.141	0.212	5	40	
27	-0.012	-0.148	-0.43	2	68		77	0.0149	0.0618	-0.063	5	40	
28	-0.098	-0.201	-0.431	2	69		78	-0.015	-0.127	-0.323	5	40	
29	0.0282	0.1058	-0.22	2	74		79	0.0164	0.1119	0.0891	5	42	
30	-0.007	-0.095	0.0436	2	78		80	0.0106	0.1382	0.2703	5	42	
31	0.1008	0.303	0.1678	2	81	0.13	81	-0.013	-0.095	-0.143	5	45	
32	-0.002	-0.101	0.0954	3	25		82	0.0327	0.0237	0.296	5	56	
33	0.0246	0.1011	-0.109	3	32		83	0.0608	0.1121	0.1581	5	57	
34	0.041	0.1273	0.457	3	34		84	0.0288	0.1174	0.3086	5	59	
35	0.012	0.0747	0.51	3	36		85	0.0135	0.0383	0.1091	5	61	
36	0.0044	0.047	-0.011	3	41		86	0.0102	0.231	-0.078	5	66	
37	-0.003	-0.126	-0.07	3	42		87	0.002	0.1801	0.4548	5	74	
38	-0.04	-0.117	-0.071	3	44		88	0.0281	0.113	0.3782	5	74	
39	-0.05	-0.133	0.1072	3	44		89	0.0079	0.1509	0.1638	5	74	
40	-6E-04	-0.115	-0.028	3	44		90	0.0412	0.1225	0.4059	5	79	
41	0.0016	0.1126	-0.106	3	44		91	-0.106	-0.247	-0.198	5	79	
42	0.0151	0.1129	0.0176	3	45		92	0.0639	0.2079	0.4886	5	79	
43	-0.044	-0.173	-0.359	3	46		93	0.0011	0.1035	0.1137	5	85	
44	-0.034	-0.222	-0.399	3	47		94	-0.005	-0.093	-0.229	5	88	
45	0.038	0.132	0.1735	3	49		95	0.082	0.4191	0.2902	5	90	
46	-0.038	-0.142	-0.136	3	49		96	-0.007	-0.208	-0.113	5	88	
47	0.0309	0.1092	0.406	3	50		97	0.0229	0.2765	0.3612	5	88	
48	-0.037	-0.06	0.0137	3	51		98	0.0226	0.0451	-0.038	5	83	
49	0.0298	0.1562	0.309	3	52		99	-0.077	-0.132	0.0247	5	87	
50	-0.009	-0.152	-0.227	3	64	0.19	100	0.0125	0.2362	-0.253	5	87	0.39

No.1 group, shift=0.5SD													
No.	Average	AbA	AA5	Zone	FAP	Fq	No.	Average	AbA	AA5	Zone	FAP	Fq
1	0.0862	0.5836	0.5075	1	21		51	-0.083	-0.543	-0.653	3	26	
2	0.0589	0.4821	0.516	1	21		52	0.0106	0.4781	0.4577	3	27	
3	-0.029	-0.482	-0.376	1	22		53	-0.004	-0.451	-0.534	3	27	
4	-0.056	-0.542	-0.769	1	22		54	0.0547	0.5102	0.4197	3	27	
5	0.0028	0.455	0.4514	1	22		55	0.0409	0.5402	0.7202	3	28	
6	-0.034	-0.553	-0.329	1	22		56	0.0152	0.486	0.3704	3	28	
7	-0.037	-0.51	-0.418	1	22		57	0.0146	0.5149	0.485	3	28	
8	0.0647	0.5719	0.5748	1	23		58	0.0373	0.5085	0.5886	3	28	
9	0.0512	0.5035	0.635	1	23		59	0.0449	0.4991	0.473	3	28	
10	-0.008	-0.446	-0.82	1	24		60	-0.03	-0.533	-0.259	3	29	
11	0.0592	0.5461	0.675	1	24		61	-0.084	-0.562	-1.107	3	29	
12	0.0523	0.569	0.3441	1	25		62	-0.106	-0.544	-0.587	3	29	
13	0.0459	0.5221	0.5759	1	25		63	-0.021	-0.514	-0.578	3	29	
14	0.0716	0.569	0.596	1	25		64	0.0056	0.4797	0.6263	3	29	
15	-0.039	-0.569	-0.621	1	27		65	-0.011	-0.492	-0.603	3	30	
16	-0.036	-0.502	-0.216	1	28		66	-0.066	-0.604	-0.638	3	30	
17	-0.015	-0.479	-0.286	1	28		67	0.0057	0.4845	0.5654	3	31	
18	0.0075	0.5239	0.4277	1	30		68	-0.034	-0.515	-0.416	3	31	
19	0.1153	0.6217	0.6582	1	31	0.19	69	0.0493	0.5259	0.3082	3	31	
20	0.0065	0.47	0.338	2	21		70	-0.021	-0.501	-0.565	3	32	
21	0.0069	0.5032	0.422	2	22		71	0.041	0.5116	0.4004	3	32	
22	-0.055	-0.509	-0.41	2	23		72	-0.04	-0.508	-0.41	3	33	
23	-0.019	-0.42	-0.131	2	23		73	-0.028	-0.526	-0.22	3	34	
24	0.0494	0.5514	0.6564	2	24		74	0.0379	0.5256	0.5431	3	34	
25	-0.01	-0.483	-0.394	2	24		75	0.0367	0.5006	0.469	3	35	
26	-0.022	-0.501	-0.099	2	25		76	-0.013	-0.5	-0.925	3	36	
27	-0.021	-0.497	-0.474	2	26		77	-0.018	-0.496	-0.279	3	39	
28	0.0237	0.5399	0.2883	2	27		78	0.0729	0.5826	0.6737	3	41	0.45
29	-0.003	-0.533	-0.493	2	30		79	-0.018	-0.487	-0.562	4	23	
30	0.0043	0.5193	0.4864	2	35		80	-0.006	-0.428	-0.622	4	23	
31	0.0087	0.5358	0.5023	2	36		81	-0.012	-0.494	-0.273	4	24	0.03
32	-0.033	-0.476	-0.418	2	39		82	-0.059	-0.528	-0.484	5	22	
33	-0.097	-0.585	-0.411	2	40	0.14	83	0.0419	0.5305	0.6293	5	23	
34	0.098	0.5329	0.4958	3	22		84	-0.033	-0.521	-0.393	5	25	
35	0.0301	0.4935	0.9604	3	24		85	0.0763	0.5357	0.2736	5	25	
36	-0.003	-0.485	-0.604	3	24		86	0.026	0.509	0.7656	5	26	
37	0.0753	0.538	0.2772	3	24		87	0.0407	0.4913	0.269	5	27	
38	-0.006	-0.488	-0.269	3	24		88	0.0226	0.5008	0.6903	5	28	
39	0.0715	0.5419	0.6145	3	25		89	0.0737	0.5428	0.7429	5	28	
40	-0.037	-0.516	-0.687	3	25		90	-0.032	-0.505	-0.478	5	28	
41	0.1023	0.5487	0.2075	3	25		91	-0.069	-0.572	-0.686	5	28	
42	0.0597	0.5097	0.2695	3	25		92	0.0395	0.5419	0.4603	5	28	
43	0.0013	0.4501	0.3268	3	25		93	0.0383	0.5125	0.4329	5	28	
44	0.0825	0.5649	0.2541	3	25		94	-0.061	-0.491	-0.538	5	29	
45	-0.105	-0.574	-0.271	3	25		95	-0.052	-0.549	-0.476	5	32	
46	-0.015	-0.449	-0.245	3	26		96	-0.05	-0.624	-0.693	5	35	
47	-0.065	-0.538	-0.356	3	26		97	0.0473	0.5839	0.6528	5	37	
48	-0.039	-0.485	-0.366	3	26		98	-0.029	-0.454	-0.464	5	39	
49	-0.01	-0.523	-0.618	3	26		99	-0.102	-0.635	-0.422	5	42	
50	-0.023	-0.514	-0.34	3	26		100	0.0292	0.5854	0.7661	5	45	0.19

No.1 group, shift=1.0SD													
No.	Average	AbA	AA5	Zone	FAP	Fq	No.	Average	AbA	AA5	Zone	FAP	Fq
1	-0.055	-0.959	-0.784	1	21		51	0.0547	0.9861	1.2897	1	23	
2	0.0715	1.0401	1.1843	1	21		52	-0.105	-1.051	-0.877	1	23	
3	0.0473	1.0246	0.7234	1	21		53	-0.033	-0.996	-0.987	1	24	
4	0.0069	1.0077	1.1246	1	21		54	-0.012	-0.941	-0.72	1	24	
5	0.0065	0.9428	0.8108	1	21		55	0.0373	0.9856	0.9127	1	24	
6	-0.021	-0.968	-0.736	1	21		56	0.1153	1.116	0.9823	1	24	
7	0.0862	1.0501	0.9741	1	21		57	-0.021	-0.961	-1.092	1	24	
8	-0.018	-0.969	-0.97	1	21		58	-0.106	-1.007	-0.962	1	25	
9	-0.011	-0.966	-0.907	1	21		59	0.0237	1.0382	0.9589	1	26	0.59
10	0.0647	1.056	1.0489	1	21		60	0.0449	0.9506	1.0016	2	21	
11	0.0292	1.01	0.414	1	21		61	0.0409	1.0334	1.0843	2	22	
12	0.0729	1.0343	0.8841	1	21		62	0.0146	1.0029	1.0002	2	22	
13	-0.029	-0.917	-0.794	1	21		63	-0.037	-0.988	-1.046	2	22	
14	0.098	0.9784	1.0304	1	21		64	-0.04	-0.989	-0.912	2	22	
15	0.0106	0.9606	0.9021	1	21		65	0.0301	0.9636	1.2365	2	22	
16	0.0753	0.9986	1.1145	1	21		66	-0.006	-0.872	-1.086	2	22	
17	0.0597	0.9847	0.9888	1	21		67	0.0226	0.9792	1.1096	2	22	
18	-0.036	-0.942	-0.852	1	21		68	-0.066	-1.084	-1.074	2	22	
19	0.0057	0.9532	0.7081	1	21		69	-0.065	-1.015	-1.113	2	22	
20	-0.034	-0.986	-0.732	1	21		70	0.1023	0.9964	0.8631	2	22	
21	0.0592	1.0242	1.3306	1	21		71	0.0523	1.0609	1.2732	2	22	
22	0.0512	0.9739	1.3113	1	21		72	-0.069	-1.075	-1.082	2	22	
23	-0.021	-0.975	-0.787	1	21		73	0.0763	1.0286	0.8513	2	22	
24	0.0028	0.9334	0.9011	1	21		74	0.0419	1.0308	0.9626	2	22	
25	0.0459	0.9743	1.0816	1	21		75	-0.018	-0.936	-1.011	2	23	
26	0.0056	0.9575	0.7381	1	21		76	-0.039	-1.05	-1.092	2	23	
27	0.0825	1.0512	1.1444	1	21		77	0.0075	1.0038	0.8411	2	23	
28	0.0589	0.9313	0.9652	1	21		78	-0.003	-0.943	-1.185	2	23	
29	0.0043	0.9632	0.7967	1	22		79	-0.008	-0.902	-1.509	2	23	
30	-0.052	-1.016	-0.703	1	22		80	0.0367	0.9564	0.816	2	23	
31	-0.015	-0.92	-0.896	1	22		81	-0.023	-0.98	-0.912	2	23	
32	-0.029	-0.966	-0.86	1	22		82	0.0013	0.9245	0.9091	2	23	
33	-0.061	-0.921	-0.847	1	22		83	-0.097	-1.032	-0.809	2	24	
34	0.0737	1.0416	1.0758	1	22		84	0.0493	1.0286	1.148	2	24	
35	-0.056	-1.014	-1.241	1	22		85	-0.003	-0.994	-0.834	2	24	
36	0.0087	0.9655	0.8832	1	22		86	-0.015	-0.936	-1.027	2	24	
37	-0.006	-0.959	-0.904	1	22		87	0.0152	0.9599	0.9612	2	25	
38	-0.034	-1.029	-0.805	1	22		88	-0.03	-1.002	-0.959	2	26	0.29
39	0.0494	1.0364	1.1954	1	22		89	-0.022	-0.987	-0.823	3	22	
40	-0.019	-0.877	-0.834	1	22		90	-0.033	-0.929	-0.777	3	22	
41	-0.01	-1.042	-1.085	1	22		91	0.0379	1.0418	0.9479	3	22	
42	-0.013	-0.99	-0.837	1	22		92	-0.028	-1.003	-0.914	3	23	
43	-0.102	-1.073	-0.73	1	22		93	0.0716	1.0148	1.0861	3	23	
44	-0.037	-0.967	-0.875	1	22		94	0.041	0.9324	0.6779	3	24	
45	-0.01	-0.936	-1.055	1	22		95	-0.032	-1.002	-1.218	3	24	
46	0.0407	0.9611	0.9689	1	23		96	0.0395	0.9995	0.8669	3	24	
47	0.026	0.9856	0.7862	1	23		97	0.0383	0.9912	1.013	3	24	
48	-0.039	-0.95	-0.951	1	23		98	-0.05	-1.068	-0.947	3	25	
49	-0.004	-0.934	-1.153	1	23		99	-0.084	-1.019	-1.297	3	25	0.11
50	-0.083	-1.043	-1.27	1	23		100	-0.059	-1.013	-0.969	5	22	0.01

A.2 Modified FDPASL data

No.1 group, shift=0.1SD													
No.	Average	AbA	AA5	Zone	FAP	Fq	No.	Average	AbA	AA5	Zone	FAP	Fq
1	0.0328	0.1655	0.4018	3	24		51	0.0149	0.0618	-0.063	5	40	
2	0.0614	0.1853	0.5773	3	30		52	-0.015	-0.127	-0.323	5	40	
3	-0.071	-0.195	-0.316	3	34		53	0.0044	0.047	-0.011	5	41	
4	0.0046	0.1132	-0.029	3	35		54	0.0185	0.1271	0.0444	5	42	
5	0.0339	0.116	0.687	3	35		55	-0.003	-0.126	-0.07	5	42	
6	0.012	0.0747	0.51	3	36		56	0.0164	0.1119	0.0891	5	42	
7	0.0173	0.1066	0.4209	3	37		57	-0.04	-0.117	-0.071	5	44	
8	0.0106	0.1382	0.2703	3	42		58	-0.05	-0.133	0.1072	5	44	
9	-0.034	-0.222	-0.399	3	47		59	-6E-04	-0.115	-0.028	5	44	
10	0.0736	0.1314	0.374	3	50		60	0.0016	0.1126	-0.106	5	44	
11	-0.089	-0.114	-0.288	3	67		61	0.0151	0.1129	0.0176	5	45	
12	-0.012	-0.148	-0.43	3	68		62	-0.013	-0.095	-0.143	5	45	
13	-0.098	-0.201	-0.431	3	69		63	-0.044	-0.173	-0.359	5	46	
14	0.0229	0.2765	0.3612	3	88		64	-0.018	-0.15	-0.17	5	47	
15	0.082	0.4191	0.2902	3	90	0.15	65	0.038	0.132	0.1735	5	49	
16	0.053	0.1211	0.1865	4	26		66	-0.038	-0.142	-0.136	5	49	
17	-0.08	-0.184	-0.172	4	31		67	0.0309	0.1092	0.406	5	50	
18	-0.061	-0.208	-0.26	4	38		68	-0.037	-0.06	0.0137	5	51	
19	0.0529	0.1284	0.016	4	48		69	0.0298	0.1562	0.309	5	52	
20	-0.004	-0.053	-0.32	4	54		70	-0.021	-0.103	-0.092	5	52	
21	0.0436	0.241	-0.332	4	64		71	0.0134	0.1244	-0.003	5	52	
22	0.0772	0.2395	-0.042	4	70		72	-0.032	-0.173	-0.331	5	56	
23	0.0198	0.1134	0.106	4	78		73	0.0327	0.0237	0.296	5	56	
24	-0.057	-0.121	-0.341	4	79		74	0.0176	0.1887	0.3013	5	57	
25	-0.014	-0.148	0.1524	4	87		75	0.0608	0.1121	0.1581	5	57	
26	-0.012	-0.058	-0.109	4	89	0.11	76	0.0288	0.1174	0.3086	5	59	
27	0.03	0.0882	0.0327	5	21		77	0.0135	0.0383	0.1091	5	61	
28	-0.083	-0.184	0.2037	5	22		78	0.0832	0.1657	0.2313	5	63	
29	-0.035	-0.092	-0.08	5	23		79	0.0067	0.1467	0.2275	5	64	
30	-0.047	-0.138	0.0012	5	24		80	-0.009	-0.152	-0.227	5	64	
31	-0.002	-0.101	0.0954	5	25		81	0.0102	0.231	-0.078	5	66	
32	-0.055	-0.092	-0.012	5	25		82	0.0107	0.1484	-0.062	5	69	
33	0.0398	0.1034	0.1483	5	25		83	0.0282	0.1058	-0.22	5	74	
34	-0.002	-0.075	-0.08	5	26		84	0.002	0.1801	0.4548	5	74	
35	-0.046	-0.11	-0.204	5	26		85	0.0281	0.113	0.3782	5	74	
36	-0.06	-0.134	0.1018	5	26		86	0.0079	0.1509	0.1638	5	74	
37	-0.049	-0.109	0.1189	5	27		87	0.0068	0.227	0.088	5	78	
38	0.0157	0.1282	0.0027	5	28		88	-0.007	-0.095	0.0436	5	78	
39	0.0235	0.1047	0.4187	5	29		89	0.0412	0.1225	0.4059	5	79	
40	-0.004	-0.102	-0.432	5	31		90	-0.106	-0.247	-0.198	5	79	
41	-0.021	-0.129	0.1271	5	31		91	0.0639	0.2079	0.4886	5	79	
42	0.0499	0.1428	-0.09	5	32		92	0.1008	0.303	0.1678	5	81	
43	0.0246	0.1011	-0.109	5	32		93	0.0226	0.0451	-0.038	5	83	
44	0.0468	0.155	-0.032	5	32		94	0.0011	0.1035	0.1137	5	85	
45	0.041	0.1273	0.457	5	34		95	-0.077	-0.132	0.0247	5	87	
46	0.0122	0.1343	-0.168	5	35		96	0.0125	0.2362	-0.253	5	87	
47	-0.037	-0.11	-0.083	5	35		97	-0.005	-0.093	-0.229	5	88	
48	0.1238	0.2315	0.3497	5	35		98	-0.007	-0.208	-0.113	5	88	
49	-0.065	-0.156	-0.129	5	37		99	0.0246	0.1414	-0.075	5	89	
50	-0.054	-0.141	0.212	5	40		100	0.0817	0.3341	0.2721	5	89	0.74

No.1 group, shift=0.5SD

No.	Average	AbA	AA5	Zone	FAP	Fq	No.	Average	AbA	AA5	Zone	FAP	Fq
1	0.0301	0.4935	0.9604	1	24		51	0.0152	0.486	0.3704	3	28	
2	-0.084	-0.562	-1.107	1	29		52	0.0146	0.5149	0.485	3	28	
3	-0.013	-0.5	-0.925	1	36	0.03	53	0.0373	0.5085	0.5886	3	28	
4	-0.056	-0.542	-0.769	2	22		54	0.0449	0.4991	0.473	3	28	
5	-0.008	-0.446	-0.82	2	24		55	0.0226	0.5008	0.5903	3	28	
6	0.0494	0.5514	0.6564	2	24		56	0.0737	0.5428	0.5021	3	28	
7	0.0409	0.5402	0.7202	2	28		57	-0.032	-0.505	-0.478	3	28	
8	-0.069	-0.572	-0.686	2	28		58	0.0395	0.5419	0.4603	3	28	
9	-0.066	-0.604	-0.638	2	30		59	0.0383	0.5125	0.4329	3	28	
10	0.1153	0.6217	0.6582	2	31		60	-0.106	-0.544	-0.587	3	29	
11	-0.05	-0.624	-0.693	2	35		61	-0.021	-0.514	-0.578	3	29	
12	0.0087	0.5358	0.5023	2	36	0.09	62	0.0056	0.4797	0.5336	3	29	
13	0.0862	0.5836	0.5075	3	21		63	-0.061	-0.491	-0.538	3	29	
14	0.0589	0.4821	0.516	3	21		64	0.0075	0.5239	0.4277	3	30	
15	0.0065	0.47	0.338	3	21		65	-0.003	-0.533	-0.493	3	30	
16	-0.029	-0.482	-0.376	3	22		66	-0.011	-0.492	0.5397	3	30	
17	0.0028	0.455	0.4514	3	22		67	0.0057	0.4845	0.5654	3	31	
18	-0.034	-0.553	-0.329	3	22		68	-0.034	-0.515	-0.416	3	31	
19	-0.037	-0.51	-0.418	3	22		69	0.0493	0.5259	0.3082	3	31	
20	0.0069	0.5032	0.422	3	22		70	-0.021	-0.501	-0.565	3	32	
21	0.098	0.5329	0.4958	3	22		71	0.041	0.5116	0.4004	3	32	
22	-0.059	-0.528	-0.484	3	22		72	-0.052	-0.549	-0.476	3	32	
23	0.0647	0.5719	0.5748	3	23		73	-0.04	-0.508	-0.41	3	33	
24	0.0512	0.5035	0.595	3	23		74	0.0379	0.5256	0.5431	3	34	
25	-0.055	-0.509	-0.41	3	23		75	0.0043	0.5193	0.4864	3	35	
26	-0.018	-0.487	-0.562	3	23		76	0.0367	0.5006	0.469	3	35	
27	-0.006	-0.428	0.5215	3	23		77	0.0473	0.5839	0.5232	3	37	
28	0.0419	0.5305	0.5621	3	23		78	-0.033	-0.476	-0.418	3	39	
29	0.0592	0.5461	0.575	3	24		79	-0.029	-0.454	-0.464	3	39	
30	-0.01	-0.483	-0.394	3	24		80	-0.097	-0.585	-0.411	3	40	
31	-0.003	-0.485	0.5921	3	24		81	0.0729	0.5826	0.5112	3	41	
32	-0.012	-0.494	-0.273	3	24		82	-0.102	-0.635	-0.422	3	42	
33	0.0523	0.569	0.3441	3	25		83	0.0292	0.5854	0.557	3	45	0.71
34	0.0459	0.5221	0.5759	3	25		84	-0.019	-0.42	-0.131	5	23	
35	0.0716	0.569	0.596	3	25		85	0.0753	0.538	0.2772	5	24	
36	0.0715	0.5419	0.573	3	25		86	-0.006	-0.488	-0.269	5	24	
37	-0.037	-0.516	0.5913	3	25		87	-0.022	-0.501	-0.099	5	25	
38	0.0013	0.4501	0.3268	3	25		88	0.1023	0.5487	0.2075	5	25	
39	-0.033	-0.521	-0.393	3	25		89	0.0597	0.5097	0.2695	5	25	
40	-0.021	-0.497	-0.474	3	26		90	0.0825	0.5649	0.2541	5	25	
41	-0.065	-0.538	-0.356	3	26		91	-0.105	-0.574	-0.271	5	25	
42	-0.039	-0.485	-0.366	3	26		92	0.0763	0.5357	0.2736	5	25	
43	-0.01	-0.523	-0.569	3	26		93	-0.015	-0.449	-0.245	5	26	
44	-0.023	-0.514	-0.34	3	26		94	0.0237	0.5399	0.2883	5	27	
45	-0.083	-0.543	0.5992	3	26		95	0.0407	0.4913	0.269	5	27	
46	0.026	0.509	0.5732	3	26		96	-0.036	-0.502	-0.216	5	28	
47	-0.039	-0.569	-0.583	3	27		97	-0.015	-0.479	-0.286	5	28	
48	0.0106	0.4781	0.4577	3	27		98	-0.03	-0.533	-0.259	5	29	
49	-0.004	-0.451	-0.534	3	27		99	-0.028	-0.526	-0.22	5	34	
50	0.0547	0.5102	0.4197	3	27		100	-0.018	-0.496	-0.279	5	39	0.17

No.1 group, shift=1.0SD

No.	Average	AbA	AA5	Zone	FAP	Fq	No.	Average	AbA	AA5	Zone	FAP	Fq
1	0.0715	1.0401	1.1843	1	21		51	-0.033	-0.996	-0.987	1	24	
2	0.0069	1.0077	1.1246	1	21		52	0.0373	0.9856	0.9127	1	24	
3	0.0862	1.0501	0.9741	1	21		53	0.1153	1.116	0.9823	1	24	
4	-0.018	-0.969	-0.97	1	21		54	-0.021	-0.961	-1.092	1	24	
5	-0.011	-0.966	-0.907	1	21		55	0.0493	1.0286	1.148	1	24	
6	0.0647	1.056	1.0489	1	21		56	-0.015	-0.936	-1.027	1	24	
7	0.098	0.9784	1.0304	1	21		57	-0.032	-1.002	-1.218	1	24	
8	0.0106	0.9606	0.9021	1	21		58	0.0383	0.9912	1.013	1	24	
9	0.0753	0.9986	1.1145	1	21		59	-0.106	-1.007	-0.962	1	25	
10	0.0597	0.9847	0.9888	1	21		60	0.0152	0.9599	0.9612	1	25	
11	0.0592	1.0242	1.3306	1	21		61	-0.05	-1.068	-0.947	1	25	
12	0.0512	0.9739	1.3113	1	21		62	-0.084	-1.019	-1.297	1	25	
13	0.0028	0.9334	0.9011	1	21		63	0.0237	1.0382	0.9589	1	26	
14	0.0459	0.9743	1.0816	1	21		64	-0.03	-1.002	-0.959	1	26	0.64
15	0.0825	1.0512	1.1444	1	21		65	0.0065	0.9428	0.8108	2	21	
16	0.0589	0.9313	0.9652	1	21		66	0.0729	1.0343	0.8841	2	21	
17	0.0449	0.9506	1.0016	1	21		67	-0.036	-0.942	-0.852	2	21	
18	0.0737	1.0416	1.0758	1	22		68	-0.055	-0.959	-0.784	2	21	
19	-0.056	-1.014	-1.241	1	22		69	0.0473	1.0246	0.7234	2	21	
20	-0.006	-0.959	-0.904	1	22		70	-0.021	-0.968	-0.736	2	21	
21	0.0494	1.0364	1.1954	1	22		71	-0.029	-0.917	-0.794	2	21	
22	-0.01	-1.042	-1.085	1	22		72	0.0057	0.9532	0.7081	2	21	
23	-0.01	-0.936	-1.055	1	22		73	-0.034	-0.986	-0.732	2	21	
24	0.0409	1.0334	1.0843	1	22		74	-0.021	-0.975	-0.787	2	21	
25	0.0146	1.0029	1.0002	1	22		75	0.0056	0.9575	0.7381	2	21	
26	-0.037	-0.988	-1.046	1	22		76	-0.015	-0.92	-0.896	2	22	
27	0.0301	0.9636	1.2365	1	22		77	-0.029	-0.966	-0.86	2	22	
28	-0.006	-0.872	-1.086	1	22		78	-0.061	-0.921	-0.847	2	22	
29	0.0226	0.9792	1.1096	1	22		79	0.0087	0.9655	0.8832	2	22	
30	-0.066	-1.084	-1.074	1	22		80	-0.034	-1.029	-0.805	2	22	
31	-0.065	-1.015	-1.113	1	22		81	-0.019	-0.877	-0.834	2	22	
32	0.0523	1.0609	1.2732	1	22		82	-0.013	-0.99	-0.837	2	22	
33	-0.069	-1.075	-1.082	1	22		83	-0.037	-0.967	-0.875	2	22	
34	-0.04	-0.989	-0.912	1	22		84	0.0043	0.9632	0.7967	2	22	
35	0.0419	1.0308	0.9626	1	22		85	-0.052	-1.016	-0.703	2	22	
36	0.0379	1.0418	0.9479	1	22		86	-0.102	-1.073	-0.73	2	22	
37	-0.059	-1.013	-0.969	1	22		87	0.1023	0.9964	0.8631	2	22	
38	0.0407	0.9611	0.9689	1	23		88	0.0763	1.0286	0.8513	2	22	
39	-0.039	-0.95	-0.951	1	23		89	-0.022	-0.987	-0.823	2	22	
40	-0.004	-0.934	-1.153	1	23		90	-0.033	-0.929	-0.777	2	22	
41	-0.083	-1.043	-1.27	1	23		91	-0.105	-1.051	-0.877	2	23	
42	0.0547	0.9861	1.2897	1	23		92	0.026	0.9856	0.7862	2	23	
43	-0.018	-0.936	-1.011	1	23		93	0.0075	1.0038	0.8411	2	23	
44	-0.039	-1.05	-1.092	1	23		94	0.0367	0.9564	0.816	2	23	
45	-0.003	-0.943	-1.185	1	23		95	-0.012	-0.941	-0.72	2	24	
46	-0.008	-0.902	-1.509	1	23		96	-0.097	-1.032	-0.809	2	24	
47	-0.023	-0.98	-0.912	1	23		97	-0.003	-0.994	-0.834	2	24	
48	0.0013	0.9245	0.9091	1	23		98	0.0395	0.9995	0.8669	2	24	
49	-0.028	-1.003	-0.914	1	23		99	0.041	0.9324	0.6779	2	24	0.35
50	0.0716	1.0148	1.0861	1	23		100	0.0292	1.01	0.414	3	21	0.01

Appendix B: Fuzzy system notions and design

B.1 The permutation of order of operation

If the *max-min* method is used for composition, the order of operation for composition and union can be permuted.

$$C' = (A' \, and \, B') \circ \bigcup_{j=1}^{m} R_j = (A' \, and \, B') \circ \bigcup_{j=1}^{m} [(A_j \, and \, B_j) \rightarrow C_j)]$$

$$\begin{aligned}
\mu_{C'}(z) &= [\mu_{A'}(x) \, and \, \mu_{B'}(y)] \circ \max[\mu_{R_1}(x,y,z),...,\mu_{R_m}(x,y,z)] \\
&= \max_{x,y} \min\{[\mu_{A'}(x) \, and \, \mu_{B'}(y)], \max[\mu_{R_1}(x,y,z),...,\mu_{R_m}(x,y,z)]\} \\
&= \max_{x,y} \max\{\min[(\mu_{A'}(x) \, and \, \mu_{B'}(y)), \mu_{R_1}(x,y,z)],...,\min[(\mu_{A'}(x) \, and \, \mu_{B'}(y)), \mu_{R_m}(x,y \\
&= \max\{[(\mu_{A'}(x) \, and \, \mu_{B'}(y)) \circ \mu_{R_1}(x,y,z)],...,[(\mu_{A'}(x) \, and \, \mu_{B'}(y)) \circ \mu_{R_m}(x,y,z)]\}
\end{aligned}$$

That is,

$$\begin{aligned}
C' &= [(A' \times B') \circ R_1] \cup \cdots \cup [(A' \times B') \circ R_m] \\
&= \bigcup_{j=1}^{m} [(A' \times B') \circ R_j] \\
&= \bigcup_{j=1}^{m} C'_j
\end{aligned}$$

B.2 The decompositions of composition and implication

Suppose the *max-min* is used for composition, *min* is used for intersection, R_C or R_P is used for implication.

$$C'_j = (A' \, and \, B') \circ [(A_j \, and \, B_j) \rightarrow C_j]$$

$$\begin{aligned}
\mu_{C'_j} &= [\mu_{A'}(x) and \, \mu_{B'}(y)] \circ [\mu_{A_j \times B_j}(x,y) \rightarrow \mu_{C_j}(z)] \\
&= [\mu_{A'}(x) and \, \mu_{B'}(y)] \circ \{\min[\mu_{A_j}(x), \mu_{B_j}(y)] \rightarrow \mu_{C_j}(z)\} \\
&= [\mu_{A'}(x) and \, \mu_{B'}(y)] \circ \min\{[\mu_{A_j}(x) \rightarrow \mu_{C_j}(z)], [\mu_{B_j}(y) \rightarrow \mu_{C_j}(z)]\} \\
&= \max_{x,y} \min\{\min[\mu_{A'}(x), \mu_{B'}(y)], \min[(\mu_{A_j}(x) \rightarrow \mu_{C_j}(z)), (\mu_{B_j}(y) \rightarrow \mu_{C_j}(z))]\} \\
&= \max_{x,y} \min\{\min[\mu_{A'}(x), (\mu_{A_j}(x) \rightarrow \mu_{C_j}(z))], \min[\mu_{B'}(y), (\mu_{B_j}(y) \rightarrow \mu_{C_j}(z))]\} \\
&= \min\{[\mu_{A'}(x) \circ (\mu_{A_j}(x) \rightarrow \mu_{C_j}(z))], [\mu_{B'}(y) \circ (\mu_{B_j}(y) \rightarrow \mu_{C_j}(z))]\}
\end{aligned}$$

That is

$$C'_j = [A' \circ (A_j \rightarrow C_j)] \cap [B' \circ (B_j \rightarrow C_j)]$$

B.3 The proof for matching degree (1)

Suppose R_C is used for implication, *max-min* is used for composition and it is quoting from appendix 3.2:

$$C'_j = (A' \times B') \circ [(A_j \times B_j) \rightarrow C_j) = [A' \circ (A_j \rightarrow C_j)] \cap [B' \circ (B_j \rightarrow C_j)]$$

$$\begin{aligned}
\mu_{C_j}(z) &= \min\{\max_x \min[\mu_{A'}(x), (\mu_{A_j}(x) \rightarrow \mu_{C_j}(z))], \max_y \min[\mu_{B'}(y), (\mu_{B_j}(y) \rightarrow \mu_{C_j}(z))]\} \\
&= \min\{\max_x \min[\mu_{A'}(x), \mu_{A_j}(x) \wedge \mu_{C_j}(z)], \max_y \min[\mu_{B'}(y), \mu_{B_j}(y) \wedge \mu_{C_j}(z)]\} \\
&= \min\{\max_x [\mu_{A'}(x) \wedge \mu_{A_j}(x) \wedge \mu_{C_j}(z)], \max_y [\mu_{B'}(y) \wedge \mu_{B_j}(y) \wedge \mu_{C_j}(z)]\} \\
&= \{\max_x [\mu_{A'}(x) \wedge \mu_{A_j}(x) \wedge \mu_{C_j}(z)]\} \wedge \{\max_y [\mu_{B'}(y) \wedge \mu_{B_j}(y) \wedge \mu_{C_j}(z)]\} \\
&= \{\max_x [\mu_{A'}(x) \wedge \mu_{A_j}(x)]\} \wedge \{\max_y [\mu_{B'}(y) \wedge \mu_{B_j}(y)]\} \wedge \mu_{C_j}(z) \\
&= \alpha_j \wedge \mu_{C_j}(z)
\end{aligned}$$

B.4 The proof for matching degree (2)

Suppose R_P is used for implication, *max-min* is used for composition and it is quoting from appendix 3.2:

$$C'_j = (A' \times B') \circ [(A_j \times B_j) \rightarrow C_j) = [A' \circ (A_j \rightarrow C_j)] \bigcap [B' \circ (B_j \rightarrow C_j)]$$

$$\begin{aligned}
\mu_{C_j}(z) &= \min\{\max_x \min[\mu_{A'}(x), (\mu_{A_j}(x) \rightarrow \mu_{C_j}(z))], \max_y \min[\mu_{B'}(y), (\mu_{B_j}(y) \rightarrow \mu_{C_j}(z))]\} \\
&= \min\{\max_x \min[\mu_{A'}(x), \mu_{A_j}(x)\mu_{C_j}(z)], \max_y \min[\mu_{B'}(y), \mu_{B_j}(y)\mu_{C_j}(z)]\} \\
&= \min\{\max_x [\mu_{A'}(x) \wedge \mu_{A_j}(x)\mu_{C_j}(z)], \max_y [\mu_{B'}(y) \wedge \mu_{B_j}(y)\mu_{C_j}(z)]\} \\
&= \{\max_x [\mu_{A'}(x) \wedge \mu_{A_j}(x)\mu_{C_j}(z)]\} \wedge \{\max_y [\mu_{B'}(y) \wedge \mu_{B_j}(y)\mu_{C_j}(z)]\} \\
&\approx \{[\max_x (\mu_{A'}(x) \wedge \mu_{A_j}(x))] \wedge [\max_y (\mu_{B'}(y) \wedge \mu_{B_j}(y))]\}\mu_{C_j}(z) \\
&= \alpha_j \mu_{C_j}(z)
\end{aligned}$$

B.5 If-then rules

For zone rule 4:

ITR_9 : If x_1 is –ZA and x_2 is ZC and x_3 is ZC and x_4 is ZC and x_5 is ZC and x_6 is ZC then z is NM;

ITR_{10} : If x_1 is -ZB and x_2 is –ZB and x_3 is –ZB and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC then z is NS;

ITR_{11} : If x_1 is –ZB and x_2 is ZC and x_3 is ZC and x_4 is ZC and x_5 is ZC and x_6 is ZC then z is NS;

ITR_{12} : If x_1 is –ZB and x_2 is –ZB and x_3 is –ZB and x_4 is ZB and x_5 is ZB and x_6 is ZB then z is NS;

ITR_{13} : If x_1 is –ZC and x_2 is –ZC and x_3 is –ZC and x_4 is ZB and x_5 is ZB and x_6 is ZB then z is NZ;

ITR_{14} : If x_1 is –ZC and x_2 is ZC and x_3 is ZC and x_4 is ZC and x_5 is ZC and x_6 is ZC then z is NZ;

ITR_{15} : If x_1 is C and x_2 is C and x_3 is C and x_4 is B and x_5 is B and x_6 is B then z is PZ;

ITR_{16} : If x_1 is –ZA and x_2 is –ZB and x_3 is -ZB and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC then z is NM;

ITR_{17} : If x_1 is –ZA and x_2 is –ZB and x_3 is –ZB and x_4 is ZC and x_5 is ZC and x_6 is ZC then z is NM;

ITR_{18} : If x_1 is –ZA and x_2 is -ZB and x_3 is -ZB and x_4 is ZB and x_5 is ZB and x_6 is ZB then z is NM;

ITR_{19} : If x_1 is –ZA and x_2 is -ZC and x_3 is -ZC and x_4 is ZC and x_5 is ZC and x_6 is ZC then z is NM;

ITR_{20} : If x_1 is –ZA and x_2 is -ZC and x_3 is -ZC and x_4 is ZB and x_5 is ZB and x_6 is ZB then z is NM;

ITR_{21} : If x_1 is –ZA and x_2 is -ZC and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is

ZA then z is NM;

ITR_{22} : If x_1 is -ZA and x_2 is ZC and x_3 is ZC and x_4 is ZB and x_5 is ZB and x_6 is ZB then z is NM;

ITR_{23} : If x_1 is –ZA and x_2 is ZC and x_3 is ZC and x_4 is ZC and x_5 is ZC and x_6 is ZA then z is NM;

ITR_{24} : If x_1 is –ZB and x_2 is –ZC and x_3 is –ZC and x_4 is ZC and x_5 is ZC and x_6 is ZC then z is NS;

ITR_{25} : If x_1 is –ZB and x_2 is –ZC and x_3 is –ZC and x_4 is ZB and x_5 is ZB and x_6 is ZB then z is NS;

ITR_{26} : If x_1 is –ZB and x_2 is -ZC and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is ZA then z is NS;

ITR_{27} : If x_1 is -ZB and x_2 is ZC and x_3 is ZC and x_4 is ZC and x_5 is ZC and x_6 is ZB then z is NS;

ITR_{28} : If x_1 is –ZB and x_2 is ZC and x_3 is ZC and x_4 is ZC and x_5 is ZC and x_6 is ZA then z is NS;

ITR_{29} : If x_1 is –ZB and x_2 is –ZB and x_3 is –ZB and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NS;

ITR_{30} : If x_1 is –ZC and x_2 is -ZC and x_3 is -ZC and x_4 is ZC and x_5 is ZC and x_6 is ZA then z is NZ;

ITR_{31} : If x_1 is –ZC and x_2 is -ZC and x_3 is -ZC and x_4 is ZC and x_5 is ZC and x_6 is ZB then z is NZ;

ITR_{32} :If x_1 is –ZC and x_2 is -ZC and x_3 is ZB and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NZ;

ITR_{33} :If x_1 is C and x_2 is C and x_3 is C and x_4 is B and x_5 is B and x_6 is A then z is PZ;

ITR_{34} :If x_1 is -ZA and x_2 is –ZB and x_3 is –ZC and x_4 is -ZC and x_5 is ZC and x_6 is ZC then z is NM;

ITR_{35} :If x_1 is –ZA and x_2 is -ZB and x_3 is -ZC and x_4 is -ZC and x_5 is ZB and x_6 is ZB then z is NM;

ITR_{36} : If x_1 is –ZA and x_2 is –ZB and x_3 is –ZC and x_4 is -ZC and x_5 is -ZC and x_6 is ZA then z is NM;

ITR_{37} :If x_1 is –ZA and x_2 is –ZB and x_3 is –ZB and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NM;

ITR_{38} :If x_1 is –ZA and x_2 is -ZB and x_3 is -ZB and x_4 is ZC and x_5 is ZC and x_6 is ZB then z is NM;

ITR_{39} : If x_1 is -ZA and x_2 is -ZB and x_3 is -ZB and x_4 is ZC and x_5 is ZC and x_6 is ZA then z is NM;

ITR_{40} : If x_1 is –ZA and x_2 is –ZC and x_3 is -ZC and x_4 is ZC and x_5 is ZC and x_6 is

ZB then z is NM;

ITR_{41} : If x_1 is –ZA and x_2 is –ZC and x_3 is –ZC and x_4 is ZC and x_5 is ZC and x_6 is ZA then z is NM;

ITR_{42} : If x_1 is –ZA and x_2 is -ZC and x_3 is -ZC and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NM;

ITR_{43} : If x_1 is –ZA and x_2 is ZC and x_3 is ZC and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NM;

ITR_{44} : If x_1 is –ZB and x_2 is -ZC and x_3 is -ZC and x_4 is ZC and x_5 is ZC and x_6 is ZB then z is NS;

ITR_{45} : If x_1 is –ZB and x_2 is -ZC and x_3 is -ZC and x_4 is ZC and x_5 is ZC and x_6 is ZA then z is NS;

ITR_{46} : If x_1 is -ZB and x_2 is –ZC and x_3 is –ZC and x_4 is ZB and x_5 is ZB and x_6 is -ZA then z is NS;

ITR_{47} : If x_1 is –ZB and x_2 is ZC and x_3 is ZC and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NS;

ITR_{48} : If x_1 is –ZC and x_2 is ZC and x_3 is ZC and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NZ;

ITR_{49} : If x_1 is –ZA and x_2 is –ZB and x_3 is –ZC and x_4 is ZC and x_5 is ZC and x_6 is ZB then z is NM;

ITR_{50} : If x_1 is –ZA and x_2 is -ZB and x_3 is -ZC and x_4 is ZC and x_5 is ZC and x_6 is ZA then z is NM;

ITR_{51} : If x_1 is -ZA and x_2 is -ZB and x_3 is -ZC and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NM;

ITR_{52} :If x_1 is –ZA and x_2 is –ZB and x_3 is ZC and x_4 is ZB and x_5 is ZB and x_6 is ZA then z is NM;

ITR_{53} :If x_1 is –ZA and x_2 is –ZC and x_3 is ZC and x_4 is ZC and x_5 is ZB and x_6 is ZA then z is NM;

ITR_{54} :If x_1 is –ZB and x_2 is -ZC and x_3 is ZC and x_4 is ZC and x_5 is ZB and x_6 is ZA then z is NS;

ITR_{55} :If x_1 is –ZA and x_2 is -ZB and x_3 is -ZC and x_4 is ZC and x_5 is ZB and x_6 is ZA then z is NM;

$ITR_{56} \sim ITR_{102}$: Same with $ITR_9 \sim ITR_{55}$, if reverse the sign of linguistic term in antecedent and replace "N" with "P" in consequent.

For zone rule 5:

ITR_{103} : If x_1 is –ZA and x_2 is –ZB and x_3 is –ZB and x_4 is –ZB and x_5 is –ZC and x_6

is –ZC and x_7 is –ZC and x_8 is –ZC then z is NM;

ITR_{104} : If x_1 is –ZA and x_2 is –ZC and x_3 is –ZC and x_4 is –ZC and x_5 is –ZC and x_6 is –ZB and x_7 is –ZB and x_8 is –ZB then z is NM;

ITR_{105} : If x_1 is –ZB and x_2 is –ZC and x_3 is –ZC and x_4 is –ZC and x_5 is –ZC and x_6 is –ZC and x_7 is –ZC and x_8 is –ZA then z is NS;

ITR_{106} : If x_1 is -ZB and x_2 is -ZA and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC and x_7 is -ZC and x_8 is -ZC then z is NS;

ITR_{107} : If x_1 is -ZC and x_2 is -ZC and x_3 is -ZC and x_4 is -ZC and x_5 is -ZB and x_6 is -ZB and x_7 is -ZB and x_8 is ZA then z is NZ;

ITR_{108} : If x_1 is -ZC and x_2 is -ZC and x_3 is -ZC and x_4 is -ZC and x_5 is -ZA and x_6 is -ZB and x_7 is -ZB and x_8 is -ZB then z is NZ;

ITR_{109} : If x_1 is -ZA and x_2 is -ZC and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC and x_7 is -ZC and x_8 is -ZC then z is NM;

ITR_{110} : If x_1 is -ZB and x_2 is -ZC and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC and x_7 is -ZC and x_8 is -ZC then z is NS;

ITR_{111} : If x_1 is -ZC and x_2 is -ZA and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC and x_7 is -ZC and x_8 is -ZC then z is NZ;

ITR_{112} : If x_1 is -ZC and x_2 is -ZB and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC and x_7 is -ZC and x_8 is -ZC then z is NZ;

ITR_{113} : If x_1 is -ZC and x_2 is -ZC and x_3 is -ZC and x_4 is -ZC and x_5 is -ZC and x_6 is -ZC and x_7 is -ZC and x_8 is -ZC then z is NZ;

$ITR_{114} \sim ITR_{124}$: Same with $ITR_{103} \sim ITR_{113}$, if reverse the sign of linguistic term in antecedent and replace "N" with "P" in consequent.

B.6 The *t*-test for effect of membership functions

No.	MF1	MF2	MF3	MF4	MF5	FM6	FM7
1	0.5105	0.5053	0.5220	0.5030	0.5239	0.5135	0.5112
2	0.5145	0.5098	0.5265	0.5042	0.5403	0.5145	0.5144
3	0.5143	0.5008	0.5245	0.5063	0.5270	0.5165	0.5145
4	0.4961	0.4910	0.5119	0.4853	0.5116	0.4989	0.4957
5	0.5102	0.5047	0.5214	0.5022	0.5257	0.5122	0.5104
6	0.5150	0.5072	0.5320	0.5110	0.5273	0.5156	0.5163
7	0.4926	0.4817	0.5065	0.4837	0.5090	0.4925	0.4923
8	0.5091	0.5002	0.5198	0.5000	0.5226	0.5093	0.5127
9	0.5125	0.5078	0.5145	0.5010	0.5233	0.5125	0.5123
10	0.5134	0.5039	0.5246	0.5036	0.5253	0.5161	0.5136
Average	0.5088	0.5012	0.5204	0.5001	0.5236	0.5102	0.50934
SD	0.0079	0.0087	0.0075	0.0087	0.0086	0.0081	0.00830
t0		2.0475	-3.3686	2.3708	-3.9872	-0.3634	-0.1380

Appendix C: Forecast function

C.1 The comparison of abnormal process average (ABA) and EWMA

C.1.1 Shift=SD/2

	θ=0.9			θ=0.85			θ=0.8		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	-0.4629	-0.3276	0.1353	0.5324	0.5406	0.0082	0.5066	0.6533	0.1467
2	0.5066	0.4662	0.0404	-0.4613	-0.4176	0.0437	-0.56	-0.4604	0.0996
3	-0.5600	-0.2551	0.3049	0.5196	0.4240	0.0956	0.5122	0.5110	0.0012
4	0.5122	0.2534	0.2588	-0.4658	-0.4578	0.0080	0.4917	0.5938	0.1021
5	0.4917	0.4279	0.0638	-0.4926	-0.4082	0.0844	-0.4659	-0.4354	0.0305
6	-0.4659	-0.279	0.1869	0.5188	0.4193	0.0995	-0.5601	-0.4159	0.1442
7	-0.5601	-0.2488	0.3113	0.4805	0.3406	0.1399	0.5193	0.6133	0.0940
8	0.5193	0.4049	0.1144	-0.6083	-0.4855	0.1228	-0.5692	-0.5871	0.0179
9	-0.5692	-0.3834	0.1858	-0.5014	-0.3997	0.1017	-0.5148	-0.4592	0.0556
10	-0.5148	-0.2662	0.2486	0.4744	0.5158	0.0414	0.5324	0.6318	0.0994
Average	-0.1103	-0.0208	0.1850	-0.0004	0.0072	0.0745	-0.0108	0.0645	0.0791
SD	0.5330	0.3581	0.0960	0.5346	0.4685	0.0464	0.5524	0.5681	0.0507

	θ=0.75			θ=0.7			θ=0.6		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	-0.4629	-0.4882	0.0253	-0.4613	-0.6641	0.2028	-0.5166	-0.865	0.3484
2	0.5066	0.7084	0.2018	0.5196	0.5985	0.0789	-0.5001	-0.7259	0.2258
3	-0.56	-0.5398	0.0202	-0.4658	-0.6861	0.2203	0.5185	0.5244	0.0059
4	0.5122	0.604	0.0918	-0.4926	-0.6248	0.1322	-0.4985	-0.6846	0.1861
5	0.4917	0.65	0.1583	0.5188	0.6474	0.1286	0.5272	0.95	0.4228
6	-0.4659	-0.4923	0.0264	0.4805	0.4703	0.0102	-0.4997	-0.6694	0.1697
7	-0.5601	-0.474	0.0861	-0.6083	-0.5787	0.0296	-0.4773	-0.4957	0.0184
8	0.5193	0.6858	0.1665	-0.5014	-0.5678	0.0664	0.5085	0.6247	0.1162
9	-0.5692	-0.6685	0.0993	0.4744	0.7184	0.244	-0.5153	-0.1013	0.414
10	-0.5148	-0.5327	0.0179	0.5118	0.6634	0.1516	-0.4985	-0.728	0.2295
Average	-0.1103	-0.0547	0.0894	-0.0024	-0.0024	0.1265	-0.1952	-0.2171	0.2137
SD	0.5330	0.6198	0.0678	0.5324	0.6595	0.0802	0.4923	0.6728	0.1475

	θ=0.5			θ=0.3			θ=0.1		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	0.5031	0.9218	0.4187	-0.4739	-0.5681	0.0942	0.5542	0.7238	0.1696
2	0.5140	0.9888	0.4748	0.5543	0.9032	0.3489	-0.4704	-1.0455	0.5751
3	0.5385	0.4946	0.0439	-0.5105	-0.7847	0.2742	0.5240	0.7775	0.2535
4	0.5017	0.8567	0.3550	0.4880	0.9717	0.4837	-0.4850	-1.1194	0.6344
5	-0.4888	-0.9099	0.4211	0.5071	0.9114	0.4043	-0.5134	-0.6752	0.1618
6	0.5084	0.6337	0.1253	-0.4965	-0.5130	0.0165	0.4923	1.0729	0.5806
7	-0.4925	-0.3623	0.1302	-0.5453	-0.6179	0.0726	0.5140	0.4386	0.0754
8	0.5193	0.8134	0.2941	0.5428	0.9292	0.3864	-0.5509	-1.0126	0.4617
9	0.5520	0.7930	0.2410	-0.4878	-0.9353	0.4475	-0.5184	-0.5170	0.0014
10	-0.4776	-0.5311	0.0535	-0.4989	-0.8199	0.321	0.5301	1.1149	0.5848
Average	0.2178	0.3699	0.2558	-0.0921	-0.0523	0.2849	0.0077	-0.0242	0.3498
SD	0.4861	0.6969	0.1605	0.5300	0.8536	0.1665	0.5438	0.9312	0.2419

C.1.2 Shift=SD

	θ=0.9			θ=0.8			θ=0.75		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	-0.9772	-0.4341	0.5431	0.9940	0.7642	0.2298	1.0132	0.7201	0.2931
2	-0.9491	-0.4498	0.4993	0.9253	0.7156	0.2097	-0.9772	-0.9087	0.0685
3	0.9063	0.4604	0.4459	-1.0269	-0.8440	0.1829	-0.9491	-0.8828	0.0663
4	-0.9423	-0.1015	0.8408	0.9312	0.5786	0.3526	0.9063	0.8294	0.0769
5	-0.9590	-0.3876	0.5714	1.0135	0.4647	0.5488	-0.9423	-0.1533	0.7890
6	0.9134	0.3683	0.5451	0.8663	0.5375	0.3288	-0.9590	-0.6806	0.2784
7	0.9290	0.4010	0.5280	-1.0018	-0.5786	0.4232	0.9134	0.5972	0.3162
8	1.0351	0.4132	0.6219	-0.9775	-0.6072	0.3703	0.9290	0.6958	0.2332
9	-1.0009	-0.4871	0.5138	0.9670	0.7497	0.2173	1.0351	0.8061	0.2290
10	-0.9725	-0.4506	0.5219	-0.9671	-0.5221	0.4450	-1.0009	-0.9814	0.0195
Average	-0.2017	-0.0668	0.5631	0.1724	0.1258	0.3308	-0.0032	0.0042	0.2370
SD	0.9885	0.4251	0.1078	1.0042	0.6690	0.1205	1.0155	0.7994	0.2227

	θ=0.7			θ=0.6			θ=0.5		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	-0.9725	-0.9432	0.0293	-0.8926	-0.6976	0.1950	1.0308	1.1269	0.0961
2	1.0438	0.8030	0.2408	1.0043	1.1125	0.1082	-0.9119	-1.2731	0.3612
3	1.0125	0.6097	0.4028	-0.9935	-0.3612	0.6323	1.0034	1.0619	0.0585
4	-0.9132	-0.5432	0.3700	0.9787	0.9007	0.0780	-0.9434	-1.0235	0.0801
5	-1.0026	-0.6582	0.3444	0.9624	0.7742	0.1882	-0.9897	-1.2254	0.2357
6	-0.9536	-0.7689	0.1847	-0.9545	-1.1372	0.1827	0.9953	0.8298	0.1655
7	-1.0679	-0.9841	0.0838	-1.0573	-0.8652	0.1921	0.9530	1.0139	0.0609
8	-1.0032	-0.8367	0.1665	0.9784	1.1197	0.1413	-1.0364	-0.6721	0.3643
9	0.9105	0.6297	0.2808	-0.9787	-0.8972	0.0815	-0.9559	-1.1938	0.2379
10	0.9863	0.9106	0.0757	-0.9701	-0.8298	0.1403	0.9436	1.1427	0.1991
Average	-0.1960	-0.1781	0.2179	-0.1923	-0.0881	0.1940	0.0089	-0.0213	0.1859
SD	1.0205	0.8030	0.1314	1.0106	0.9414	0.1604	1.0299	1.1285	0.1152

	θ=0.3			θ=0.1		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	1.0132	1.1105	0.0973	-0.9119	-1.4896	0.5777
2	-0.9772	-1.4873	0.5101	1.0034	1.2762	0.2728
3	-0.9491	-1.3944	0.4453	-0.9434	-1.1210	0.1776
4	0.9063	0.9485	0.0422	-0.9897	-1.4904	0.5007
5	-0.9423	-0.1967	0.7456	0.9953	1.2257	0.2304
6	-0.9590	-0.7275	0.2315	0.9530	1.0957	0.1427
7	0.9134	0.8011	0.1123	-1.0364	-0.6337	0.4027
8	0.9290	1.0359	0.1069	-0.9559	-1.1194	0.1635
9	1.0351	1.0943	0.0592	0.9436	1.1129	0.1693
10	-1.0009	-1.5536	0.5527	0.9940	1.2025	0.2085
Average	-0.0032	-0.0369	0.2903	0.0052	0.0059	0.2846
SD	1.0155	1.1628	0.2516	1.0259	1.2635	0.1546

C.1.3 Shift=SD/3

	θ=0.9			θ=0.85			θ=0.8		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	-0.2978	-0.2563	0.0415	0.3720	0.4779	0.1059	-0.3249	-0.4204	0.0955
2	0.4401	0.4108	0.0293	-0.3293	-0.1601	0.1692	-0.3710	-0.4488	0.0778
3	-0.2871	-0.1941	0.0930	0.3278	0.2379	0.0899	0.3799	0.5109	0.1310
4	-0.3619	-0.2093	0.1526	0.4295	0.5631	0.1336	-0.3177	0.2465	0.5642
5	-0.2718	-0.2295	0.0423	0.3480	0.3268	0.0212	-0.3342	-0.5597	0.2255
6	-0.3277	-0.2234	0.1043	0.3123	0.3744	0.0621	0.3791	0.5086	0.1295
7	-0.2949	-0.3696	0.0747	-0.3447	-0.4044	0.0597	-0.3086	-0.4022	0.0936
8	-0.3549	-0.2469	0.1080	-0.3354	-0.2739	0.0615	0.3519	0.3026	0.0493
9	-0.3048	-0.4735	0.1687	-0.4294	-0.5383	0.1089	-0.3149	-0.4148	0.0999
10	-0.3232	-0.1919	0.1313	0.3689	0.4829	0.1140	-0.3397	-0.5259	0.1862
Average	-0.2384	-0.1984	0.0946	0.0720	0.1086	0.0926	-0.1200	-0.1203	0.1653
SD	0.2401	0.2318	0.0480	0.3738	0.4107	0.0429	0.3389	0.4508	0.1495

	θ=0.7			θ=0.6			θ=0.5		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	-0.3177	-0.5786	0.2609	-0.3150	-0.4696	0.1546	-0.3150	-0.4451	0.1301
2	-0.3142	-0.8615	0.5473	0.3824	0.6569	0.2745	0.3824	0.6902	0.3078
3	0.3791	0.1806	0.1985	-0.3439	-0.6682	0.3243	-0.3439	-0.7275	0.3836
4	-0.3086	-0.4622	0.1536	0.3224	0.3612	0.0388	0.3224	0.3117	0.0107
5	0.3519	0.4499	0.0980	0.3373	0.2504	0.0869	0.3373	0.2252	0.1121
6	-0.3149	-0.4835	0.1686	-0.3249	-0.4602	0.1353	-0.3249	-0.4550	0.1301
7	-0.3397	-0.6675	0.3278	-0.3710	-0.6172	0.2462	-0.3710	-0.6805	0.3095
8	0.3196	0.3943	0.0747	0.3799	0.6863	0.3064	0.3799	0.7310	0.3511
9	0.3458	0.4655	0.1197	-0.3177	-0.6415	0.3238	-0.3177	-0.6625	0.3448
10	-0.3755	-0.6863	0.3108	-0.3342	-0.9872	0.6530	-0.3342	-1.0694	0.7352
Average	-0.0574	-0.2249	0.2260	-0.0585	-0.1889	0.2544	-0.0585	-0.2082	0.2815
SD	0.3508	0.5315	0.1422	0.3571	0.6133	0.1733	0.3571	0.6416	0.2040

	θ=0.3			θ=0.1		
Items	ABA	EWMA	|Error|	ABA	EWMA	|Error|
1	0.3791	-0.3714	0.7505	-0.3716	-0.7776	0.4060
2	-0.3086	-0.7599	0.4513	0.3642	0.2057	0.1585
3	0.3519	0.5658	0.2139	0.3318	0.6182	0.2864
4	-0.3149	-0.5082	0.1933	0.3400	0.3709	0.0309
5	-0.3497	-1.2049	0.8552	0.3264	0.8795	0.5531
6	0.3196	0.3225	0.0029	-0.3404	-1.3164	0.9760
7	0.3458	0.6132	0.2674	0.3232	-0.5748	0.8980
8	-0.3754	-0.7691	0.3937	0.3375	1.2489	0.9114
9	-0.3539	-0.5524	0.1985	-0.3700	-0.6790	0.3090
10	0.3655	0.6883	0.3228	-0.4115	-0.9703	0.5588
Average	0.0059	-0.1976	0.3649	0.0530	-0.0995	0.5088
SD	0.3660	0.6836	0.2624	0.3675	0.8735	0.3312

C.2 Forecast results for abnormal processes $\overline{X}$ averages shifted

C.2.1 Shift=SD/3

Forecast results (shift=SD/3, θ=0.85)							
AbA	AbSD	EA	ESD	Fpo	FAP	ER	|ER|
0.3567	0.5637	0.4213	0.0683	27	26	-0.0310	0.0310
-0.4500	0.6454	-0.4790	0.0199	59	63	-0.0470	0.0470
0.3494	0.5690	0.3140	0.0578	64	31	-0.0740	0.0740
0.3244	0.4936	0.3365	0.0263	32	24	0.0470	0.0470
-0.4310	0.6351	-0.4680	0.0063	54	56	-0.0280	0.0280
-0.3490	0.5615	-0.3370	0.0484	20	24	-0.0490	0.0490
0.3083	0.4987	0.2900	0.0487	25	27	-0.0210	0.0210
-0.3750	0.5728	-0.3590	0.0718	19	22	0.0150	0.0150
0.3143	0.5918	0.2920	0.0903	23	26	-0.0130	0.0130
-0.3220	0.5386	-0.3150	0.0378	32	35	0.0168	0.0168
Average				35.5	33.4	-0.0184	0.0342
Standard Deviation				16.939	14.3465	0.0362	0.0196

C.2.2 Shift=SD/2

Forecast results (shift=SD/2, θ=0.85)							
AbA	AbSD	EA	ESD	Fpo	FAP	ER	|ER|
-0.4620	0.6279	-0.4400	0.0324	37	31	0.0331	0.0331
0.5065	0.6194	0.5204	0.0552	26	25	-0.0390	0.0390
-0.5590	0.7129	-0.5580	0.0283	32	31	0.0287	0.0287
0.5122	0.6752	0.4840	0.0427	30	23	-0.0800	0.0800
0.4917	0.6829	0.4891	0.0548	47	34	-0.0360	0.0360
-0.5600	0.6917	-0.5410	0.0532	28	24	0.0308	0.0308
0.5193	0.6254	0.5033	0.0477	21	25	0.0298	0.0298
-0.5690	0.7298	-0.5680	0.0500	49	42	0.0433	0.0433
-0.5140	0.6954	-0.5460	0.0841	33	26	0.0408	0.0408
-0.4610	0.6095	-0.4570	0.0464	30	24	0.0423	0.0423
Average				33.3	28.5	0.0094	0.0404
Standard deviation				8.8450	6.0231	0.0440	0.0149

C.2.3 Shift=SD

Forecast results (shift=SD, θ=0.73)							
AbA	AbSD	EA	ESD	Fpo	FAP	ER	|ER|
-0.9650	1.0409	-1.0340	0.0734	23	21	0.0497	0.0497
-0.9880	1.0654	-1.0290	0.0846	18	22	0.0079	0.0079
-0.9880	1.0439	-1.0060	0.0768	20	22	-0.0340	0.0340
1.0037	1.1031	1.0782	0.0914	20	23	-0.0410	0.0410
-0.8710	0.9738	-0.9020	0.0525	18	22	-0.0350	0.0350
-0.9190	1.0536	-0.9520	0.0940	21	22	0.0150	0.0150
-0.9650	1.0629	-1.0070	0.0906	20	22	0.0529	0.0529
-1.0010	1.0548	-1.0160	0.0998	36	24	-0.0660	0.0660
0.9964	1.0124	0.9902	0.1116	23	22	-0.0620	0.0620
1.0416	1.0565	1.0919	0.0753	33	22	0.0114	0.0114
Average				23.2	22.2	-0.0101	0.0375
Standard deviation				6.2325	0.7888	0.0434	0.0208

C.3 Forecast results for abnormal R average spread

C.3.1 *Spread=1.2σ*

Forecast results (spread=1.2σ, θ=0.85)							
AbA	AbSD	EA	ESD	Fpo	FAP	ER	|ER|
2.5225	1.0219	2.5748	0.1521	27	22	0.19	0.19
2.9759	1.3225	3.0643	0.0765	41	44	0.03	0.03
2.7165	1.1755	2.7321	0.0974	38	40	0.0422	0.0422
2.8139	1.0644	2.9587	0.0865	26	31	0.1782	0.1782
2.8798	1.1497	3.0237	0.0806	27	28	0.0228	0.0228
2.711	1.1452	2.8339	0.1002	44	36	0.1334	0.1334
2.9043	1.3645	2.933	0.0437	61	54	0.1161	0.1161
2.8647	1.2124	2.9569	0.0962	30	28	-0.0634	0.0634
2.8451	1.2416	2.9387	0.1859	41	30	-0.1726	0.1726
2.9206	1.2109	3.1836	0.1034	30	44	0.1646	0.1646
Average				36.5	35.7	0.06413	0.11133
Standard deviation				10.92652	9.730251	0.11648	0.066073

C.3.2 *Spread=1.3σ*

Forecast results (spread=1.3σ, θ=0.85)							
AbA	AbSD	EA	ESD	Fpo	FAP	ER	|ER|
3.1306	1.3401	3.1553	0.1008	45	49	-0.0015	0.0015
3.3013	1.2792	3.5723	0.0562	53	57	0.1856	0.1856
3.0786	1.3399	3.1825	0.0659	49	35	0.238	0.238
3.0713	1.2345	3.3053	0.064	22	45	0.1839	0.1839
2.9192	1.2124	3.0862	0.1593	53	37	0.0712	0.0712
2.8898	1.3301	2.9746	0.224	21	24	-0.0293	0.0293
2.9725	1.1411	3.1691	0.0926	24	28	0.1065	0.1065
2.9525	1.3858	2.9539	0.0625	42	39	0.1215	0.1215
3.1498	1.3062	3.3195	0.1266	26	30	0.077	0.077
3.1885	1.5469	3.2425	0.2276	34	30	0.2312	0.2312
Average				36.9	37.4	0.11841	0.12457
Standard deviation				13.01666	10.34086	0.091965	0.082429

C.3.3 Spread=1.4 σ

Forecast results (spread=1.4 σ , θ =0.85)							
AbA	AbSD	EA	ESD	Fpo	FAP	ER	|ER|
3.1986	1.5149	3.4186	0.3502	24	28	0.0808	0.0808
3.2216	1.6757	3.2866	0.1614	35	39	-0.0743	0.0743
3.1994	1.5052	3.2408	0.0459	26	29	0.0477	0.0477
3.3198	1.7046	3.5370	0.1634	41	41	-0.0192	0.0192
3.3860	1.6760	3.6530	0.2463	34	31	-0.1963	0.1963
3.0659	1.4588	3.2342	0.1972	89	32	0.1530	0.1530
3.1598	1.5664	3.1457	0.2411	39	28	-0.1309	0.1309
3.4439	1.6416	3.4003	0.0985	53	47	-0.2163	0.2163
3.2493	1.5452	3.2475	0.1541	26	30	0.0436	0.0436
3.4018	1.5850	3.5551	0.1599	46	43	-0.0720	0.0720
Average				41.3	34.8	-0.0384	0.1034
• Standard deviation				19.1488	7.0206	0.1215	0.0673

C.3.4 Spread=1.5 σ

Forecast results (spread=1.5 σ , θ =0.85)							
AbA	AbSD	EA	ESD	Fpo	FAP	ER	|ER|
3.6125	1.8300	3.8593	0.2203	25	29	0.0216	0.0216
3.4827	1.8415	3.6371	0.0873	34	36	0.0244	0.0244
3.5129	1.8464	3.6938	0.1828	26	25	-0.0945	0.0945
4.0199	2.0986	3.9447	0.1692	85	36	-0.0988	0.0988
3.3234	1.6397	3.3734	0.0581	28	27	-0.0494	0.0494
3.5068	1.8112	3.6129	0.2538	41	26	-0.1532	0.1532
3.6827	1.8069	3.8072	0.1382	30	34	0.1003	0.1003
3.5148	1.8820	3.7030	0.1852	28	32	0.0576	0.0576
3.6792	1.7019	3.7909	0.1323	29	33	0.1424	0.1424
3.2586	1.6683	3.4041	0.1958	28	26	-0.0868	0.0868
Average				35.4	30.4	-0.0136	0.0829
Standard deviation				18.0259	4.2999	0.0972	0.0449

C.4 Control results of R-fuzzy controller with forecast function

C.4.1 Spread=1.2 σ

Number	Normal	Abnormal	controlled	UCL	|N-A|	|N-C|	|N-C|*2/UCL
1	2.3925	2.7731	2.3650	5.0601	0.3806	0.0275	0.0109
2	2.2600	2.6107	2.0817	4.7799	0.3507	0.1783	0.0746
3	2.2877	2.6467	2.3198	4.8278	0.3590	0.0321	0.0133
4	2.3925	2.7731	2.2650	5.0601	0.3806	0.1275	0.0504
5	2.2322	2.6043	2.3698	4.7210	0.3721	0.1376	0.0583
6	2.3506	2.7076	2.1718	4.9715	0.3570	0.1788	0.0719
7	2.2652	2.6273	2.1190	4.7910	0.3621	0.1462	0.0610
8	2.4460	2.8354	2.3083	5.1732	0.3894	0.1377	0.0532
9	2.2095	2.5708	2.3187	4.6732	0.3613	0.1092	0.0467
10	2.1262	2.4648	2.1880	4.4970	0.3386	0.0618	0.0275
Average	2.2962	2.6614	2.2507	4.8555	0.3651	0.1137	0.0468
SD	0.0982	0.1113	0.1034	0.2078	0.0155	0.0554	0.0226

C.4.2 Spread=1.3 σ

Number	Normal	Abnormal	controlled	UCL	|N-A|	|N-C|	|N-C|*2/UCL
1	2.1262	2.6341	2.2934	4.4970	0.5079	0.1672	0.0744
2	2.3925	2.9634	2.3046	5.0601	0.5709	0.0879	0.0347
3	2.2600	2.7860	2.1110	4.7799	0.5260	0.1490	0.0623
4	2.2827	2.8288	2.2580	4.8278	0.5461	0.0247	0.0102
5	2.3259	2.8762	2.2805	4.9193	0.5503	0.0454	0.0185
6	2.3775	2.9326	2.5002	5.0285	0.5551	0.1227	0.0488
7	2.2322	2.7904	2.2014	4.7210	0.5582	0.0308	0.0130
8	2.3506	2.8860	2.1984	4.9715	0.5354	0.1522	0.0612
9	2.2652	2.8084	2.2379	4.7910	0.5432	0.0273	0.0114
10	2.4460	3.0300	2.3507	5.1732	0.5840	0.0953	0.0368
Average	2.3059	2.8536	2.2736	4.8769	0.5477	0.0903	0.0371
SD	0.0923	0.1107	0.1041	0.1952	0.0217	0.0559	0.0237

C.4.3 Spread=1.4 σ

Number	Normal	Abnormal	Controlled	UCL	|N-A|	|N-C|	|N-C|*2/UCL
1	2.1262	2.8033	2.3567	4.4970	0.6771	0.2305	0.1025
2	2.3925	3.1536	2.4186	5.0601	0.7611	0.0261	0.0103
3	2.2600	2.9613	2.2403	4.7799	0.7013	0.0197	0.0082
4	2.2827	3.0108	2.3231	4.8278	0.7281	0.0404	0.0167
5	2.3259	3.0597	2.4181	4.9193	0.7338	0.0922	0.0375
6	2.3775	3.1176	2.2291	5.0285	0.7401	0.1484	0.0590
7	2.4303	3.1964	2.4252	5.1402	0.7661	0.0051	0.0020
8	2.2322	2.9765	2.3529	4.7210	0.7443	0.1207	0.0511
9	2.3506	3.0645	2.2249	4.9715	0.7139	0.1257	0.0506
10	2.2652	2.9894	2.3858	4.7910	0.7242	0.1206	0.0503
Average	2.3043	3.0333	2.3375	4.8736	0.7290	0.0929	0.0388
SD	0.0898	0.1123	0.0801	0.1898	0.0269	0.0706	0.0307

C.4.4 Spread=1.5 σ

Number	Normal	Abnormal	Controlled	UCL	|N-A|	|N-C|	|N-C|*2/UCL
1	2.1262	2.9726	2.3510	4.4970	0.8464	0.2248	0.1000
2	2.3925	3.3439	2.5134	5.0601	0.9514	0.1209	0.0478
3	2.2600	3.1367	2.3696	4.7799	0.8767	0.1096	0.0459
4	2.2827	3.1929	2.2156	4.8278	0.9102	0.0671	0.0278
5	2.3259	3.2431	2.3413	4.9193	0.9172	0.0154	0.0063
6	2.3775	3.3026	2.3588	5.0285	0.9251	0.0187	0.0074
7	2.4303	3.3880	2.5330	5.1402	0.9577	0.1027	0.0400
8	2.2322	3.1626	2.4688	4.7210	0.9304	0.2366	0.1002
9	2.3506	3.2430	2.3515	4.9715	0.8924	0.0009	0.0004
10	2.2652	3.1705	2.3238	4.7910	0.9053	0.0586	0.0245
Average	2.3043	3.2156	2.3827	4.8736	0.9113	0.0955	0.0400
SD	0.0898	0.1183	0.0960	0.1898	0.0336	0.0823	0.0357

C.5 Control results of $\overline{X}$-fuzzy controller with forecast function

C.5.1 Shift=SD/3

Number	Normal	Abnormal	Controlled	U-L	|N-A|	|N-C|	(N-C)*2/(U-L)
1	-0.0232	-0.2545	-0.1102	2.4752	0.2313	0.0870	0.0703
2	0.0126	0.2559	-0.0398	2.6043	0.2433	0.0524	0.0402
3	0.0252	0.2761	0.0200	2.6862	0.2509	0.0052	0.0039
4	-0.0188	-0.2704	-0.0254	2.6930	0.2516	0.0066	0.0049
5	0.0340	0.2877	0.1073	2.7157	0.2537	0.0733	0.0540
6	0.0313	0.2926	0.1018	2.7965	0.2613	0.0705	0.0504
7	-0.0190	-0.2702	0.0124	2.6893	0.2512	0.0314	0.0234
8	-0.0005	-0.2941	-0.0286	2.6606	0.2936	0.0281	0.0211
9	0.0237	0.2740	0.0500	2.6794	0.2503	0.0263	0.0196
10	0.0412	0.2871	0.0696	2.6327	0.2459	0.0284	0.0216
Average	0.0107	0.0584	0.0157	2.6633	0.2533	0.0409	0.0310
SD	0.0243	0.2850	0.0687	0.0834	0.0162	0.0284	0.0219

C.5.2 Shift=SD/2

Number	Normal	Abnormal	Controlled	U-L	|N-A|	|N-C|	|N-C|*2/(U-L)
1	0.0801	0.4389	-0.0183	2.5600	0.3588	0.0984	0.0769
2	-0.0037	-0.3838	0.0030	2.7126	0.3801	0.0067	0.0049
3	-0.0309	-0.3950	0.0339	2.5985	0.3641	0.0648	0.0499
4	-0.0369	-0.4066	0.0316	2.6384	0.3697	0.0685	0.0519
5	0.0173	0.3841	-0.0554	2.6171	0.3668	0.0727	0.0556
6	0.0236	0.4032	0.0221	2.7089	0.3796	0.0015	0.0011
7	-0.0079	-0.3926	-0.0242	2.7452	0.3847	0.0163	0.0119
8	-0.0485	-0.4413	-0.1012	2.8031	0.3928	0.0527	0.0376
9	0.0801	0.4389	0.0184	2.5605	0.3588	0.0617	0.0482
10	-0.0037	-0.3838	0.0030	2.7126	0.3801	0.0067	0.0049
Average	0.0070	-0.0738	-0.0087	2.6657	0.3736	0.0450	0.0343
SD	0.0447	0.4224	0.0428	0.0825	0.0116	0.0342	0.0266

C.5.3 Shift=SD/1.5

Number	Normal	Abnormal	Controlled	U-L	|N-A|	|N-C|	|N-C|*2/(U-L)
1	0.0236	0.5297	-0.0039	2.7089	0.5061	0.0275	0.0203
2	-0.0485	-0.5723	0.0121	2.8031	0.5238	0.0606	0.0432
3	0.0801	0.5585	0.0266	2.5605	0.4784	0.0535	0.0418
4	-0.0309	-0.5164	-0.0088	2.5985	0.4855	0.0221	0.0170
5	0.0549	0.5819	0.1370	2.8207	0.5270	0.0821	0.0582
6	-0.0113	-0.4849	0.0188	2.5352	0.4736	0.0301	0.0237
7	0.0335	0.5489	0.0822	2.7586	0.5154	0.0487	0.0353
8	0.0002	0.5177	-0.0635	2.7697	0.5175	0.0637	0.0460
9	0.0355	0.5349	-0.0490	2.6733	0.4994	0.0845	0.0632
10	-0.0506	-0.5791	-0.1514	2.8286	0.5285	0.1008	0.0713
Average	0.0087	0.1119	1E-05	2.7057	0.5055	0.0574	0.0420
SD	0.0442	0.5604	0.0790	0.1092	0.0204	0.0264	0.0184

C.5.4 Shift=SD

Number	Normal	Abnormal	Controlled	U-L	|N-A|	|N-C|	|N-C|*2/(U-L)
1	-0.0309	-0.7592	0.0173	2.5985	0.7283	0.0482	0.0371
2	0.0549	0.8454	0.0223	2.8207	0.7905	0.0326	0.0231
3	0.0335	0.8066	0.0106	2.7586	0.7731	0.0229	0.0166
4	0.0002	0.7764	0.0123	2.7697	0.7762	0.0121	0.0087
5	0.0355	0.7847	-0.0643	2.6733	0.7492	0.0998	0.0747
6	-0.0683	-0.8435	-0.0568	2.7661	0.7752	0.0115	0.0083
7	-0.0509	-0.7781	0.0642	2.5948	0.7272	0.1151	0.0887
8	0.0625	0.7898	0.0781	2.5950	0.7273	0.0156	0.0120
9	0.0512	0.8134	0.0399	2.7197	0.7622	0.0113	0.0083
10	0.0437	0.7503	-0.0947	2.5211	0.7066	0.1384	0.1098
Average	0.0131	0.3186	0.0029	2.6818	0.7516	0.0508	0.0387
SD	0.0475	0.7682	0.0569	0.0997	0.0279	0.0485	0.0381

Appendix D: Published papers

This Appendix lists papers published during the course of this research.

Hefin Rowlands and Li Ren Wang, (2000), An Approach of Fuzzy Logic Evaluation and Control in SPC. *Quality and Reliability Engineering International,* **16,** pp.91-98.

Li Ren Wang and Hefin Rowlands, (2000), An Approach to a NN-Fuzzy-SPC Controller. *The 7th Mechatronics Forum International Conference,* USA, (CD-ROM).

Li Ren Wang and Hefin Rowlands, (1999), The Evaluation and Control of SPC in Fuzzy Logic and Neural Network. *Workshop on European Scientific and Industrial Collaboration (WESIC '99),* UK, pp. 391-398.

Li Ren Wang and Hefin Rowlands, (1999), A Fuzzy Logic Application in SPC Evaluation and Control. 1999 *7th IEEE International Conference on Emerging Technologies and Factory Automation (ETFA '99),* Spain, pp. 679-684.

AN APPROACH OF FUZZY LOGIC EVALUATION AND CONTROL IN SPC

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Summary

Quality Control plays an important part in most industrial systems. Its role in providing relevant and timely data to management for decision marking purposes is vital. A method that uses statistical techniques to monitor and control product quality is called Statistical Process Control (SPC) where control charts are test tools frequently used for monitoring the manufacturing process. Engineers or managers can evaluate abnormal process by using SPC Zone Rules in control charts.

In the conventional use of the zone rules, the user is only able to determine whether or not the process is out of control. What actions should be taken to adjust the process is uncertain and is evaluated based on knowledge of the system and past experiences.

This paper explores the integration of Fuzzy Logic and control charts, to create and design a Fuzzy - SPC Evaluation and Control (FSEC) method based on the application of fuzzy logic to the SPC Zone Rules. A simulation programme implementing FSEC was written in Borland C++ 5.0 and simulation results obtained and analysed. The abnormal processes simulated were automatically adjusted for each of the zone rules tested and showed an improved performance after the control action thus confirming the merit of the technique as a special method with the specific numerical control action based on a quality evaluation criteria.

Key words:
Statistical Process Control, Control Chart, Zone Rule, Fuzzy Logic, Simulation System.

1. Introduction to Statistical Process Control (SPC)

Statistical process Control (**SPC**) is a method that uses statistical techniques to measure, interpret and ultimately control product quality. Improving quality not only decreases cost but also produces more consistent products which will in turn lead to greater customer satisfaction. SPC is directed toward the identification and ultimate removal of the underlying causes of the problem. Hence the central focus of the SPC approach is that both quality and productivity will be enhanced [1,2].

1.1 Control Charts

In general, control charts consists of a Centre Line (CL), Lower Control Limit (LCL) and Upper Control Limit (UCL), and are based on normal distribution theory. The centre line represents an estimate of the process average and the control limits indicate the range of normal variability (i.e. $\pm 3\sigma$).

Points plotted on the chart represent average values of samples drawn from the process. In brief, a point outside the control limits indicates that some assignable cause is present and suggests the need for corrective action (point *a* in Fig.1); points falling at random between the control limits indicate no abnormal conditions and require no action.

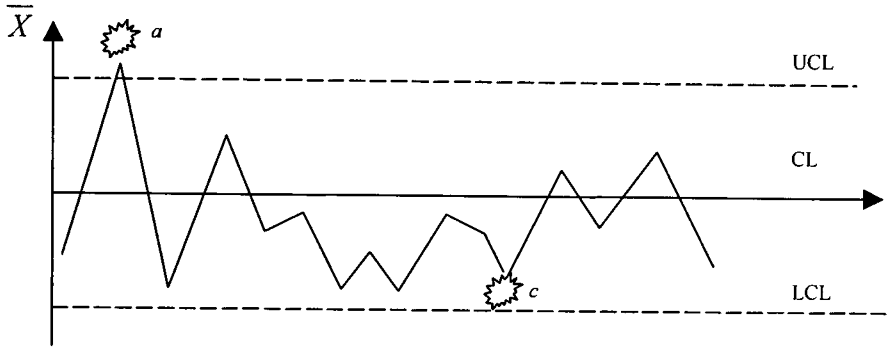

Fig.1 $\overline{X}$ control chart

1.2 SPC Process and Evaluation

The SPC process consists of observation, evaluation, diagnosis, decision and implementation [1]. In the evaluation stage, the control chart is a statistical model that shows how the data should behave if the process is stable and in control. In a control chart, successive data collected from a process under statistical control will exhibit variability in the quality characteristic of interest due to a constant set of common causes. The sample points will be shown as random points on the control chart. When a major disturbance affects the process, the ensuing measurement will be seen not to conform to this common model and the process mean and / or standard deviation will be shifted from the common cause model. Zone rules were developed as a method for process measurement and evaluation to identify abnormal disturbances in the process [1].

For example:

- Zone Rule1: The existence of a single point beyond a control limit (point *a* in Fig.1).
- Zone Rule2: The existence of two of any three successive points in zone A or beyond (point *a* and *b* inFig.2).
- Zone Rule3: The existence of four of any five successive points in zone B or beyond (point *c* and *d* in Fig.2).
- Zone Rule4: Six successive points increasing or decreasing continuously, signaling a systematic trend in the process.
- Zone Rule5: Eight or more successive points either strictly above or strictly below the centreline indicating that the process mean (in $\overline{X}$ chart) or variability (in R chart) has shifted from the centreline (point *c* in Fig.1).

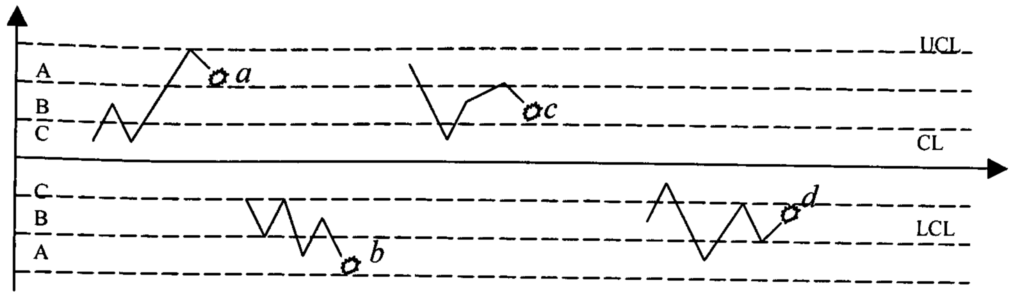

Fig.2 zone rules 2 and 3

Other zone rules have also been developed and used to identify abnormal disturbances in the process.

1.3 Current research in SPC

Research has been carried out to explore the use of fuzzy set theory to build control charts with linguistic data. An intelligent methods was proposed by Wang and TZVI[3,4] in which the control charts are constructed using linguistic data suitable for situations where quality characteristics can not be measured numerically. The centre line and control limits were transferred to the fuzzy subsets associated with the linguistic data. The linguistic approach was applied to p charts, and was verified using results obtained from simulated data. The results suggested that control chart based on linguistic data are significantly more sensitive to process shifts than are conventional p charts.

New control charts were developed by Kanagawa et al for linguistic variables based on using the concept of probability density functions (p.d.f.) existing behind the linguistic data in order to control the process variability and process average [5]. The p.d.f. was assumed to exist behind the linguistic data and represented by the Gram-Charlier series.

By combining traditional SPC and traditional automatic process control techniques, the minimum cost feedback control scheme was developed by Box [6] and Wiel [7]. A classic Proportional – Integral (PI) controller was discussed and the SPC method employed in analyzing the disturbances in the feedback control system.

This paper uses fuzzy subset theory but in a different way to previous work which concentrated on p – chart interpretation. The traditional control chart concepts are kept but fuzzy logic is used to interpret the zone rules. This forms the basis of a new Fuzzy – SPC Evaluation and Control system which results in a numerical control action which can be used as the output instruction in a high level controller in a process control or a supervisory control system. The fuzzy – SPC evaluation that results in an automatically generated control action is described in detail in this paper. The use of the numeric control action to adjust the process mean to obtain an improved quality performance of the system is illustrated. The development of the automatic controller will be discussed in future papers.

2. Fuzzy System and Design

2.1 A Brief Overview of Fuzzy Logic

Fuzzy Logic is a method of common sense or inference based on natural language [8]. It is based on the concept of a Fuzzy set which is a set without a crisp, clearly defined boundary. It describes vague concepts (e.g. fast runner, hot weather,). Fuzzy sets can

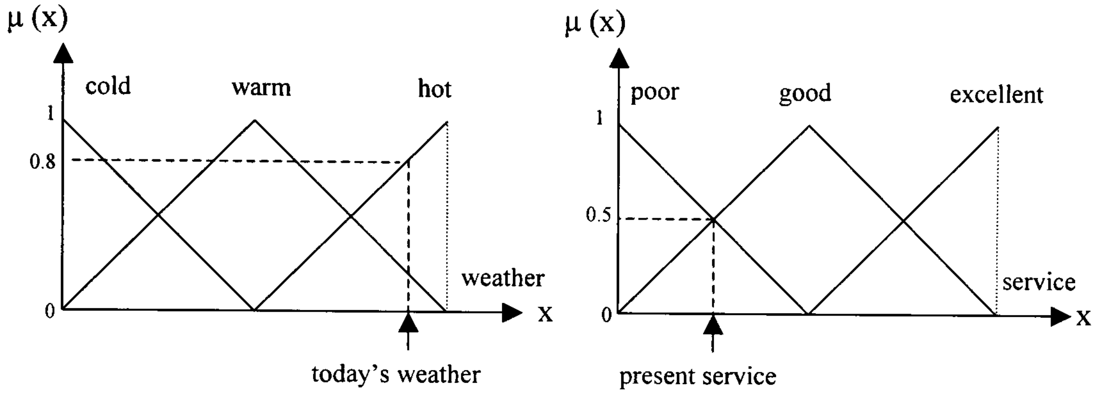

Fig.3 membership function

contain elements with only a partial degree of membership and admits the possibility of partial membership within it (e.g. the weather is rather hot, poor service) where this membership takes on a value between 0 and 1 (e.g. the weather is hot to the degree 0.8, the service is poor to the degree 0.5 . This is illustrated in Fig.3).

In more general terms, fuzzy logic operators are defined as Intersection (AND), Union (OR) and Complement (NOT) [9].

Fuzzy Reasoning is an application of fuzzy relationships [10] and forms the basis of fuzzy set analysis. It uses "if – then" rules to explain fuzzy reasoning and to build the fuzzy base. For example "If x is A then y is B" , where A and B are linguistic values defined by fuzzy sets on the ranges (universes of discourse) of X and Y respectively. The if-part of the rule " x is A" is called the antecedent or premise, while the then-part of the rule "y is B" is called the consequent or conclusion that is calculated by a fuzzy Compositional Operator [11].

Fuzzy Inference is the actual process of mapping from a given input to an output using fuzzy logic. This process involves membership functions, fuzzy logic operators and if-then rules.

2.2 Design and Calculation of the Fuzzy Base for SPC Evaluation and Control

The fuzzy logic method is employed in conjunction with SPC to develop a new method to represent the characteristic of a product, to analyse the behaviour and tendency of the process, to evaluate whether the process is out of control, and to output instructions to the operator, engineer or to an automatic control system where appropriate. This provides improvement on existing techniques of evaluating abnormal behaviours using zone rules by providing a numeric evaluation of the abnormal behaviours.

2.2.1 Input conversion and normalization

Data sampled from different processes can have different values and can be distributed in different ways. Generally, the data of a sample input signal should be converted to suitable linguistic terms which are defined by the fuzzy set [11] in order to simplify the calculations in fuzzy logic.

If the process data input is x' and $x'_{\min} \le x' \le x'_{\max}$, the universe is $[x_{\min}, x_{\max}]$, and data point x ($x_{\min} \le x \le x_{\max}$) is converted from x', where x is used in the Fuzzy Inference. The conversion equations used are[12]:

$$x = \frac{x_{\min} + x_{\max}}{2} + k(x' - \frac{x'_{\min} + x'_{\max}}{2}) \quad \text{(1)}$$

$$\text{where } k = \frac{x_{\max} - x_{\min}}{x'_{\max} - x'_{\min}} \quad \text{(2)}$$

2.2.2 Fuzzy Subset and Membership function

Fuzzy membership can take on a value between 0 and 1 and x takes in the interval [-14, +14] (this interval was chosen for convenience and ease of calculation) that is based on $\pm$ A, $\pm$ B, $\pm$ C and $\pm$ OUT as linguistic terms. The triangular membership function type is suitable to represent the random variables in SPC [12], which is used to represent the input and output. Fig.4 illustrates eight possible fuzzy subsets associated with the terms " -OUT, -A, -B, -C, C, B, A, OUT " corresponding to zones A, B, and C in SPC Zone Rules described previously in Figure 2. The eight fuzzy subsets cover the horizontal position in the universe of discourse, which ranges from −14 to +14. The membership functions associated with each linguistic term are defined for input variables from the process.

In Fig.4, normalized data points $X \in \{$ -14, -13,...-1,0,1, ..., 13, 14$\}$ are represented by the following discrete ranges:

X ={[-14.5, 13.5], [13.5, 12.5],...[-1.5, -0.5], [-0.5, 0.5], [0.5, 1.5], ... [12.5, 13.5], [13.5, 14.5]}

where linguistic term T(x)= { -OUT, -A, -B, -C, C, B, A, OUT }
For the output variables, the same types of membership functions as in Fig. 4 are defined:

$Z \in \{$ -14, -13,...-1,0,1, ..., 13, 14$\}$

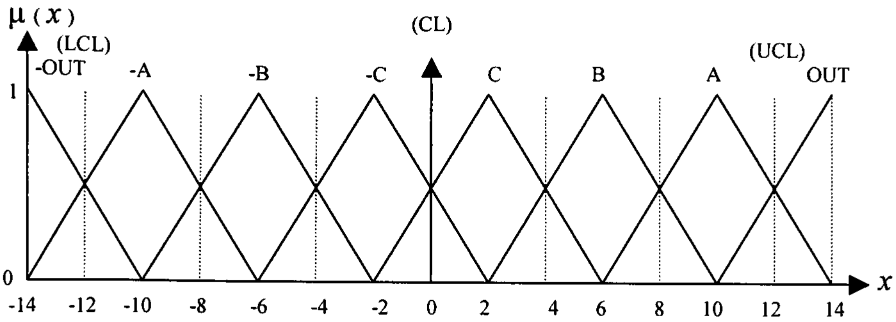

Fig.4 membership function for SPC Zone Rule

where linguistic term T(z) is given by:

T (z) = { NB, NM, NS, NZ, PZ, PS, PM, PB },
where
NB: Negative Big; NM: Negative Medium; NS: Negative Small; NZ: Negative Zero; PZ: Positive Zero; PS: Positive Small; PM: Positive Medium; PB: Positive Big.

The linguistic terms T (z) are the fuzzy subsets that are used to describe different output values from the fuzzy system.

2.2.3 If – Then Rules / Fuzzy Reasoning and Defuzzification

To illustrate the method of fuzzy reasoning, SPC Zone Rules 1 to 5 described in section 1 are represented by the Fuzzy Inference System as a Multiple Input / Single Output (MISO) system. The fuzzy reasoning / if-then rules are defined as:

(1) If point1 (x_1) is $\pm$ OUT then state (z) is PB/NB (zone rule 1);

(2) If point1 is $\pm$ A and point2 (x_2) is $\pm$ A then state is PM/NM (zone rule 2);

(3) If point1 is $\pm$ B (or $\pm$ A) and point2 is $\pm$ B and point3 (x_3) is $\pm$ B and point4 (x_4) is $\pm$ then state is PS/NS (zone rule 3);

(4) If point1 is A (or B or C), and point1> point2>…>point6 (x_6), then state is PM (or PS PZ) (zone rule 4);

(5) If point1 is -A (or -B or -C), and point1< point2<…<point6, then state is NM (or NS c NZ) (zone rule 4).

(6) If point1 is A (or B or C), and point2 >= CL and point3 >= CL and … and point8 (x_8) CL, then state is PM (or PS or PZ) (zone rule 5);

(7) If point1 is -A (or -B or -C), and point2 < CL and point3 < CL and … and point8< CL then state is NM (or NS or NZ) (zone rule 5);

The process program considers all possible combinations to fully describe the 5 zone rules. The linguistic terms (NB, NM,...) are chosen based on the characteristic of the zone rules.

The rule editor is used for editing the list of rules that define the behavior of the system. It can contain a large editable text field for displaying and editing the rules. Usually, the if - then rules can be designed by an experienced expert. In fact, SPC Zone Rules represent a summary of people's experiences from the manufacturing processes, but also have a statistical basis.

2.2.4 Defuzzification

Defuzzification is a calculation method used to convert the value that is described in the fuzzy set to a crisp output value. The crisp variable z_0 is given by:

$$z_0 = \frac{\sum_{i=1}^{k} z_i \, \mu_{S'}(z_i)}{\sum_{i=1}^{k} \mu_{S'}(z_i)} \qquad (3)$$

where

z_i --- fuzzy inference output variable value discussed;

and

$\mu_{S'}(z_i)$ --- membership function of fuzzy set S' of fuzzy inference output z_i.

For each zone rule a defuzzified control table is generated. The Control Table (Table1) gives the crisp variable $|z_0|$ which is the output from the fuzzy inference system. Table 1 illustrates the control table for zone rule 2. In table1, point1 and point2 which are described in section 2.2.3 are defined by $|x_1|$ and $|x_2|$ which are evaluated in the range [0, 14].

$\lvert x_2\rvert$ \ $\lvert x_1\rvert$	0 ~ 7	8	9	10	11	12	13 ~ 14
0 ~ 7	0	0	0	0	0	0	0
8	0	8.23	8.84	10.00	10.25	10.73	0
9	0	8.84	8.84	10.00	10.25	10.73	0
10	0	10.00	10.00	10.00	10.25	10.73	0
11	0	10.25	10.25	10.25	10.25	10.73	0
12	0	10.73	10.73	10.73	10.73	10.73	0
13 ~ 14	0	0	0	0	0	0	0

Table 1 Control table ($|z_0|$) for zone rule2

2.3 Programming in C++ to simulate the Fuzzy - SPC control process

There are some Statistical Process Control software packages without the functions of SPC Zone Rules on the market, and as such are not suitable for the approach adopted in this paper. Therefore to cater for all types of SPC software packages a preliminary simulation system was written in Borland C++.

This system can simulate a random process, generate different data in any running time with different values, uncontrolled $\overline{X}$ chart and zones can be drawn and abnormal points marked by sample number. The controlled pattern can be shown as the results of the process after applying the fuzzy inference system

The main functions of the program are:

- To create data to represent a normal and abnormal process;
- To calculate the distribution of the data;
- To create related data text files on disk for analysis;
- To calculate the average, standard deviation, UCL, LCL and boundaries between zones A, B and C;
- Inspect and interpret the process data by zone rule 1, 2, 3, 4 and 5;
- Produce an automatic control signal from the control table and transfer it to the SPC control chart;
- Plot $\overline{X}$ charts with and without controlled action for comparison.

An outline of the main functions of the program is shown in a process flow diagram in appendix 1.

A sample of nine $\overline{X}$ charts in Figs. 5~9 illustrate the executed outputs of the fuzzy system in which an abnormal process is simulated, tested and adjusted by the fuzzy inference system. In the upper charts of Fig. 5 ~ 9, abnormal points (i.e., points which contravene the zone rules) are marked by double rings, a vertical line and the number of the sample. The upper charts are uncontrolled and the lower charts are controlled and adjusted by control outputs of the fuzzy inference at the first abnormal point which are marked by the large vertical lines. It can be seen that to the right of the large vertical lines, subsequent abnormal points (marked with double rings) are improved to normal data in the lower charts.

In this simulation study, 500 random data points were generated by RAND () function for each run and the subgroup size chosen was 5 with the $\overline{X}$ chart plotted for 100 $\overline{X}$ values. The abnormal process is software generated and the system tests and adjusts it automatically. The results show that the fuzzy – SPC system successfully adjusts and improve the process for the simulated data.

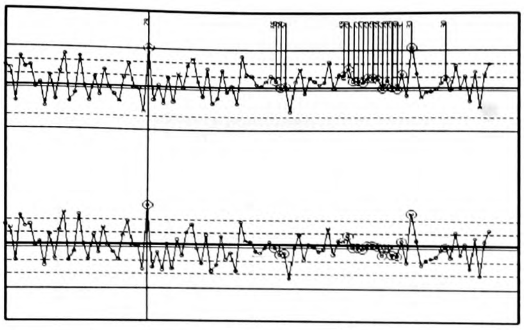

Fig.5 Control in zone rule 1

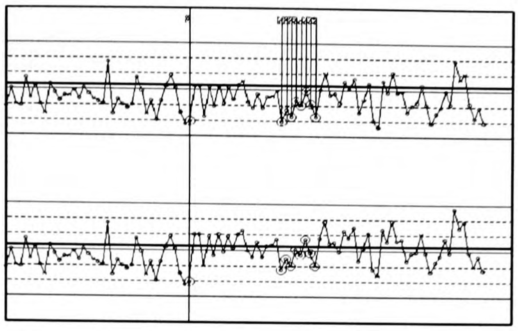

Fig.6 Control in zone rule2

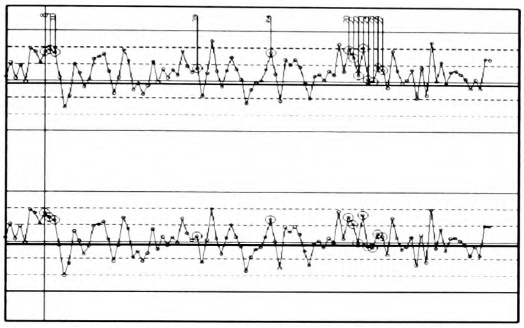

Fig.7 Control in zone rule 3

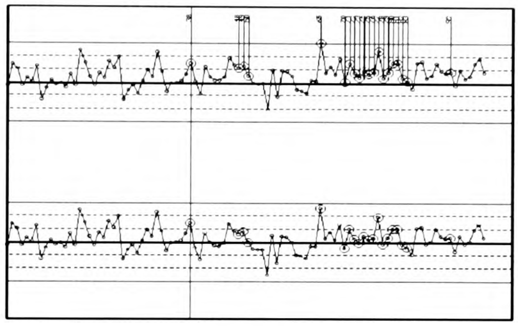

Fig.8 Control in zone rule 4

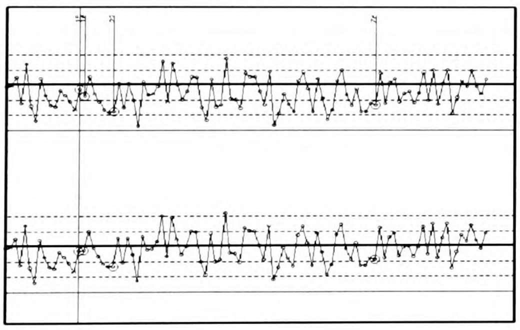

Fig.9 Control in zone rule 5

Abnormal data from Figure 5~ Figure 9 are summarized in table 2. The sample number for the first abnormal point (control point) and successive abnormal points are given in columns 2 and 3 and the zone rule number that is used for the appropriate test is indicated in column 4.

Figure	Number of First abnormal point	Numbers of Successive Abnormal points	Zone rules
5	28		1
5		83	1
5		55, 56, 57, 69~81	4
5		90	5
6	38		2
6		57~65	4
7	8		3, 4
7		9, 10,	3, 4
7		39, 70~77	4
7		54	5
8	15		4
8		16	4
8		22	5
8		76	3
9	38		5
9		49	3, 4
9		48, 50, 92, 70~83	4

Table 2 Summary of simulation results

3. Conclusion

In this paper, a method of Fuzzy - SPC Evaluation and Control (FSEC) has been described which combines traditional SPC methodology with an intelligent systems approach. Fuzzy logic is employed to evaluate SPC zone rules1, 2, 3, 4 and 5, which lead to the special membership functions and fuzzy if - then rules. FSEC is based on a MISO fuzzy system. Its five dimensional control table was generated as a result of the approach of fuzzy logic evaluation that is calculated according to SPC Zone Rules. The output of the fuzzy inference is based on the if – then rules, which is used by the FSEC simulation.

In this approach, abnormal processes were simulated in the software and were shown to be detected and adjusted successfully. This new approach is a unique application of the fuzzy logic technique for SPC control chart evaluation. However further work needs to be conducted on the choice of different membership functions (different slope, width and position of triangle) with comparisons to identify improved control stability and accuracy of the method. Further tests need to be conducted on real data and the number of tests will be extended for analysis and improved simulation.

This approach can provide the following benefits. Firstly, In the conventional use of the zone rules, the user is only able to identify whether or not the process is out of control.

What actions should be taken to adjust the process is uncertain and is evaluated based on knowledge of the system and past experiences. The fuzzy – SPC Evaluation and Control can be used in an SPC process to improve the accuracy and consistency of interpretation of the data, where the numeric output from the fuzzy system indicates what specific action should be taken if the process is out-of-control. Secondly, the Fuzzy Logic Evaluation can be employed in an automatic closed-loop SPC control process in the future. The simulation results have illustrated how this can be achieved. The fuzzy logic evaluation based on SPC control chart zone rules is a new method for a quality based high-level control system for supervisory control.

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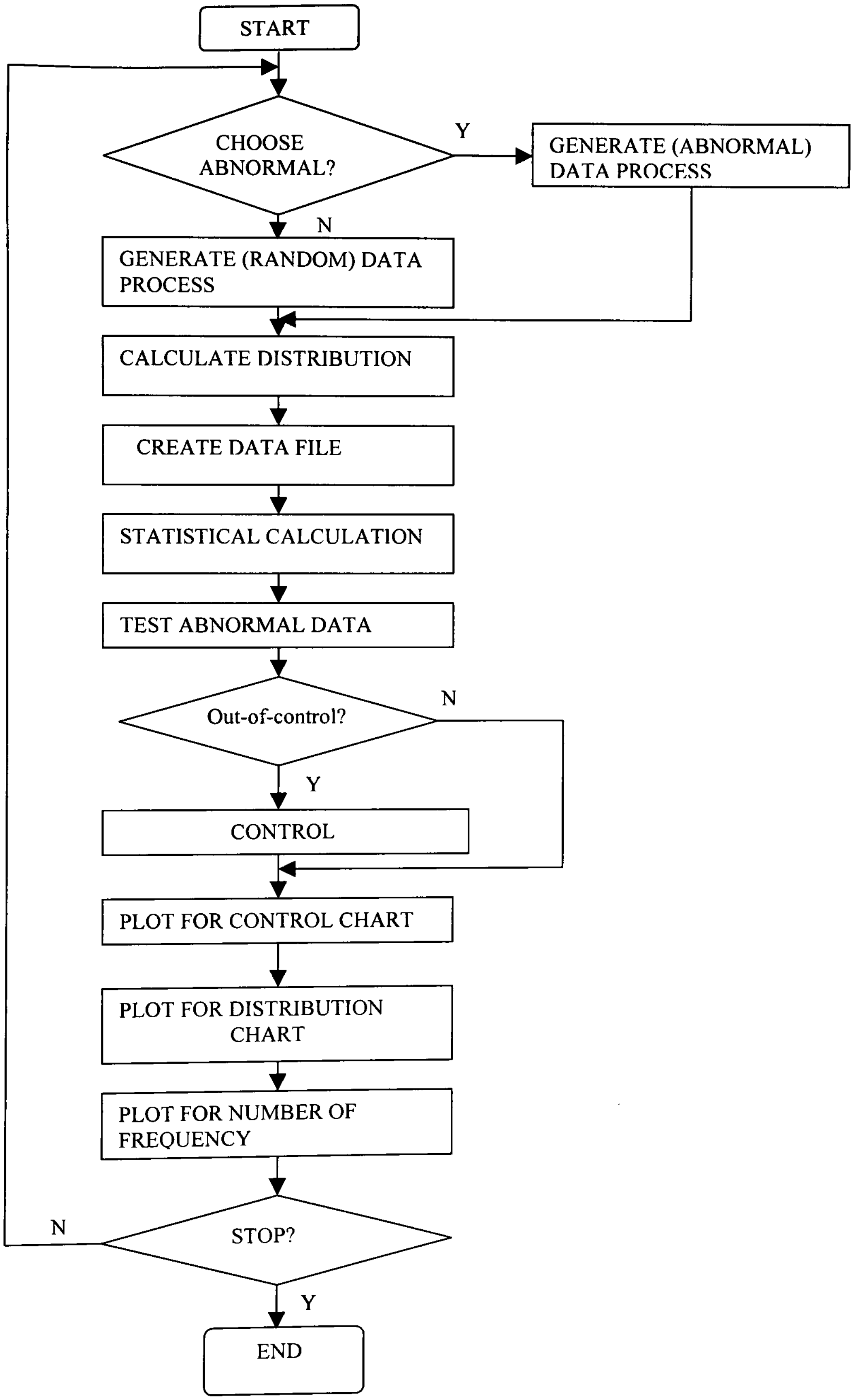
Appendix 1 --- Process Flowgraph
Main function:



AN APPROACH TO A NN – FUZZY – SPC CONTROLLER

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ABSTRACT

Statistical process Control (SPC) is a method that uses statistical techniques to measure and interpret product quality. In the conventional use of SPC, the user may just need to know whether or not the process is "out of control". What actions should be taken to adjust the process is uncertain. In this paper, fuzzy logic and neural network (NN) techniques are employed to develop a new NN - Fuzzy - SPC evaluation and control method, in order to generate specific numeric control actions to adjust the abnormal process. The neural network optimises the membership functions in the controller until the best control results are obtained.

1. INTRODUCTION TO CONTROL CHARTS AND ZONE RULES

One important tool in Statistical Process Control (SPC) is the Control Chart. Control charts for sample average and sample range are often referred to as $\overline{X}$ chart and R chart. The $\overline{X}$ chart is used to detect the shift in the process mean and the R chart is used to test the changes in the amount of process variability (Devor et al., 1992).

Generally a control chart consists of a Centre Line (CL), Lower Control Limit (LCL) and Upper Control Limit (UCL). The centreline represents the sample average and the control limits indicate the range of sample variability. When points fall on the chart at random positions between the control limits, common causes and no abnormal conditions are indicated and require no control action. When a point falls outside the control limits or a group of points are drawn as some regular patterns, this indicates that some assignable cause or special cause was present and suggests the need for corrective action.

Often, these regular or unnatural patterns contain extreme points, such as too many points near the control limits or points in a run above or below the centreline. They can often be identified by a cursory examination of the charts. Eight specific tests called zone rules have been developed for identifying the presence of unnatural patterns in the charts. The tests are performed by dividing the distance between the upper and lower control limits on the $\overline{X}$ chart into zones defined by $\pm\sigma, \pm 2\sigma$ and $\pm 3\sigma$ where $\pm\sigma$ is described as zone C, $\pm 2\sigma$ is described as zone B and $\pm 3\sigma$ is described as zone A. If the process mean level has shifted, it can be tested by the zone rules which are described below. (Only 5 of 8 zone rules are used in this paper). For example, the existence of a single point beyond a control limit (zone rule1), the existence of two out of any three successive points in zone A or beyond (zone rule2), the existence of four of any five successive points in zone B or beyond (zone rule3), six successive points increasing or decreasing continuously (zone rule4), eight or more successive points either strictly above or strictly below the centre line (zone rule5).

The zones are only used on the $\overline{X}$ chart making rule3 1~5 appropriate in this case. Zone rule 1, 4 and 5 can be used without zones, which are appropriate for the R chart (Devor et al., 1992).

In the conventional use of SPC, zone rules are used to identify out of control conditions where the user may just need to know whether or not the process is out of control. What actions should be taken to adjust the process is uncertain. This is a major drawback of control charts and as a consequence the user can not employ SPC techniques to automatically control the process.

A new Fuzzy-SPC evaluation and control method has been developed known as the fuzzy-SPC controller ($\overline{X}$ controller and R controller). Fuzzy logic is applied to $\overline{X}$ control charts for the $\overline{X}$ controller and R charts for the R controller to create a fuzzy inference system in order to obtain the specific numerical control actions. The fuzzy control output of the fuzzy inference system is based on the zone rules described above. Some drawbacks of this approach are highlighted and as a result an improved method where fuzzy membership functions are optimised by a neural network in order to obtain the best control result is implemented. Simulation examples using C++ and MATLAB are used to illustrate the Fuzzy – SPC controller and the NN - Fuzzy – SPC controller described above.

2. FUZZY LOGIC IN SPC

2.1 Fuzzy logic

Fuzzy Logic is a method of common sense or inference based on natural language (Gulley and Jang, 1995). Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set is a set without a crisp, clearly defined boundary and describes vague concepts (e.g. fast runner, hot weather). Fuzzy sets can contain elements with only a partial degree of membership and admits the possibility of partial membership within it where this membership takes on a value between 0 and 1.

In more general terms, fuzzy logic operators are defined as Intersection (AND), Union (OR) and Complement (NOT) (Ross, 1995). The If-Then Rules are also very important for a fuzzy system. They are called Fuzzy Reasoning which are applications of fuzzy relations (Toshiro, 1991). For example "If x is A then y is B", where A and B are linguistic values defined by fuzzy sets on the ranges (universes of discourse) of X and Y respectively. The if-part of the rule " x is A" is called the antecedent or premise, while the then-part of the rule "y is B" is called the consequent or conclusion that is calculated by a fuzzy Compositional Operator (Yan et al., 1994). Fuzzy Inference is the actual process of mapping from a given input to an output using fuzzy logic. This process involves membership functions, fuzzy logic operators and if-then rules.

Fuzzy logic can be applied widely in industry, e.g. process control, signal and information process, image processing, model identification and pattern recognition (Yen and Langari, 1998).

2.2 Fuzzy logic in the SPC

In this paper, fuzzy logic has been used for SPC control and adjustment. The controller has 2 input variables and 1 output variable which are designed to represent Zone Rules 1~5 as described in section 1 (Zone rules 6~8 will be implemented in the future). One input variable describes the zone rule and other describes the position of the current group of sample data.

For example, for the $\overline{X}$ controller, membership takes on a value between 0 and 1 and the input variable x takes in the interval [0,1] that is based on $\pm$ A, $\pm$ B, $\pm$ C and $\pm$ OUT as linguistic terms. The triangular membership function type is suitable to represent random variables (Sun, 1997), which is used to represent the antecedent and the consequent. The antecedent membership function has eight possible fuzzy subsets associated with the terms " –OUT, -A, -B, -C, C, B, A, OUT" which correspond to SPC Zone Rules. For the output variable y, similar type of membership functions are defined, where:

$y \in [0, 1]$ ....(1)

and the linguistic terms **T** (y) is given by:

T (y) = {NB,NM,NS,NZ,PZ,PS,PM,PB} .... (2)

where

NB: Negative Big; NM: Negative Medium; NS: Negative Small; NZ: Negative Zero; PZ: Positive Zero; PS: Positive Small; PM: Positive Medium; PB: Positive Big.

A simulation study was carried out in C++ to identify the effect of changing membership functions on the performance of the Fuzzy – SPC system. Seven different (slope, position and width) consequent membership functions were chosen and described in Fig.1 for the investigation. The simulation generates normal and abnormal process data. The abnormal data are tested by the Fuzzy - SPC controller which uses zone rules and fuzzy inference. The control action can be obtained from the controller to adjust or control the abnormal process.

The simulation was run many times to see the effect of the fuzzy control action on different zone rules. In every SPC zone rule for one adjustment or control action, the abnormal process was controlled successfully by the fuzzy - SPC controller for each of the seven membership functions.

The errors between normal process average and abnormal process average to be controlled were |0.0043|~ |0.0528| (Wang and Rowlands, 1999).

When the process average is shifted by a small amount (i.e. 0~1/3 standard deviation), for particular zone rule, the best control result appears in different membership functions described in Fig. 1.

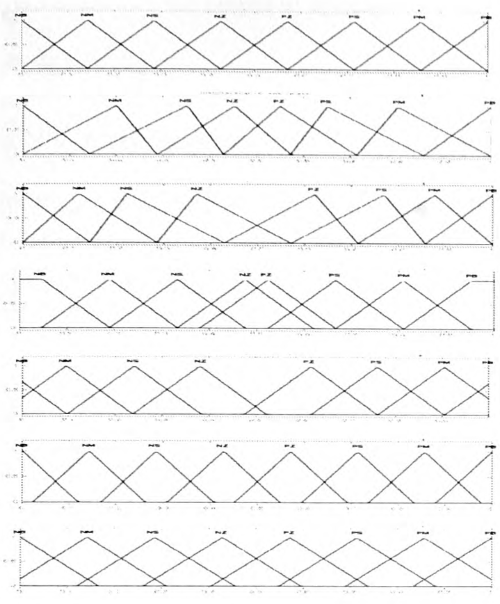

Fig. 1 Membership function used in the Fuzzy-SPC system

When the process average is shifted by a larger amount (i.e. larger than 1/3 standard deviation), the best control result can be not obtained after one control action. The fuzzy-SPC controller should control this abnormal process step by step until the control error is less than a predetermined limit.

In the next section, a control system is designed to optimise a dynamic membership function automatically for every zone rule in the random process, in order to overcome the above two weaknesses and to obtain the best controlled result after one implementation step (i.e. one control action).

3. NN-FUZZY MODEL

The Takagi-Sugeno (T-S) model was introduced in 1984 (Yen, 1998). The main motivation for developing this model is to reduce the number of if – then rules required, especially for complex and high-dimensional problems. This model replaces the consequent (then part) fuzzy sets with a linear equation of the input variables.

For i th rule, the T-S model have the form

IF x_1 is A_{i1} and ... and x_r is A_{ir},
THEN
$y = f_i(x_1, x_2, ..., x_r) = b_{i0} + b_{i1}x_1 + ... + b_{ir}x_r$ ......(3)

Where $x_j (j = 1,2,...,r)$ are input variables, $A_{ij} (i = 1,2,...,m)$ are fuzzy sets, m is number of rules, f_i is the linear model equation and $b_{ij} (j = 0,1,...,r)$ are real-valued parameters.

The Zero-Order T-S model is described in the neural network (NN-Fuzzy Model) in Figure 2, where **X** is the input vector, **A** is the fuzzy set and y is the system output.

In the NN - fuzzy model:
layer1 calculates the membership $\mu_A(x)$,

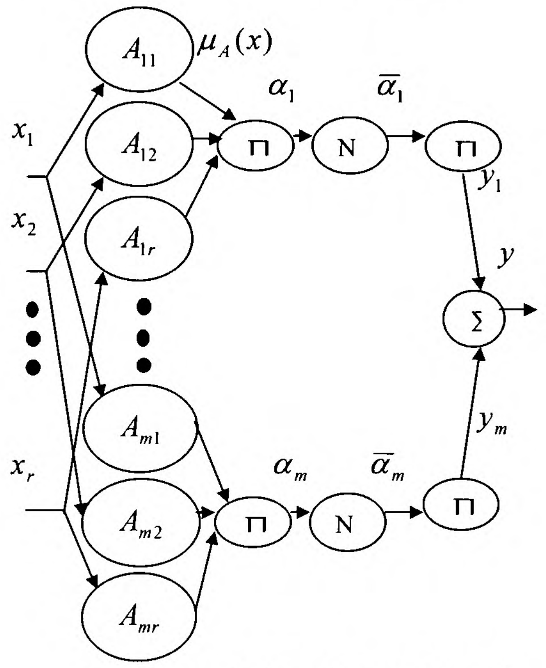

Fig.2 NN-fuzzy model

layer2 calculates the matching degree:

$\alpha_i = \prod_{j=1}^{r} \mu_{A_{ij}}(x_j) = \mu_{A_{i1}}(x_1) \wedge \mu_{A_{i2}}(x_2) .. \wedge \mu_{A_{ir}}(x_r)$ ....(4)

layer3 calculate the normalized matching degree:

$\bar{\alpha}_i = \dfrac{\alpha_i}{\sum_{i=1}^{m} \alpha_i}$ .... (5)

layer4 calculates the output of the i th rule:

$$y_i = \overline{\alpha}_i f_i \text{ .... (6)}$$

f_i is the linear equation mentioned before, while can be considered as a weight in the neural network. In this zero-order T-S model, singleton membership function r_i is defined for output:

$$f_i = r_i \text{ .... (7)}$$

so

$$y_i = \overline{\alpha}_i f_i = \overline{\alpha}_i r_i \text{ .... (8)}$$

layer5 calculates the output of system:

$$y = \sum_{i=1}^{m} y_i \text{ .... (9)}$$

The NN- fuzzy model in figure2 is a multiple feed forward network, and the Back Propagation algorithm is used to train the input / output relation via optimisation of the weight coefficients W_{ij} (can be membership functions in layer4). Normally the gradient descent technique is applied to calculate the minimal objective error function E (Sun, 1997):

$$\Delta W_{ij} = -\eta \frac{\partial E}{\partial W_{ij}} \text{ .... (10)}$$

and

$$E = \frac{1}{2}(y - y')^2 \text{ .... (11)}$$

where y is the fuzzy system output, y' is target output and η is learning rate.

4. NN-FUZZY-SPC SYSTEM AND ITS SIMULATION

Figure3 illustrates the structure of the NN-Fuzzy-SPC system. The inputs are process settings or targets (process average μ and standard deviation SD) and outputs are abnormal process average μ' and standard deviation SD' to be controlled. The disturbance includes the average shift and the change in standard deviation. Process signals can be tested, classified and transferred to the R - Fuzzy controller and the $\overline{X}$ - Fuzzy controller separately, in order to generate control actions to adjust the abnormal process. Every controller (R or $\overline{X}$) has two inputs: current sample value and average of previous samples. The controlled result is tested in standard deviation and average by the NN-Fuzzy model (1) and (2) (see fig. 3), If the errors are larger than predefined limits, the neural networks will optimise the consequent membership functions automatically in the R - Fuzzy controller and the $\overline{X}$-Fuzzy controller until the best control results are obtained.

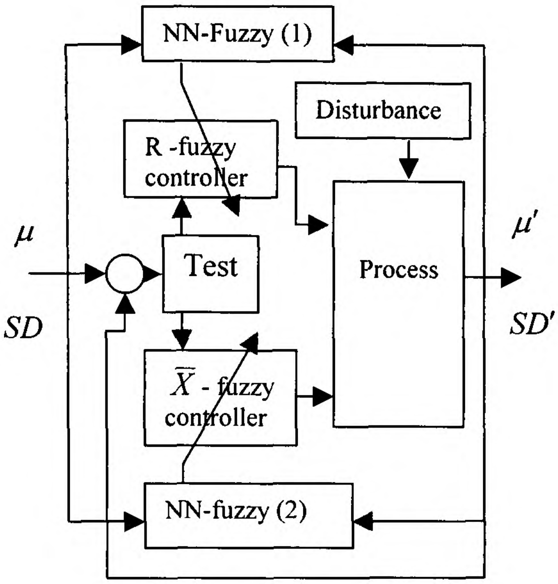

Fig.3 NN-Fuzzy-SPC System

Figure 4~6 illustrate the results of the control simulation. In figure 4, the upper two pictures describe a normal process in $\overline{X}$ and R charts, where the lower two pictures describe an abnormal process (process average shifted by one SD value and range spread 4 times). Abnormal data are marked by large circles.

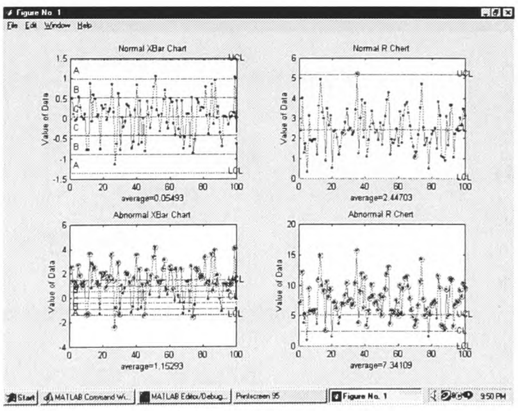

Fig. 4 Normal and abnormal processes

The abnormal process is initially controlled in a one step control action by an unoptimised controller and the process still shows abnormal data (figure 5). The control error is still large.

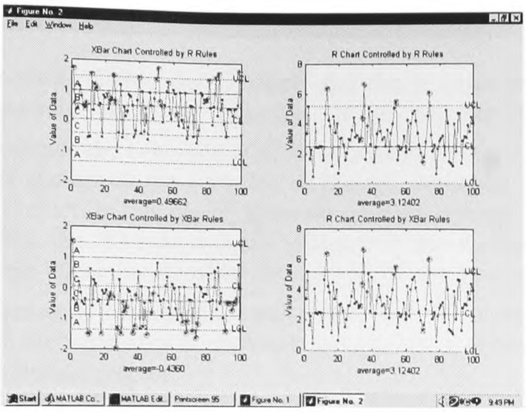

Fig. 5 A one step controlled by fuzzy controller

Figure 6 shows the control results in a one step control action using optimised R and $\overline{X}$ controller. In this NN – Fuzzy – SPC system, the triangle membership functions are used to describe the antecedent, and the consequent membership function are simplified to crisp singletons which correspond to the zero-order T-S model mentioned in section 3. The final training sum of squared error (SSE) was less than 0.01.

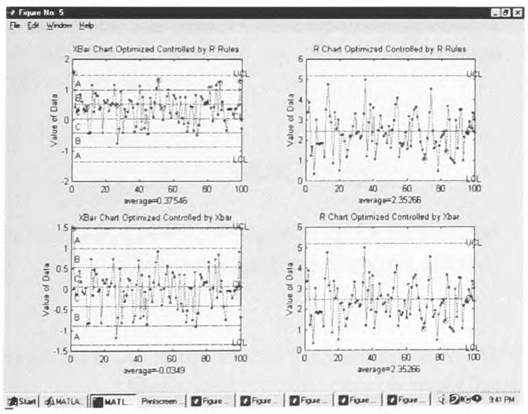

Fig.6 Controlled process by optimised fuzzy controllers

Table 1 summarises the control results, which are shown in figure 4 ~ figure 6. In table 1, $\overline{\overline{X}}$ (averages of $\overline{X}$ values) and $\overline{R}$ (average of R values) are much improved after a one step control action which is unoptimised. However, the best results are obtained after a one step optimised control action with the average results being close to the normal values.

Table 1 Summary of control results

	$\overline{\overline{X}}$	$\overline{R}$
Normal	0.0549	2.4470
Abnormal	1.1529	7.3411
After one control action	-0.4360	3.1240
After one optimised control action	-0.0349	2.3527

If the value of $(UCL - LCL)$ is viewed as full - scale deflection (f.s.d.) in $\overline{X}$ and R charts, the relative errors can be calculated as a percentage of f.s.d. (Bentley, 1995).

Therefore, for $\overline{X}$ and unoptimised general control,

$$e_1 = \left|\frac{-0.4360 - 0.0549}{UCL - LCL}\right| \times 100\%$$
$$= \left|\frac{-0.4360 - 0.0549}{1.4603 - (-1.3001)}\right| \times 100\% \qquad \text{............(12)}$$
$$= 17.78\%$$

Similarly, for $\overline{X}$ and optimised control,

$$e_2 = \left|\frac{-0.0349 - 0.0549}{1.4603 - (-1.3001)}\right| \times 100\% \qquad \text{............ (13)}$$
$$= 3.25\%$$

For R and unoptimised general control,

$$e_3 = \left|\frac{3.1240 - 2.4470}{UCL - LCL}\right| \times 100\%$$
$$= \left|\frac{3.1240 - 2.4470}{5.2110}\right| \times 100\% \qquad \text{...............(14)}$$
$$= 12.99\%$$

Similarly, for R and optimised control,

$$e_4 = \left|\frac{2.3527 - 2.4470}{5.2110}\right| \times 100\% \qquad \text{...............(15)}$$
$$= 1.81\%$$

The optimised control results are seen to be much improved from general unoptimised control results.

5. CONCLUSION

In control charts, the $\overline{X}$ chart and the R chart are commonly used in industry. It is suitable to investigate the behavior of process average in the $\overline{X}$ chart and check the variation of process deviation in the R chart. In this paper, when the process average is shifted, the R value was not changed in the R chart. On the other hand, when the spread of the process deviation is increased, it will affect the $\overline{X}$ value. Both are necessary to measure and control a normal distribution process.

A NN-Fuzzy model has been successfully applied though the use of simulated examples. The results were obtained with short epoch training times. Optimised consequent (then part) membership functions are used in the control of abnormal process to provide the ideal control actions. After a one step adjustment by the tuned R-fuzzy controller and $\overline{X}$-fuzzy controller, deviation spread and shifted average can be returned to a normal situation.

In the 20 experiments implemented, different shift levels ($\sigma/3$ ~ σ) and different spread levels (2σ~4σ) were made to the abnormal process. The NN-Fuzzy model provided a satisfactory optimisation procedure (error curve decay) and ideal consequent membership function. The error range for the abnormal process $\overline{X}$ average was 2.08%~5.78%, and for the R average of abnormal process was 1.58%~2.09%.

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THE EVALUATION AND CONTROL OF SPC IN FUZZY LOGIC AND NEURAL NETWORK

Abstract: *Control chart pattern recognition is an important method of statistical process control (SPC) for quality management. In the conventional use of the SPC, zone rules are used to identify out of control conditions where the user may just need to know whether or not the process is out of control. What actions should be taken to adjust the process is uncertain. This is a major drawback of control charts and as a consequence the user can not employ SPC techniques to automatically control the process. This paper provides a new Fuzzy / Neural Network – SPC evaluation and control method. Fuzzy logic is applied to control charts to create a fuzzy inference system. Different membership functions are chosen and their fuzzy control bases generated as outputs of the fuzzy inference system based on zone rules. They are executed in a simulation system that is written in Visual C++. A neural network is trained by process data and appropriate fuzzy membership function is generated from the neural network after completion of the learning period. As a result, the control quality can be optimised and improved.*

1 Introduction

Control charts are important tools in statistical process control (SPC), the idea is to monitor the process by periodically drawing small samples from a production process and estimating the process mean and process variability from the sample by the sample mean $\overline{X}$ and the range R of the sample.

To manage any process and reduce variation, the variation must be traced back to its source – common causes or special causes. Common causes refer to the many sources of chance variation that are always present in varying degrees in different processes. The output of a process which contains only common causes of variation from a pattern which is stable over time is predictable and therefore, provides the basis for subsequent process improvement. Special causes refer to any assignable factors, which are often irregular, unstable and unpredictable.

Control charts consist of a centreline, upper control limit and lower control limit. The centreline represents the process average and control limits indicate the range of process variability (DEVOR *et al*, 1992). In brief, when points fall on the chart at random positions between control limits, common causes and no abnormal conditions are indicated and require no control action. When a point falls outside the control limits or a group of points are drawn as some regular patterns, this indicates that some assignable cause or special cause was present and suggests the need for corrective action.

Often, these regular or unnatural patterns contain extreme points, such as too many points near the control limits or points in a run above or below the centreline. They can often be identified by a cursory examination of the charts. Eight specific tests called zone rules have been developed for identifying the presence of unnatural patterns in the charts. The tests are performed by dividing the distance between the upper and lower control limits on the $\overline{X}$ or R charts into zones defined by $\pm\sigma, \pm 2\sigma$ and $\pm 3\sigma$ where $\pm\sigma$ is described as zone C, $\pm 2\sigma$ is described as zone B and $\pm 3\sigma$ is described zone A. If the process mean level has shifted, it can be tested by the zone rules described below. For example, the existence of a single point beyond a control limit (zone rule1), the existence of two out of any three successive points in zone A or beyond (zone rule2), the existence of four of any five successive points in zone B or beyond (zone rule3), six successive points increase or decrease continuously (zone rule4), eight or more successive points either strictly above or strictly below (zone rule5).

This paper employs fuzzy subset theory and neural networks to improve SPC evaluation. The traditional $\overline{X}$ control chart is used, fuzzy logic is applied in the SPC zone rules and a neural network is used to choose the ideal membership function. A fuzzy/neural network – SPC controller is approached as a high level controller in a process control system, which can be used in a supervisor control or adaptive control systems. Section 2 discusses the design of the fuzzy inference system, section 3 describes the main functions of the simulation system, section 4 contain the neural network training and its executed results and finally some conclusions are made in section 5.

2 Fuzzy Logic and the Application of Fuzzy Logic Toolbox in MATLAB

2.1 Fuzzy Logic

Fuzzy Logic is a method of common sense or inference based on natural language (GULLEY and JANG, 1995). Fuzzy logic starts with the concept of a Fuzzy set. A Fuzzy set is a set without a crisp, clearly defined boundary. It describes vague concepts (e.g. fast runner, hot weather). Fuzzy sets can contain elements with only a partial degree of membership and admits the possibility of partial membership within it(e.g. the weather is rather hot, poor service) and this membership takes on a value between 0 and 1 (e.g. the weather is hot to the degree 0.8, the service is poor to the degree 0.5).

In more general terms, fuzzy logic operators are defined as Intersection (AND), Union (OR) and Complement (NOT) (ROSS, 1995).

The If-Then Rules are also very important for a fuzzy system. They are called Fuzzy Reasoning which are applications of fuzzy relations (TOSHIRO, 1991). For example "If x is A then y is B" , where A and B are linguistic values defined by fuzzy sets on the ranges (universes of discourse) of X and Y respectively. The if-part of the rule " x is A" is called the antecedent or premise, while the then-part of the rule "y is B" is called the consequent or conclusion that is calculated by a fuzzy Compositional Operator (YAN *et al*, 1994).

Fuzzy Inference is the actual process of mapping from a given input to an output using fuzzy logic. This process involves membership functions, fuzzy logic operators and if-then rules.

2.2 Fuzzy Inference System and Application of Fuzzy Logic Toolbox

The Fuzzy Logic Toolbox is a collection of functions built in the MATLAB numeric computing environment. It provides tools for the user to create and edit fuzzy inference systems within the framework of MATLAB. Its various functions can be used to implement several models and to compare and analyse their results (ALTROCK, 1995).

2.2.1 The Fuzzy Inference System (FIS) Editor

The FIS Editor displays general information about a fuzzy inference system. The system has 8 input variables and 1 output variable which are designed to represent Zone Rules 1~5 as described in section 1 (Zone rules 6~8 will be implemented in the future).

2.2.2 Membership Function (MF) Editor

The membership Function Editor is used to define the shapes of all the membership functions associated with each variable. Membership takes on a value between 0 and 1 and x takes in the interval [0,1] that is based on ± A, ± B, ± C and ± OUT as linguistic terms. The triangular membership function type is suitable to represent the random variables (SUN, 1997), which is used to represent the input and output. Fig.1 illustrates eight possible fuzzy subsets associated with the terms " –OUT, -A, -B, -C, C, B, A, OUT" which correspond to SPC Zone Rules.

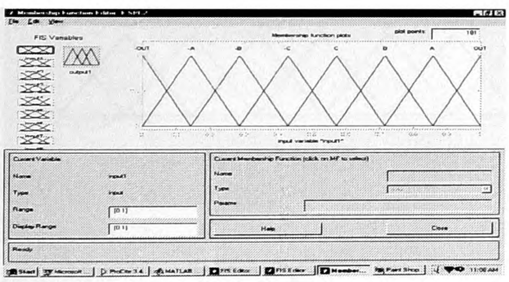

Fig.1 Input Membership Function

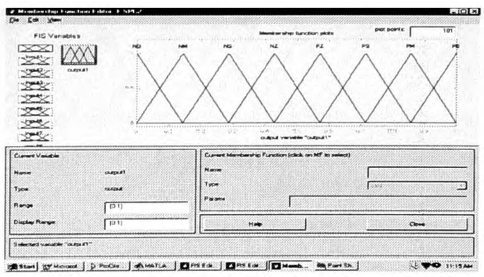

Fig.2 Output Membership Function

For the output variables, similar type of membership functions are defined (see Fig. 2), where:

$z \in [0, 1]$

and linguistic terms T(z) is given by:

T(z) = { NB, NM, NS, NZ, PZ, PS, PM, PB },

where

NB: Negative Big; NM: Negative Medium; NS: Negative Small; NZ: Negative Zero; PZ: Positive Zero; PS: Positive Small; PM: Positive Medium; PB: Positive Big.

2.2.3 Different membership functions

Figures 3 ~ 11 show nine different output membership function which are generated to determine the effect of the membership function on the simulation results. The slope, width and position of the triangles have been changed in each of these figures, in order to investigate the sensitivity of the FIS to change in the membership function. These patterns have symmetry according to SPC zone rule's characters and different slopes, widths and positions can cover a wide range of outputs from the FIS. They are used to optimise the membership functions and to training the neural network.

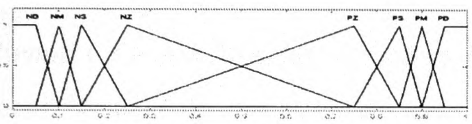

Fig.3 MF1

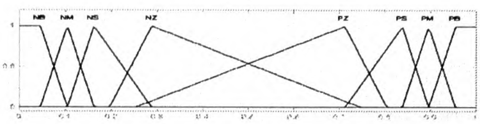

Fig.4 MF2

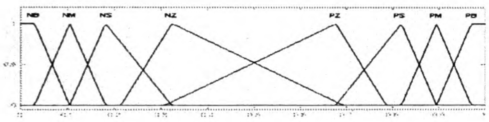

Fig.5 MF3

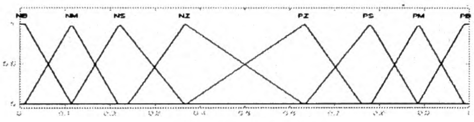

Fig.6 MF4

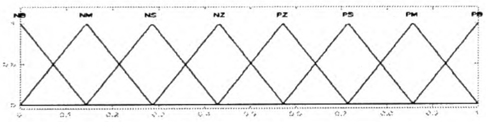

Fig.7 MF5

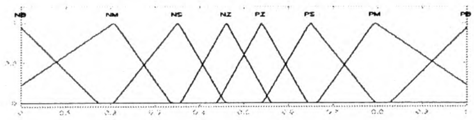

Fig.8 MF6

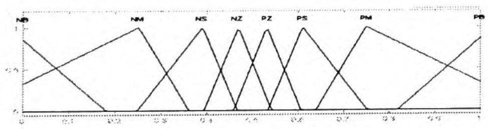

Fig.9 MF7

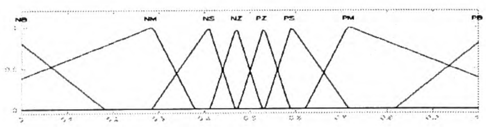

Fig.10 MF8

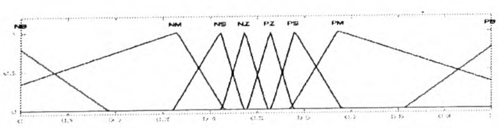

Fig.11 MF9

2.2.4 *Rule Editor*

The rule editor is for editing the list of rules that defines the behaviour of the system. It can contain a large editable text field for displaying and editing rules. Usually, the IF-THEN rules are designed by an experienced expert. In fact, SPC Zone Rules represent a summary of people's experiences from the manufacturing processes, but also have a statistical basis.

2.2.5 *Rule Viewer*

The Rule Viewer is a MATLAB-based display of the fuzzy inference program, and it is used as a diagnostic tool that can show which rules are active or how individual membership function shapes are influencing the results.

3 Simulation System Written in Visual C++ 6.0

3.1 *Visual C++ and Simulation system*

Microsoft Visual C++ is a powerful and complex tool for building 32 – bit applications for Window 95 and Window NT (GREGORY, 1998). Its Developer Studio is a completely self – contained development environment. The Microsoft Foundation Classes (MFC) are a set of predefined classes for windows programming. The Statistical Process Control simulation system was built in Visual C++ 6 Developer Studio using MFC. The simulation performs the follow operations:

- Creates data to represent normal or abnormal process;
- Calculates the distribution of the data;
- Creates related data text files on disk for analysis;
- Calculates the average, standard deviation, UCL, LCL and boundaries between zone A, B and C;
- Inspect data of the process by zone rules 1, 2, 3, 4 and 5;
- Search nine different fuzzy control bases and transfer the control instructions;
- Plot two $\overline{X}$ charts with normal and abnormal process, plot seven $\overline{X}$ charts of different control base for comparison.

3.2 *A View of the Results*

A sample of 11 $\overline{X}$ charts in (1) ~ (11) of Fig. 12 illustrate the executed outputs in which an abnormal process is simulated, tested and controlled by different output membership functions at the fuzzy zone rule5. In these figures, chart (1) represent a normal process and chart (2) is an abnormal process, abnormal points are marked by large circle, vertical line and number of zone rule used to detect the abnormality. Charts (3) ~(11) are controlled and adjusted at the first abnormal point (control point) of chart (2). The process is tested again after the first control action.

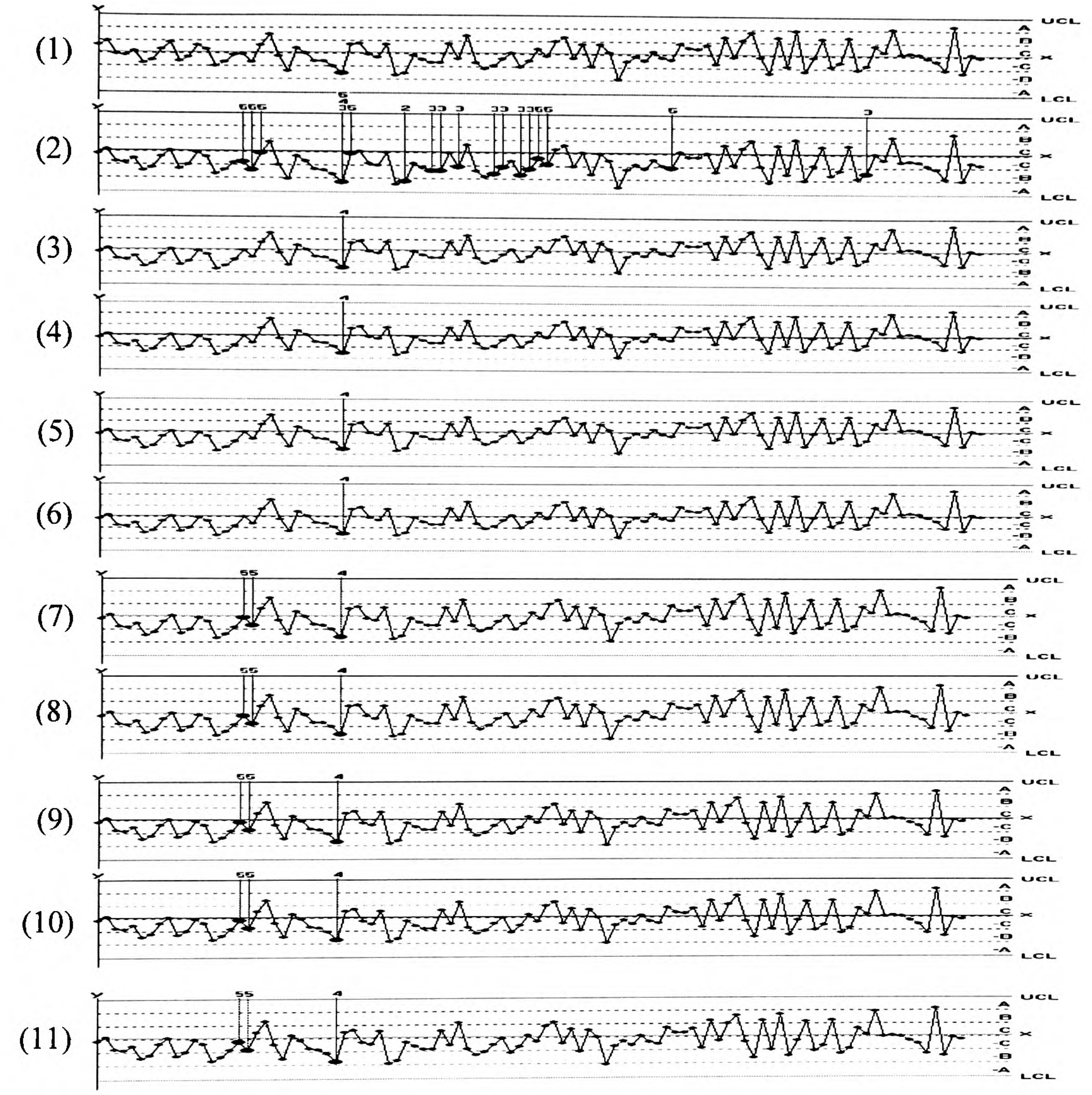

Fig.12 Display of Output of Simulation System (for Zone rule 5)

It can be seen that to the right of the control point, subsequent abnormal points are reduced and improved to normal data in charts (3) ~ (11). In this simulation study, 500 random data were generated by "RAND ()" function for each run and the subgroup size chosen was 5 with the $\overline{X}$ chart plotted for 100 $\overline{X}$ values. The abnormal process is software generated and the system will test and control it automatically. The simulation was repeated 500 times to build a large database for the training of a neural network.

4 Approach to the Neural Network Training

Artificial neural networks are computer software systems whose structure is designed in a similar way to an organic brain. They are used to model computational properties analogous to some that have been postulated for real networks of nerves, such as the ability to learn and store relationships. A neural network can efficiently approximate and interpolate multivariate data that might otherwise require huge databases; such techniques are now well accepted for nonlinear statistical fitting and prediction (OMIDVAR and ELLIOTT, 1997).

Neural networks consist of an input layer, output layer and hidden layers. They contain a set of processing nodes (neurons) interconnected in parallel. Each neuron consists of inputs, weights, a propagation function, an activation function and an output. The weights determine the influence of inputs on the neuron, the propagation function combines all inputs and make their weighted sum, the activation function computes the output of the neuron as line, step or sigmoid conversion.

The objective of a neural net is to process the information in a way that is previously trained. Training uses sample data sets of inputs and corresponding outputs. For this training, neural nets use the learning algorithms. They are used to modify the individual neurons of the net and the weight of their connections in such a way that the behaviour of the net reflects the desired one (ALTROCK, 1995).

In this study, the NeuralDesk software which is a type of neural network software is applied. To build this net, the input layer that receives the data from the control chart to be identified has 60 neurons. Each neuron represents the value of a point on the control chart. The hidden layer that extracts features from the input data comprises 20 neurons and the output layer which processes extracted features to obtain the fuzzy membership function class has 4 neurons (O1 to O4, Fig.13). For zone rule 5, the 9 control actions of MF1 to MF9 can be divided to 4 classes and the control action is derived as a result of the average from the membership functions in each class: O1 represents MF1 to MF4 (see Fig.3 to Fig.6), O2 and O3 represent MF5 (Fig.7), MF6 (Fig.8) and MF7 (Fig.9), O4 represents MF8 (Fig.10) and MF9 (Fig.11). 300 pattern and 18,000 data points have been used for the training of the neural network, with the error threshold determined as 0.05.

Fig.13 illustrates the training conditions, and the training algorithm is chosen as Stochastic Back Propagation. After 711 "Epochs", the training error value is less than 0.05. Six abnormal patterns were tested by (queried to) the trained network. The satisfied "Query Output" can be obtained (the greatest value reflects the validity of one of the "Query Outputs"). In Fig.13, O2 (on No.0 row), O3 (on No.1 row), O2 (on No.2 row), O2 (on No.3 row), O3 (on No.4 row) and O2 (on No.5 row) are greatest values. These results correspond with control actions derived from a simulation study.

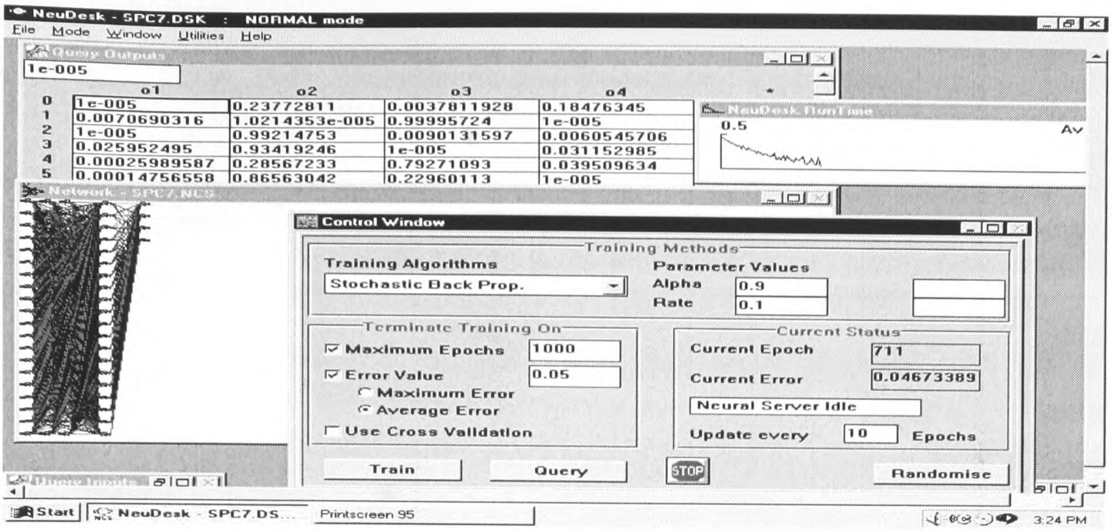

Fig. 13 Query output from neural network

5 Conclusion

In this paper, fuzzy logic is applied to represent SPC zone rule1, 2, 3, 4 and 5. Nine membership functions and fuzzy if - then rules are applied using a simulation of an abnormal process. This fuzzy calculation was completed by the "Fuzzy Logic Toolbox" in MATLAB. The output is a fuzzy inference with numerical values which can be used as a fuzzy base for the application of SPC control.

The abnormal process is software generated, tested and controlled automatically. Fig.12 illustrates this Fuzzy – SPC controlling processes which shows the ability of the fuzzy system to control the simulation process. The process were controlled by different membership functions MF1 ~ MF9 at 5 zone rules. The process averages were much improved and have different effects for each of membership functions. After training of the neural network, the optimal control actions can be gained from the network.

This approach can provide several benefits. First, it can be used in an SPC process to improve the accuracy of interpretation of the data and consistency, where the numeric output from the fuzzy system indicates what action should be taken if the process is out-of-control. It is also a useful technique to improve the management and implementation of the control process. Secondly, the Fuzzy Logic Evaluation can be employed in an automatic closed-loop SPC control process. The membership function can be changed on line in this system as the adaptive function. This feature will be explored in future work. Also a larger database will be used for training of the neural network, in order to gain more accurate results.

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A FUZZY LOGIC APPLICATION IN SPC EVALUATION AND CONTROL

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Abstract - *Control chart pattern identification is an important aspect of statistical process control (SPC). In the conventional use of SPC, the user may just need to know whether or not the process is out – of control. What actions should be taken to adjust the process is uncertain. This is a major drawback of control charts and as a consequence the user can not employ SPC technique to automatically control the process.*

This paper provides a new Fuzzy – SPC evaluation and control method. Fuzzy logic is applied to control charts to create a fuzzy inference system using the Fuzzy Logic Toolbox in MATLAB. Different membership functions were chosen and their fuzzy control bases were generated as outputs of a fuzzy inference system based on zone rules. They are executed in a simulation system written in Visual C++. The results of adjusted and controlled SPC charts are displayed on the screen. Analysis of the effect of changing the membership function is presented.

1. SHEWHART CONTROL CHART PATTERNS

1.1 A view of Control Chart Patterns

Control charts are important tools in statistical process control (SPC), the idea is to monitor the process by periodically drawing small samples from a production process and estimating the process mean and process variability from the sample by the sample mean $\overline{X}$ and the range R of the sample.

To manage any process and reduce variation, the variation must be traced back to its source – common causes or special causes. Common causes refer to the many sources of chance variation that are always present in varying degrees in different processes. The output of a process that contains only common causes of variation form a pattern that is stable over time is predictable and therefore, provides the basis for subsequent process improvement. Special causes refer to any assignable factors, which are often irregular, unstable and unpredictable.

Control charts consist of a centreline, upper control limit and lower control limit. The centreline represents the process average and control limits indicate the range of process variability [1]. In brief, when points fall on the chart at random positions between control limits, common causes and no abnormal conditions are indicated and require no control action. When a point falls outside the control limits or a group of points are drawn as some regular patterns, this indicates that some assignable cause or special cause was present and suggests the need for corrective action.

Often, these regular or unnatural patterns contain extreme points, such as too many points near the control limits or points in a run above or below the centreline. They can often be identified by a cursory examination of the charts. Eight specific tests called zone rules have been developed for identifying the presence of unnatural patterns in the charts. The tests are performed by dividing the distance between the upper and lower control limits on the $\overline{X}$ or R charts into zones defined by $\pm\sigma, \pm 2\sigma$ and $\pm 3\sigma$.

The eight rules are:
Zone Rule1: The existence of a single point beyond a control limit signals (point *a* in Fig.1).

Zone Rule2: The existence of two out of any three successive points in zone A or beyond (point *b* inFig.1).
Zone Rule3: The existence of four of any five successive points in zone B or beyond (point *c* in Fig.1).
Zone Rule4: When six successive points increase or decrease continuously, a systematic trend in the process is signalled.
Zone Rule5: Eight or more successive points either strictly above or strictly below the Centreline indicates that the process mean (in $\overline{X}$ chart) or variability (in R chart) has shifted from the centreline (point *d in* Fig.1).
Zone Rule6: When 14 successive points oscillate up and down on the chart, a systematic cyclic trend in the process is signalled.
Zone Rule7: When eight successive points occurring on either side of the centreline avoiding in zone C, an out-of control condition is signalled.
Zone Rule8: When 15 successive points in zone C only, to either side of the centreline, an out- of-control condition is signalled[1].

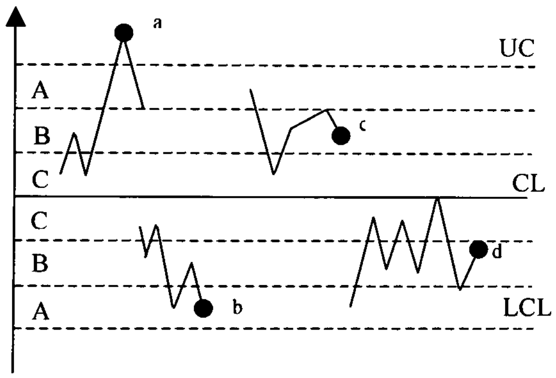

Figure 1 Zone Rule 1, 2, 3 and 5

1.2 Overview of related work

Numerous papers introduce the use of fuzzy set theory to build control charts using linguistic data. An intelligent methods was proposed by [2] and [3]. The control charts are constructed using linguistic data suitable for situations where quality characteristics can not be measured numerically. The centreline and control limits were transferred to the fuzzy subsets associated with the linguistic data. The linguistic approach was applied to *p*-charts and was verified after using results obtained from simulated data. The results suggested that control chart based on linguistic data are significantly more sensitive to process shifts than are conventional p charts.

New control charts were developed for linguistic variables based on using the concept of probability density functions (p.d.f.) existing behind the linguistic data in order to control the process variability and process average [4]. The p.d.f. was assumed to exist behind the linguistic data and represented by the Gram-Charlier series. Control charts for both process average and process variability were developed.

By combining traditional SPC and traditional automatic process control techniques, the minimum cost feedback control scheme was developed by [5] and [6] . A classic Proportional – Integral (PI) controller was discussed and the SPC method employed in analysing the disturbances in the feedback control system.

This paper employs fuzzy subset theory and feedback control to improve SPC evaluation. The traditional $\overline{X}$ control chart is used and fuzzy logic is applied in SPC zone rules. A fuzzy – SPC controller is developed as a high level controller in a process control system, which can be used in a supervisor control or adaptive control systems. Section 2 discusses the design of the fuzzy inference system, section 3 describes the main functions of the simulation system and its executed results and finally some conclusions are made in section 4.

2. FUZZY LOGIC AND APPLICATION

2.1 Fuzzy Logic

Fuzzy Logic is a method of common sense or inference based on natural language [7]. Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set is a set without a crisp, clearly defined boundary. It describes vague concepts (e.g. fast runner, hot weather,). Fuzzy sets can contain elements with only a partial degree of membership and admits the possibility of partial membership within it (e.g. the weather is rather hot, poor service) and this membership takes on a value between 0 and 1 (e.g. the weather is hot to the degree 0.8, the service is poor to the degree 0.5).

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which are applications of fuzzy relations [9]. For example "If x is A then y is B" , where A and B are linguistic values defined by fuzzy sets on the ranges (universes of discourse) of X and Y respectively. The if-part of the rule " x is A" is called the antecedent or premise, while the then-part of the rule "y is B" is called the consequent or conclusion that is calculated by a fuzzy Compositional Operator [10].

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The FIS Editor displays general information about a fuzzy inference system. The system has 8 input variables and 1 output variable which are designed to represent Zone Rules 1~5 (Zone rule 6~8 will be implemented in the future).

Membership Function Editor
The membership Function Editor is used to define the shapes of all the membership functions associated with each variable. Membership takes on a value between 0 and 1 and x takes in the interval [0,1] that is based on $\pm$ A, $\pm$ B, $\pm$ C and $\pm$ OUT as linguistic terms. The triangular membership function type is suitable to represent the random variables [11], which is used to represent the input and output. Fig.2 illustrates eight possible fuzzy subsets associated with the terms " –OUT, -A, -B, -C, C, B, A, OUT" which correspond to SPC Zone Rules.

For the output variables, similar type of membership functions (see Fig. 3) are defined:

$z \in [0, 1]$

Linguistic terms T(z):
T(z) = { NB, NM, NS, NZ, PZ, PS, PM, PB },

while NB: Negative Big; NM: Negative Medium; NS: Negative Small; NZ: Negative Zero; PZ: Positive Zero; PS: Positive Small; PM: Positive Medium; PB: Positive Big.

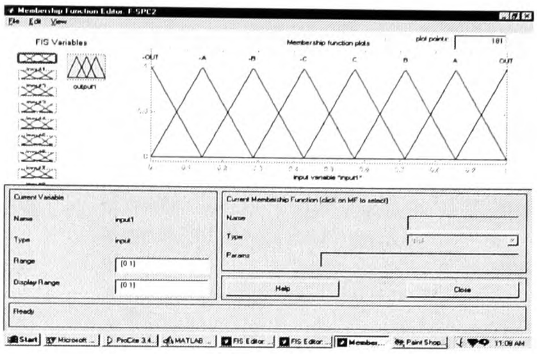


Figure 2 Input Membership Function

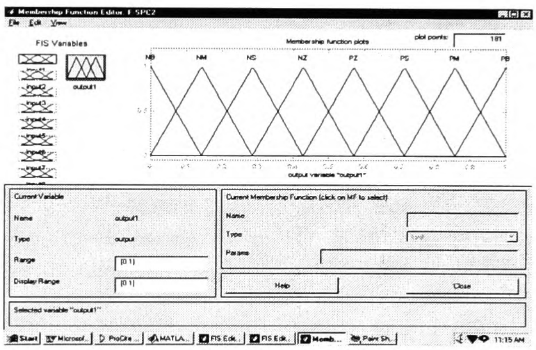


Figure 3 Output Membership Function

Different membership functions

Figure 4 ~10 illustrates seven different output membership function which are generated to determine the effect of the membership function on the simulation results. Fig.4 is a standard triangular membership function, it's slope, width and position can be changed to build different type of triangular membership functions. Fig.5 and Fig6 illustrate the different directions of the slope, Fig.7 and Fig,8 represent different positions of the functions, and Fig.9 and Fig.10 illustrate different width of the triangular functions.

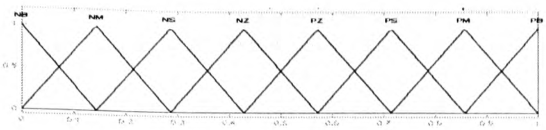


Figure 4 MF1

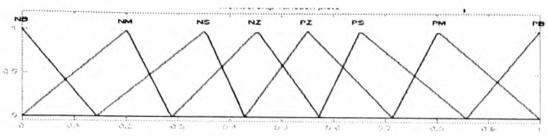


Figure 5 MF2

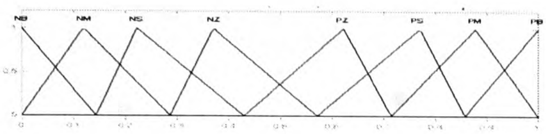


Figure 6 MF3

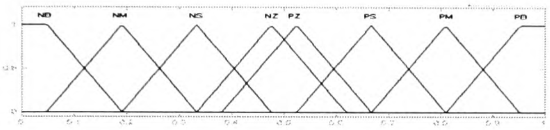


Figure 7 MF4

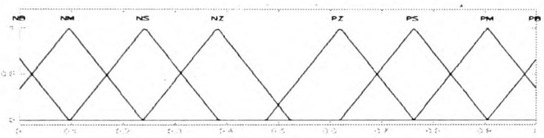


Figure 8 MF5

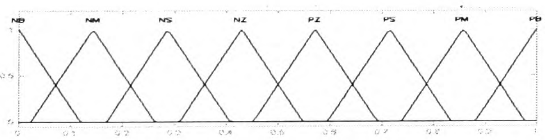


Figure 9 MF6

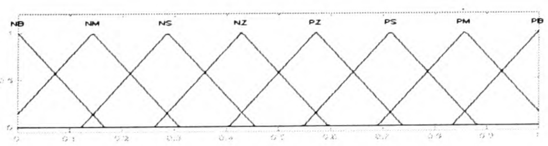


Figure 10 MF7

Rule Editor

The rule editor is for editing the list of rules that defines the behaviour of the system. It can contain a large editable text field for displaying and editing rules. Usually, the IF-THEN rules are designed by an experienced expert. In fact, SPC zone rules represent a summary of people's experiences from the manufacturing processes, but also have a statistical basis.

SPC zone rules 1 to 5, described in section 1 are represented by the Fuzzy Inference System as a Multiple Input / Single Output (MISO) controller. The fuzzy reasoning / if-then rules are defined as:

(1) If point1 is $\pm$OUT then state is PB/NB (zone rule 1);

(2) If point1 is $\pm$A and point2 is $\pm$A then state is PM/NM (zone rule 2);

(3) If point1 is $\pm$A and point3 is $\pm$A then state is PM/NM (zone rule 2);

(4) If point1 is $\pm$B (or $\pm$A) and point2 is $\pm$B and point3 is $\pm$B and point4 is $\pm$B then state is PS/NS (zone rule 3);

(5) If point1 is $\pm$B (or $\pm$A) and point3 is $\pm$B and point4 is $\pm$B and point5 is $\pm$B then state is PS/NS (zone rule 3);

(6) If point1 is $\pm$B (or $\pm$A) and point2 is $\pm$B and point4 is $\pm$B and point5 is $\pm$B then state is PS/NS (zone rule 3);

(7) If point1 is $\pm$B (or $\pm$A) and point2 is $\pm$B and point3 is $\pm$B and point5 is $\pm$B then state is PS/NS (zone rule 3);

(8) If point1 is A (or B), and point1> point2>...>point6, then state is PM (or PS) (zone rule 4);

(9) If point1 is -A (or -B), and point1< point2<...<point6, then state is NM (or NS) (zone rule 4);

(10) If point1 is A (or B or C), and point2 >= CL and point3 >= CL and ... and point8 >= CL, then state is PM (or PS) (zone rule 5);

(11) If point1 is -A (or –B or -C), and point2 < CL and point3 < CL and ... and point8< CL, then state is NM (or NS or NZ) (zone rule 5).

The linguistic terms (NB, NM,...) are chosen based on the characteristic of the zone rules. The process program considers all possible combinations to fully describe the 5 zone rules.

Rule Viewer

The Rule Viewer is a MATLAB-based display of the fuzzy inference program, it is used as a diagnostic tool and can show which rules are active or how individual membership function shapes are influencing the results.

3. SIMULATION SYSTEM WRITTEN IN VISUAL C++ 6.0

3.1 Visual C++ and Simulation system

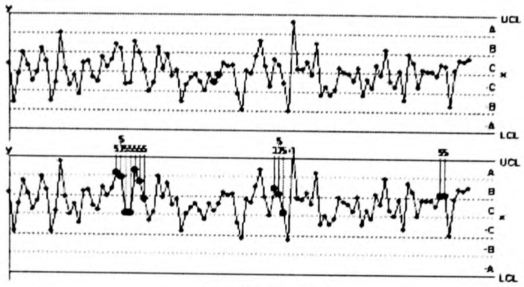
Microsoft Visual C++ is a powerful and complex tool for building 32 – bit applications for Window 95 and Window NT [12]. Its Developer Studio is a completely self – contained development environment. The Microsoft Foundation Classes (MFC) are a set of predefined classes for windows programming. The Statistical Process Control simulation system was built in Visual C++ 6 Developer Studio using MFC. The simulation performs the follow operations:

- To create data to represent normal or abnormal process;
- Calculate the distribution of the data;
- Create related data text files on disk for analysis;
- Calculate the average, standard deviation, UCL, LCL and boundaries between zone A, B and C;
- Inspect data of process by zone rule 1, 2, 3, 4 and 5;
- Search seven different fuzzy control Base and transfer the control instructions;
- Plot two $\overline{X}$ charts with normal and abnormal process, plot seven $\overline{X}$ charts of different control base for comparison.

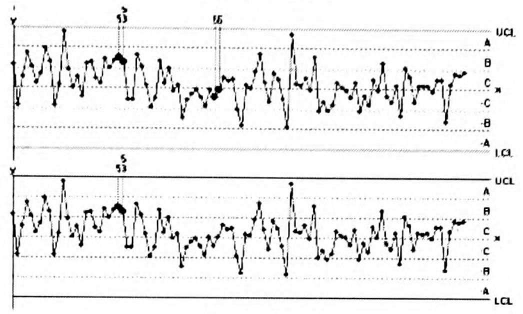
3.2 Results

A sample of nine $\overline{X}$ charts in Fig.11 illustrate the executed outputs in which an abnormal process is simulated, tested and controlled by different output membership functions at the fuzzy zone rule5. In these figures, chart 1 represent a normal process and chart 2 is an abnormal process, abnormal points are marked by large circle, vertical line and number of zone rule used to detect the abnormality. Charts 3 ~ 9 are controlled and adjusted at the first abnormal point (control point) of chart 2. The process is tested again after the first control action. It can be seen that to the right of the control point, subsequent abnormal points are reduced and improved to normal data in charts 3 ~ 9.

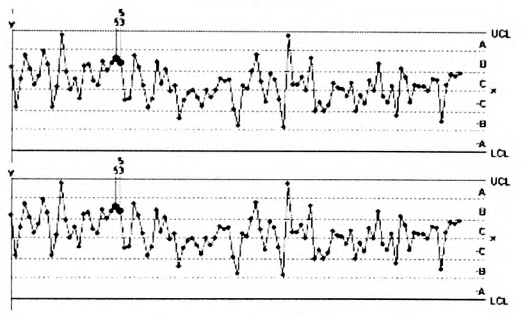
In this simulation study, 500 random data were generated by RAND () function for each run and the subgroup size chosen was 5 with the $\overline{X}$ chart plotted for 100 $\overline{X}$ values. The abnormal process is software generated and the system will test and control it automatically.



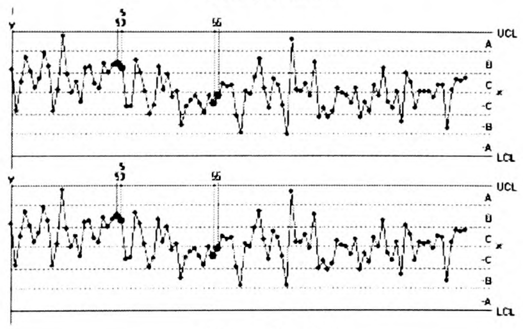
Charts 1 and 2



Charts 3 and 4



Charts 5 and 6



Charts 7 and 8

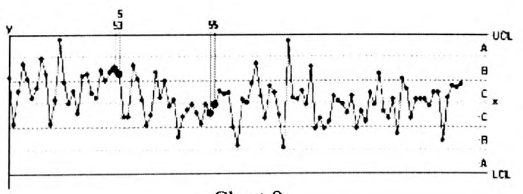


Chart 9

Figure 11 Display of Output of Simulation System (for zone rule 5)

The simulation was repeated 5 times to see the effect of zone rule 1~5 on the fuzzy control action.

Table1 summarises the process average (Aver) for the 7 membership functions. The errors (Err) are calculated from the difference between expected (exp.) values and average values. The expected values are averages of the normal processes. MF1 ~ MF7 are the seven types of membership functions (see Fig.4 ~ Fig.10).

Table 1

	Zone Rule1	Zone Rule2	Zone Rule3	Zone Rule4	Zone Rule5
	Aver (Err)	Aver (Err)	Aver (Err)	Aver (Err)	Aver (Err)
MF1	0.5073 (.0209)	0.4888 (-.0065)	0.4582 (-.0411)	0.4905 (-.0194)	0.5088 (.0076)
MF2	0.5073 (.0209)	0.4841 (-.0112)	0.4532 (-.0461)	0.4905 (-.0194)	0.5133 (.0121)
MF3	0.5073 (.0209)	0.4908 (-.0045)	0.4632 (-.0361)	0.4946 (-.0153)	0.5200 (.0188)
MF4	0.5023 (.0159)	0.4773 (-.0180)	0.4465 (-.0528)	0.4989 (-.0110)	0.5208 (.0196)
MF5	0.5112 (.0248)	0.4996 (.0043)	0.4687 (-.0306)	0.4864 (-.0235)	0.4980 (-.0032)
MF6	0.5093 (.0229)	0.4891 (-.0062)	0.4582 (-.0411)	0.4884 (-.0215)	0.5115 (.0103)
MF7	0.5053 (.0189)	0.4886 (-.0067)	0.4550 (-.0443)	0.4939 (-.0160)	0.5090 (.0078)
Exp. Value	0.4864	0.4953	0.4993	0.5099	0.5012

4. CONCLUSION

In this paper, fuzzy logic is applied to represent SPC zone rule1, 2, 3, 4 and 5. Seven membership functions and fuzzy if - then rules are applied using a simulation of an abnormal process. This fuzzy calculation was completed by the Fuzzy Logic Toolbox software in MATLAB. The output is a fuzzy inference with numerical values and it can be used as a fuzzy base for the application of SPC control.

The abnormal process is software generated, tested and controlled automatically. Fig.11 illustrates this Fuzzy – SPC controlling processes which shows the ability of the fuzzy system to control the simulation process. The process averages summarised in table1, were controlled by different membership functions MF1 ~ MF7 at each of the 5 zone rules. The process averages were much improved in all cases except when using zone rule 3. A hypothesis sign test was carried out to establish if changing the membership function led a significant effect on the process average. Each membership function was compared with the standard triangular function. The outcome was that MF3 was significantly different from MF1 with 95% confidence. This indicates that MF3 is a good membership function to use in an adaptive control system.

This approach can provide several benefits. First, it can be used in an SPC process to improve the accuracy of interpretation of the data and consistency, where the numeric output from the fuzzy system indicates what action should be taken if the process is out-of-control. It is also a useful technique to improve the management and implementation of the control process. Secondly, the Fuzzy Logic Evaluation can be employed in an automatic closed-loop SPC control process. The membership function and / or if – then rules can be changed on line in this system as the adoptive function in future work. Third, in the automatic control area, it is very important to change the parameters of a process controller when the running environment is removed, fuzzy logic evaluation based on SPC technology will be a new type of method as a high-level controller for adaptive control systems.

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